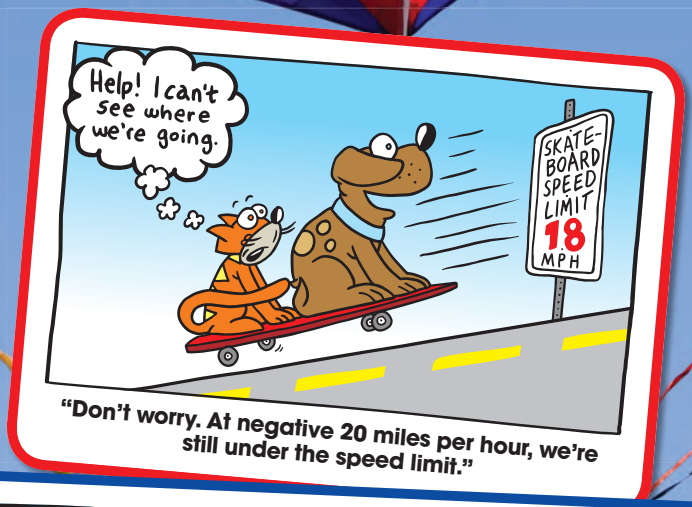
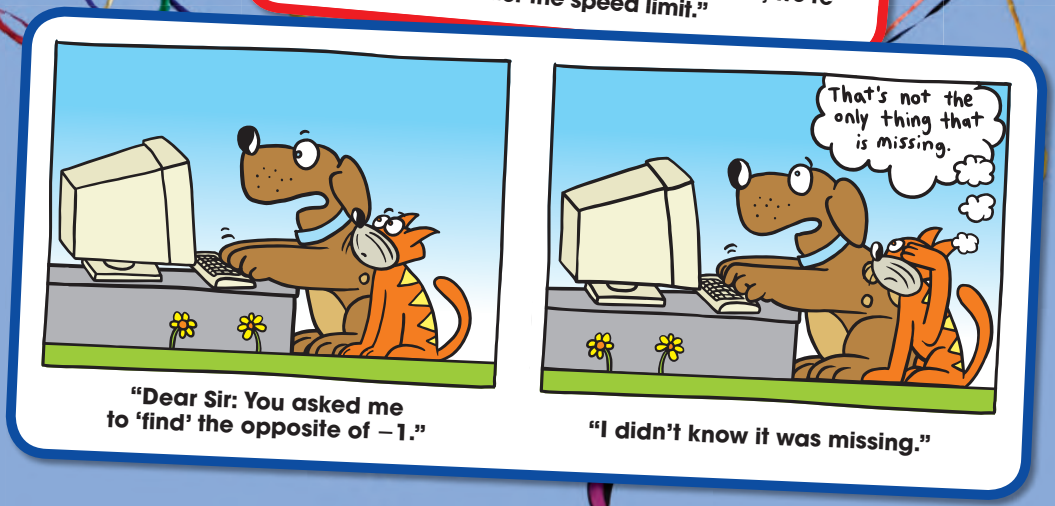
6 Integers and the Coordinate Plane





Common Core Progression

4th Grade

- Compare and order fractions.
- Compare and order decimals to hundredths place.

5th Grade

- Compare and order decimals to thousandths place.
- Generate numerical patterns, identify the relationship, and form ordered pairs.
- Graph ordered pairs in the first quadrant of the coordinate plane.

6th Grade

- Describe quantities with positive and negative numbers.
- Compare and order integers and absolute value numbers.
- Graph ordered pairs in all four quadrants of the coordinate plane.

Pacing Guide for Chapter 6

Chapter Opener	1 Day
Section 1	
Activity	1 Day
Lesson	1 Day
Section 2	
Activity	1 Day
Lesson	1 Day
Section 3	
Activity	1 Day
Lesson	1 Day
Study Help / Quiz	1 Day
Section 4	
Activity	1 Day
Lesson	1 Day
Section 5	
Activity	1 Day
Lesson	1 Day
Extension	1 Day
Chapter Review/ Chapter Tests	2 Days
Total Chapter 6	15 Days
Year-to-Date	97 Days

Chapter Summary

Section	Common Core State Standard	
6.1	Learning	6.NS.5, 6.NS.6a, 6.NS.6c
6.2	Learning	6.NS.6c, 6.NS.7a, 6.NS.7b
6.3	Learning	6.NS.5 ★, 6.NS.6a, 6.NS.6c, 6.NS.7a, 6.NS.7b
6.4	Learning	6.NS.7c, 6.NS.7d ★
6.5	Learning	6.NS.6b, 6.NS.6c ★, 6.NS.8 ★

Technology for the Teacher

BigIdeasMath.com
Chapter at a Glance
Complete Materials List
Parent Letters: English and Spanish

Common Core State Standards

4.NF.7 Compare two decimals to hundredths by reasoning about their size. . .. Record the results . . . and justify the conclusions . . . by using a visual model.

5.NBT.3b Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Additional Topics for Review

- Place Value
- Graphing Ordered Pairs in the First Quadrant

Try It Yourself

1. 0.01, 0.2, 0.42, 0.5
2. 0.75, 0.95, 1.01, 1.05
3. >
4. =
5. <
6. *Sample answer:* 8.5, 9, $\frac{19}{2}$
7. *Sample answer:* -1, 0, 1
8. *Sample answer:* 0.705, 0.749, 0.75

Record and Practice Journal Fair Game Review

1. 0.2, 0.4, 0.54, 0.61
2. 0.02, 0.11, 0.3, 0.45
3. 1.02, 1.24, 1.33, 1.7
4. 1.01, 1.06, 1.2, 1.42
5. 0.87, 0.9, 0.98, 1.23
6. 0.06, 0.5, 1.23, 1.4
7. 0.003, 0.03, 0.031, 0.033
8. 0.002, 0.02, 0.022, 0.2
9. group that brought fruit to lunch
10. <
11. >
12. <
13. =
14. >
15. <

16–22. See Additional Answers.

Math Background Notes

Vocabulary Review

- Common Denominator
- Mixed Number
- Improper Fraction

Ordering Decimals

- Students should know how to order decimals.

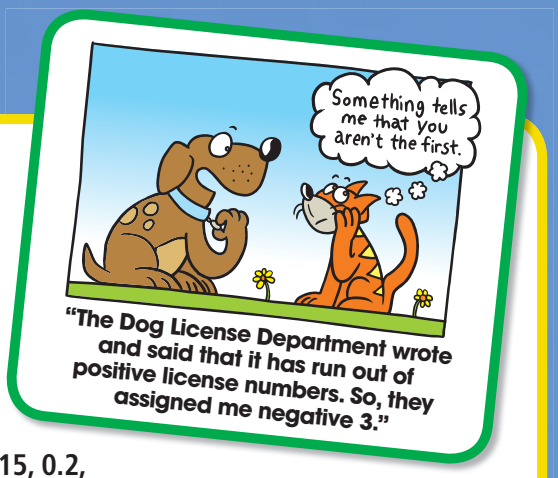
Comparing Numbers

- Students should be familiar with comparing numbers.
- Remind students that it is easier to compare numbers if they are both in the same form. In Example 3, for instance, one number is a decimal and one is a fraction. Encourage students to convert one of the numbers so that either both are decimals or both are fractions.
- **Teaching Tip:** Many students benefit from imagery when using the greater than and less than signs to compare numbers. Remind them that the greater than and less than symbols act as hungry alligators. They always want to eat the bigger number. Encourage students to be sure that the mouth of the alligator always opens towards the greater number.

Reteaching and Enrichment Strategies

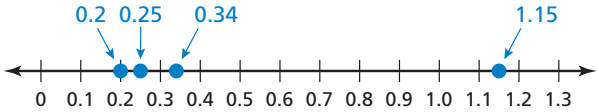
If students need help. . .	If students got it. . .
Record and Practice Journal • Fair Game Review Skills Review Handbook Lesson Tutorials	Game Closet at *BigIdeasMath.com* Start the next section

What You Learned Before



Ordering Decimals

Example 1 Use a number line to order 0.25, 1.15, 0.2, and 0.34 from least to greatest.



Try It Yourself

Use a number line to order the numbers from least to greatest.

1. 0.01, 0.42, 0.2, 0.5

2. 1.05, 0.95, 0.75, 1.01

Comparing Numbers

Complete the number sentence with <, >, or =.

Example 2 $10 \square 15$

On a number line, 10 is closer to zero than 15.

∴ So, $10 < 15$.

Example 3 $0.875 \square \frac{7}{8}$

$$0.875 = \frac{875}{1000} = \frac{875 \div 125}{1000 \div 125} = \frac{7}{8}$$

∴ So, $0.875 = \frac{7}{8}$.

Example 4 **Find three numbers that make the number sentence $1\frac{2}{5} \leq \square$ true.**

∴ *Sample answer:* $1\frac{3}{5}$, $\frac{5}{2}$, 2

Try It Yourself

Complete the number sentence with <, >, or =.

3. $2.01 \square 2.001$

4. $4.5 \square \frac{9}{2}$

5. $3.18 \square 3.2$

Find three numbers that make the number sentence true.

6. $\frac{17}{2} \leq \square$

7. $1\frac{1}{2} > \square$

8. $0.75 \geq \square$

6.1 Integers

Essential Question How can you represent numbers that are less than 0?

1 ACTIVITY: Reading Thermometers

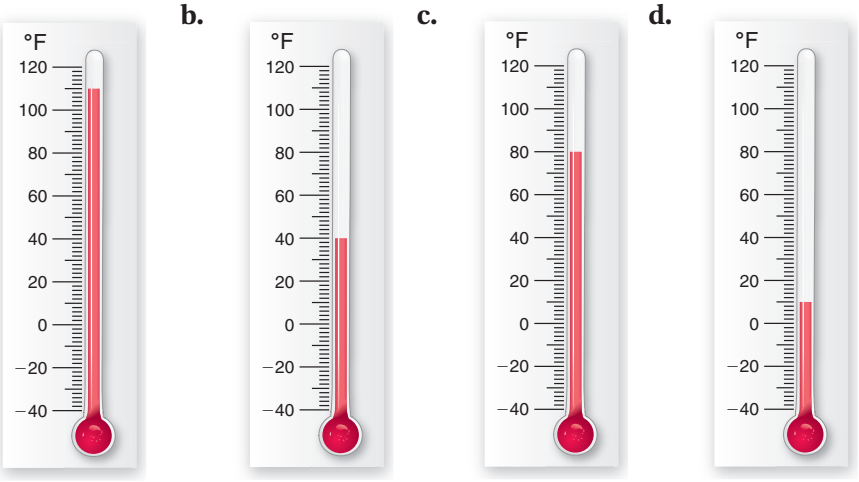
Work with a partner. The thermometers show the temperatures in four cities.

Honolulu, Hawaii *Anchorage, Alaska*

Death Valley, California *Seattle, Washington*

Write each temperature. Then match each temperature with its most appropriate location.

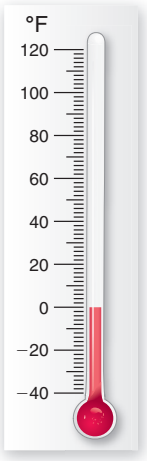
a. **b.** **c.** **d.**



e. How would you describe all the temperatures in relation to 0°F?

2 ACTIVITY: Describing a Temperature

Work with a partner. The thermometer shows the coldest temperature ever recorded in Seattle, Washington.



a. What is the temperature?

b. How do you write temperatures that are colder than this?

c. Suppose the record for the coldest temperature in Seattle is broken by 10 degrees. What is the new coldest temperature? Draw a thermometer that shows the new coldest temperature.

d. How is the new coldest temperature different from the temperatures in Activity 1?

Integers

In this lesson, you will

- understand positive and negative integers and use them to describe real-life situations.
- graph integers on a number line.

Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP5 Use Appropriate Tools Strategically:** It is important for students to create and then represent integers on a number line. A physical model that can be folded at 0 helps students to develop an understanding of the symmetric nature of integers and serves as an informal introduction to absolute value later in the chapter.

Motivate

- Have students work with partners to brainstorm types of numbers and contexts for these types of numbers. There will be three parts. Give each part a minute for brainstorming. Then, spend a minute or two for sharing.
- Part 1: Give examples of contexts where the numbers are generally greater than 100. Examples: cost of a car, population of a city
- Part 2: Give examples of contexts where the numbers are generally fractions. Examples: shoe sizes, lengths measured with a ruler
- Part 3: Give examples of contexts where the numbers are generally decimals. Examples: body temperature, track meet times

Activity Notes

Activity 1

- This activity assumes basic familiarity with different climate zones within the 50 United States.
- Working with partners, students should not have difficulty matching up the thermometers if they are familiar with the locations.

? "Do you think time of year has any bearing on the temperature readings? Explain." yes; Temperatures change through the seasons of the year.

? "What is the warmest (coldest) temperature you've experienced?" Answers will vary. This is just to start a discussion. It is likely that at least one student will mention a temperature below 0°F.

- **Discuss:** Engage students in a discussion about types of numbers. They are familiar with whole numbers, fractions, and decimals. Ask them what other types of numbers there are.

Activity 2

- This activity has students think about both sides of 0. Because the thermometer is vertical, students may start to think that there are numbers above and below zero. Number lines are generally horizontal so we also refer to left and right of zero.
- In Question 2(c), be sure that students have labeled 0 on their number line.
- Discuss Question 2(d). The new temperature, −10°F, is 10 units from 0. This idea of having two locations on the number line that are the same distance from 0 is the focus of the next activity.

Common Core State Standards

6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
6.NS.6a Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., −(−3) = 3, and that 0 is its own opposite.
6.NS.6c Find and position integers . . . on a horizontal or vertical number line diagram; . . ..

Previous Learning

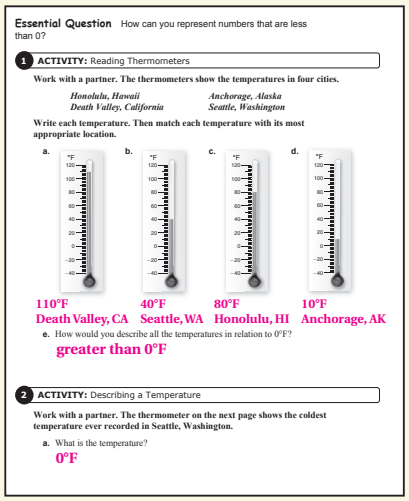
Students should know how to graph whole numbers.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

6.1 Record and Practice Journal



Essential Question How can you represent numbers that are less than 0?

1 ACTIVITY: Reading Thermometers

Work with a partner. The thermometers show the temperatures in four cities.

Honolulu, Hawaii *Anchorage, Alaska*
Death Valley, California *Seattle, Washington*

Write each temperature. Then match each temperature with its most appropriate location.

a. 110°F Death Valley, CA
b. 40°F Seattle, WA
c. 80°F Honolulu, HI
d. 10°F Anchorage, AK

e. How would you describe all the temperatures in relation to 0°F?
greater than 0°F

2 ACTIVITY: Describing a Temperature

Work with a partner. The thermometer on the next page shows the coldest temperature ever recorded in Seattle, Washington.

a. What is the temperature?
0°F

English Language Learners

Vocabulary

Students might have difficulty understanding a negative elevation. Use a vertical scale and tell them that zero represents sea level. Locations with an elevation greater than zero are above sea level. Locations with an elevation less than zero are below sea level.

6.1 Record and Practice Journal

b. How do you write temperatures that are colder than this?
Write "below zero" after temperature.

c. Suppose the record for the coldest temperature in Seattle is broken by 10 degrees. What is the new coldest temperature? Draw a thermometer that shows the new coldest temperature.
−10°F or 10 degrees below zero

d. How is the new coldest temperature different from the temperatures in Activity 1?
below zero, negative

3 ACTIVITY: Extending the System of Whole Numbers

Work with a partner.

a. Draw a number like the one shown on a sheet of paper. Complete the number line using whole numbers only.

b. Fold the paper with your number line around 0 so that the lines overlap. Make tick marks on the other side of the number line to match the tick marks for the whole numbers. **Check students' work.**

c. **STRUCTURE** Compare this number line to the thermometers from Activities 1 and 2. What do you think the new tick marks represent? How would you label them? **negative numbers**

−6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you represent numbers that are less than 0?
negative sign or "below zero"

5. Describe another real-life example that uses numbers that are less than 0?
freezing/melting points of elements

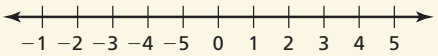
6. **REASONING** How are the temperatures shown by the thermometers at the right similar? How are they different?
both are 40° from 0°; 40°F > 0°F and −40°F < 0°F

7. **WRITING** The temperature in a town on Thursday evening is 25°F. On Sunday morning, the temperature drops below 0°F. Write a story to describe what may have happened in the town. Be sure to include the temperatures for each day.
Check students' work.

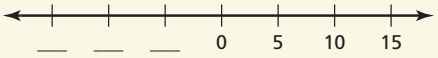
Laurie's Notes

Activity 3

- Provide students with a ruler for this activity.
- **MP5:** As they extend their number lines, there are two key features that you want to discuss with students. The first is that the tick marks are equally spaced. The second is the symmetry about 0 that is suggested in a concrete fashion by folding the number line onto itself.
- **Common Error:** Students may equally space the tick marks and then incorrectly label the negative integers.



- **Extension:** Draw a number line with increments labeled as shown. Ask students what values would be used to label the remaining tick marks.



What Is Your Answer?

- For Question 7, have students share their stories. Stories should be very different. Ask some probing questions, such as whether the temperatures could have increased.

Closure

- Refer back to the opening Motivate activity and ask students to brainstorm with their partners.
- Part 4: Give examples of contexts where the numbers are generally negative numbers. Examples: winter temperatures, overdrawn checkbooks

3 ACTIVITY: Extending the System of Whole Numbers

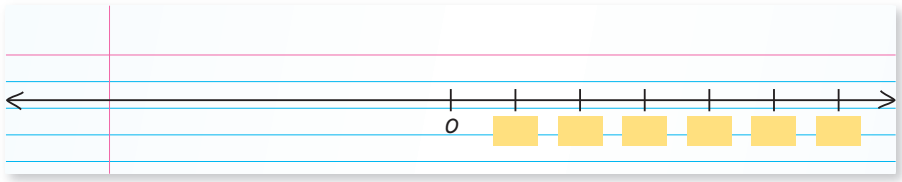
Math Practice

Maintain Oversight

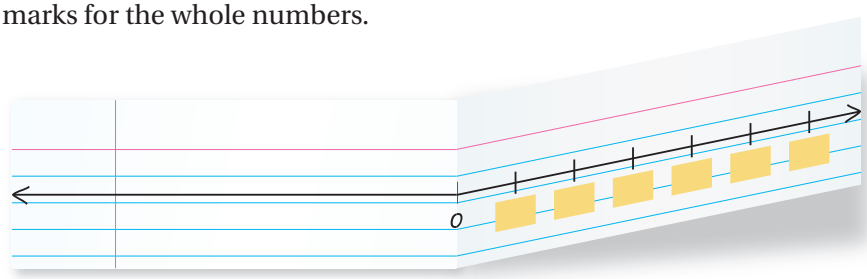
How does this activity help you represent numbers less than 0?

Work with a partner.

a. Copy and complete the number line using whole numbers only.



b. Fold the paper with your number line around 0 so that the lines overlap. Make tick marks on the other side of the number line to match the tick marks for the whole numbers.



c. **STRUCTURE** Compare this number line to the thermometers from Activities 1 and 2. What do you think the new tick marks represent? How would you label them?

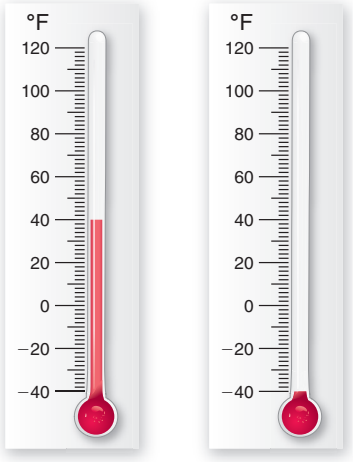
What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you represent numbers that are less than 0?

5. Describe another real-life example that uses numbers that are less than 0.

6. **REASONING** How are the temperatures shown by the thermometers at the right similar? How are they different?

7. **WRITING** The temperature in a town on Thursday evening is 25°F. On Sunday morning, the temperature drops below 0°F. Write a story to describe what may have happened in the town. Be sure to include the temperatures for each day.



Practice Use what you learned about positive and negative numbers to complete Exercises 4–7 on page 252.

6.1 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
positive numbers, *p. 250*
negative numbers, *p. 250*
opposites, *p. 250*
integers, *p. 250*

Positive numbers are greater than 0. They can be written with or without a positive sign (+).

+1 5 +20 10,000

Negative numbers are less than 0. They are written with a negative sign (−).

−1 −5 −20 −10,000

Two numbers that are the same distance from 0 on a number line, but on opposite sides of 0, are called **opposites**. The opposite of 0 is 0.

The Meaning of a Word

Opposite

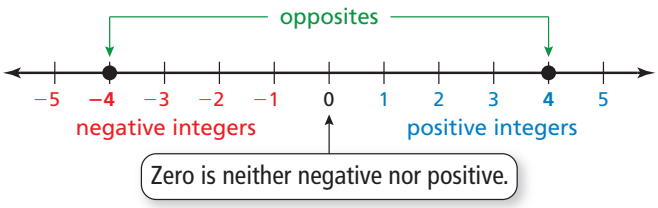
When you sit across from your friend at the lunch table, you sit **opposite** your friend.

Key Idea

Integers

Words **Integers** are the set of whole numbers and their opposites.

Graph



EXAMPLE 1 Writing Positive and Negative Integers

Write a positive or negative integer that represents the situation.

a. A contestant gains 250 points on a game show.

Gains indicates a number greater than 0. So, use a positive integer.

∴ +250, or 250

b. Gasoline freezes at 40 degrees below zero.

Below zero indicates a number less than 0. So, use a negative integer.

∴ −40

On Your Own

Now You're Ready
Exercises 8–13

Write a positive or negative integer that represents the situation.

1. A hiker climbs 900 feet up a mountain.
2. You have a debt of $24.
3. A student loses 5 points for being late to class.
4. A savings account earns $10.

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students explored a real-life application of negative numbers and gained an intuitive understanding of absolute value. (MP5)
- **Today:** Students will write and graph integers.

Motivate

- Ask for 7 volunteers to hold onto a piece of rope. They should be equally spaced. Ask the person in the middle to hold a card with the number 0 written on it.
- Say to the students:
  - "If you are 2 people away from 0, raise your hand."
  - "If you are 3 people away from 0, raise your hand."
- Now hand the card with the number 1 written on it to the person on the right of the person holding the 0 card.
- Say to the students:
  - "If you are the number −1, raise your hand."
  - "If you are the number 3, raise your hand."
  - "Keep your hand in the air if you are the least of the two numbers."
  - Repeat the instructions to compare 2 and 3, and to compare −2 and −3.
- This gives students a strong visual image of how a number line looks and where positive and negative numbers are located.

Lesson Notes

Key Idea

- Define integers. Make it clear to students that there are three sets of numbers: positive integers, negative integers, and 0. Zero is neither positive nor negative.
- Remind students that whole numbers are 0, 1, 2, 3, . . .
- Draw a number line, using two colors to differentiate positive and negative integers.
- Point out the symmetry of a number line. If the number line were drawn on a piece of paper and folded at 0, opposites would fold on top of one another. Remind students of Activity 3 yesterday with the folded paper.

Example 1

- Discuss each part of the example. The context for each part should be familiar to students.
- Point out that the + sign is optional in writing a positive number. It generally is not written.

On Your Own

- **Extension:** After student work has been checked, ask a student to state the opposite of each situation. Example: A hiker descends 900 feet down a mountain.

Goal Today's lesson is reading and graphing **integers**.

Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Differentiated Instruction

Kinesthetic

Have students use a compass to find opposites. Place the point of the compass at zero on a number line and adjust the compass so the pencil intersects a positive or negative integer. To find the opposite of that integer, draw an arc to the opposite side of the number line.

Extra Example 1

Write a positive or negative integer that represents the situation.

a. A balloon floats 5 feet above the ground. 5

b. An anchor is 15 feet underwater. −15

On Your Own

1. 900

2. −24

3. −5

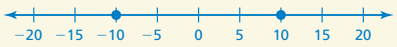
4. 10

Extra Example 2

Graph each integer and its opposite.

a. 4

b. −10



Extra Example 3

You start a game with 10 points. You gain 4 points in the 1st round. In the 2nd round, you lose 7 points.

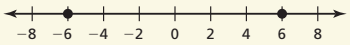
a. Write an integer that represents each point level.

You start a game. 10
You gain 4 points. +4
You lose 7 points. −7

b. After the 3rd round, your total score is 10 points. Write an integer that represents how many points you received in the 3rd round. +3

On Your Own

5.



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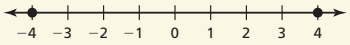

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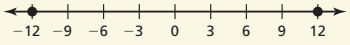

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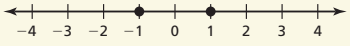


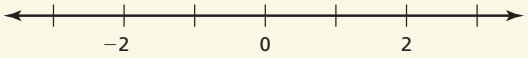
9. −7

Laurie's Notes

Example 2

? "What is the opposite of 5?" −5 "What is the opposite of −13?" 13

- **Common Error:** When asked to graph the numbers 2 and −2, students may draw a number line and label −2, 0, and 2 as shown, and write nothing else.



Make it clear to students that they should decide on a scale and then put a closed circle on the number line to graph a number.

? "What number is halfway between a number and its opposite?" 0

- **MP6 Attend to Precision:** Discuss the Reading Note in the margin. Tell students that −2 is read "negative 2" or "the opposite of 2." It should not be read "minus 2." Minus is an operation.

Example 3

- Draw a vertical number line on the board as a visual aid for students as you work through this problem.
- **FYI:** Students may ask if underground floors are really numbered using negative numbers. It is most common to use a letter such as L or P to denote a lower level or a parking level.

On Your Own

- **Think-Pair-Share:** Students should read each question independently and then work in pairs to answer the questions. When they have answered the questions, the pair should compare their answers with another group and discuss any discrepancies.

Closure

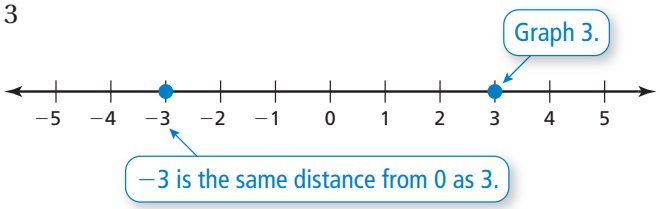
- **Exit Ticket:** Graph each integer on the same number line and write a context for each: −6 and 4.

−6 A student missed 6 questions on a quiz.
4 There are 4 new students in the class.

EXAMPLE 2 Graphing Integers

Graph each integer and its opposite.

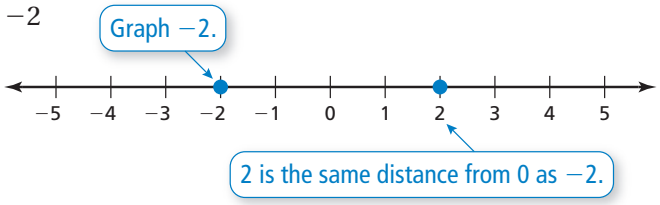
a. 3



Reading

You can think of the negative sign (−) as referring to the opposite of a number. In Example 2(b), you can read −2 as "the opposite of 2."

b. −2



EXAMPLE 3 Real-Life Application

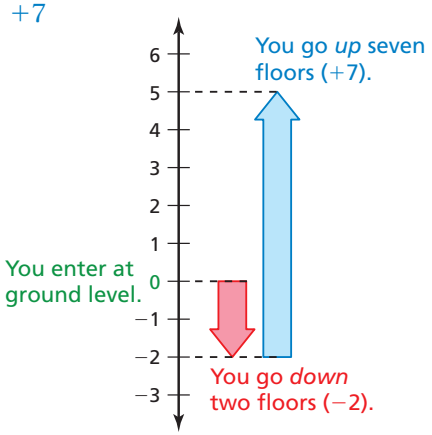
You deliver flowers to an office building. You enter at ground level and go down 2 floors to make the first delivery. Then you go up 7 floors to make the second delivery.

a. Write an integer that represents each position.

Position	*Integer*
You enter at ground level.	0
You go down 2 floors.	−2
You go up 7 floors.	+7

b. Write an integer that represents how you return to ground level.

Use a number line to model your movement, as shown.



The second delivery is on the fifth floor. You must go down 5 floors to return to ground level.

The integer representing "down 5 floors" is −5.

On Your Own

Now You're Ready
Exercises 16–23

Graph the integer and its opposite.

5. 6 **6.** −4 **7.** −12 **8.** 1

9. WHAT IF? In Example 3, you go up 9 floors to make the second delivery. Write an integer that represents how you return to ground level.

6.1 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** Which of the following numbers are integers?

$$8, -4.1, -9, \frac{1}{6}, 1.75, 22$$

2. **OPEN-ENDED** Describe a real-life example that you can represent by -1200.

3. **VOCABULARY** List three words or phrases used in real life that indicate negative numbers.

Practice and Problem Solving

Graph the number that represents the situation on a number line.

4. A football team loses 3 yards.
5. The temperature is 6 degrees below zero.
6. A person climbs 600 feet up a mountain.
7. You earn \$15 raking leaves.

Write a positive or negative integer that represents the situation.

1 8. You withdraw \$42 from an account.
9. An airplane climbs to 37,500 feet.
10. The temperature rises 17 degrees.
11. You lose 56 points in a video game.
12. A ball falls 350 centimeters.
13. You receive 5 bonus points in class.

14. **STOCK MARKET** A stock market gains 83 points. The next day, the stock market loses 47 points. Write each amount as an integer.

15. **SCUBA DIVING** The world record for scuba diving is 318 meters below sea level. Write this as an integer.

Graph the integer and its opposite.

2 16. -5
17. -8
18. 14
19. 9
20. 30
21. -150
22. -32
23. 400

24. **ERROR ANALYSIS** Describe and correct the error in describing positive integers.

> ✗ The positive integers are 0, 1, 2, 3, ...

25. **TEMPERATURE** The highest temperature in February is 25°F. The lowest temperature in February is the opposite of the highest temperature. Graph both temperatures.

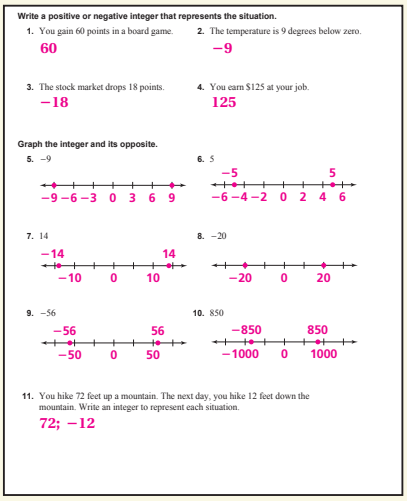
Assignment Guide and Homework Check

Level	Day 1 Activity Assignment	Day 2 Lesson Assignment	Homework Check
Basic	4–7, 33–35	1–3, 9–13 odd, 14, 15–23, odd, 24, 25, 27	9, 14, 21, 24, 27
Average	4–7, 33–35	1–3, 12–15, 16–24 even, 27, 29, 30, 31	14, 22, 24, 27, 30
Advanced	4–7, 33–35	1–3, 8–30 even, 31, 32	14, 22, 24, 26, 30

Common Errors

- **Exercises 8–13** Students may use the wrong sign when writing the integer. Tell them to look for key words, such as rise and fall, to help them determine the sign.
- **Exercises 16–23** Students may think that *opposites* are only negative numbers. For example, the opposite of -3 is -3 and the opposite of 3 is -3. Remind them that the opposite of a negative number is positive.

6.1 Record and Practice Journal



Write a positive or negative integer that represents the situation.

1. You gain 60 points in a board game. **60**
2. The temperature is 9 degrees below zero. **−9**
3. The stock market drops 18 points. **−18**
4. You earn $125 at your job. **125**

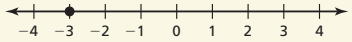
Graph the integer and its opposite.

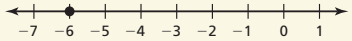
5. −9 — **−9 −6 −3 0 3 6 9**
6. 5 — **−5 5; −6 −4 −2 0 2 4 6**
7. 14 — **−14 14; −10 0 10**
8. −20 — **−20 0 20**
9. −56 — **−56 56; −50 0 50**
10. 850 — **−850 850; −1000 0 1000**
11. You hike 72 feet up a mountain. The next day, you hike 12 feet down the mountain. Write an integer to represent each situation. **72; −12**

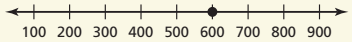
Vocabulary and Concept Check

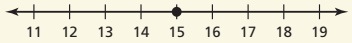
1. 8, −9, 22
2. *Sample answer:* The status of a fundraiser's goal is −$1200.
3. *Sample answer:* below, under, lose

Practice and Problem Solving

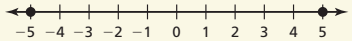
4. 

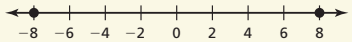
5. 

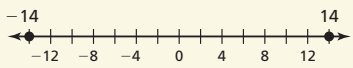
6. 

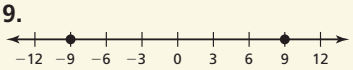
7. 

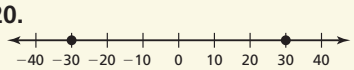
8. −42
9. 37,500
10. 17
11. −56
12. −350
13. 5
14. 83; −47
15. −318

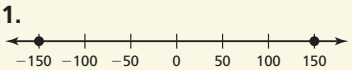
16. 

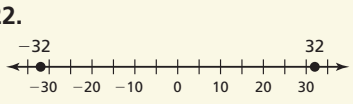
17. 

18. 

19. 

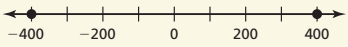
20. 

21. 

22. 

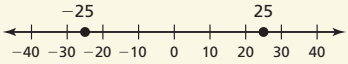
Practice and Problem Solving

23.



24. The numbers shown are whole numbers. Positive integers are 1, 2, 3, . . .

25.



26. 5 **27.** −8

28. −15 **29.** 18

30. See *Taking Math Deeper*.

31. See Additional Answers.

32. no; The flag starts at 0 and moves 8 feet right to 8 on the number line. Then the flag moves 12 feet left to −4 on the number line. Then the flag moves 13 feet right to 9 on the number line. Nine is less than 10. So, a team has not won yet.

Fair Game Review

33. $\frac{3}{8}, \frac{1}{2}, \frac{3}{4}, \frac{7}{8}$

34. 4.312, 4.316, 4.32, 4.5

35. B

Mini-Assessment

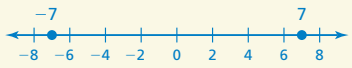
Write a positive or negative integer that represents the situation.

1. A fish is 12 meters underwater. −12

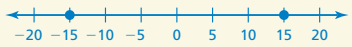
2. You deposit $25 into your savings account. 25

Graph the integer and its opposite.

3. 7



4. −15



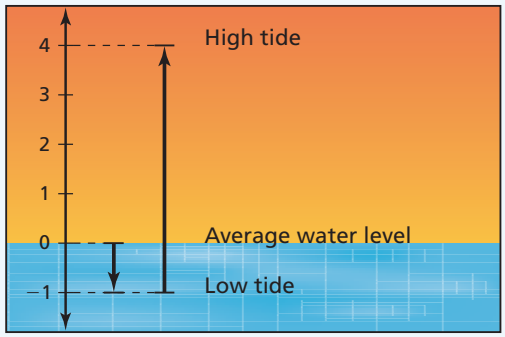
5. While playing a game, you move 6 steps forward and then 10 steps backward. Write each number as an integer. 6; −10

Taking Math Deeper

Exercise 30

This exercise involves a subtle, yet very important, use of negative numbers. The concept is that in business and science, we frequently use the number 0 as a convenient point on the number line to represent time or physical location.

1 Draw a diagram.



2 It is not *required* that we use the number 0 to represent the average water level. However, if we do, then low tide is represented by the number −1 because it is 1 foot below the average water level. High tide is represented by the number 4 because it is 5 feet higher than low tide.

High Tide = 4
Average Water Level = 0
Low tide = −1

The average water level is 4 feet below high tide. So, the average water level relative to high tide is represented by the number −4.

3 If we choose to let 0 represent high tide, then the numbering system changes. Now, we have the following.

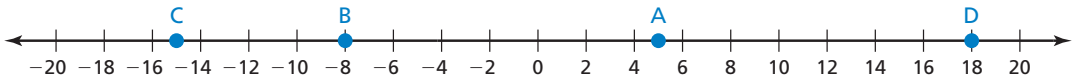
High Tide = 0
Average Water Level = −4
Low tide = −5

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Start the next section

Identify the integer represented by the point on the number line.

26. A **27.** B **28.** C **29.** D



30. TIDES Use the information below.

- Low tide is 1 foot below the average water level.
- High tide is 5 feet higher than low tide.

Write an integer that represents the average water level relative to high tide.

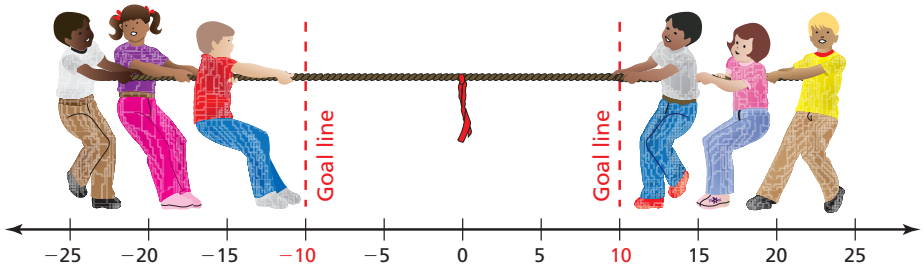
31. REPEATED REASONING Choose any positive integer.

a. Find the opposite of the integer. **b.** Find the opposite of the integer in part (a).

c. What can you conclude about the opposite of the opposite of the integer? Is this true for all integers? Use a number line to justify your answer.

d. Describe the meaning of $-(-(-6))$. Find its value.

32. Number Sense In a game of tug-of-war, a team wins by pulling the flag over its goal line. The flag begins at 0. During a game, the flag moves 8 feet to the right, 12 feet to the left, and 13 feet back to the right. Did a team win? Explain.



Fair Game Review What you learned in previous grades & lessons

Order the numbers from least to greatest. *(Skills Review Handbook)*

33. $\frac{7}{8}, \frac{1}{2}, \frac{3}{8}, \frac{3}{4}$

34. 4.5, 4.316, 4.32, 4.312

35. MULTIPLE CHOICE The height of a statue is 276 inches. What is the height of the statue in meters? Round your answer to the nearest hundredth. *(Section 5.7)*

(A) 1.09 m (B) 7.01 m (C) 108.66 m (D) 701.04 m

6.2 Comparing and Ordering Integers

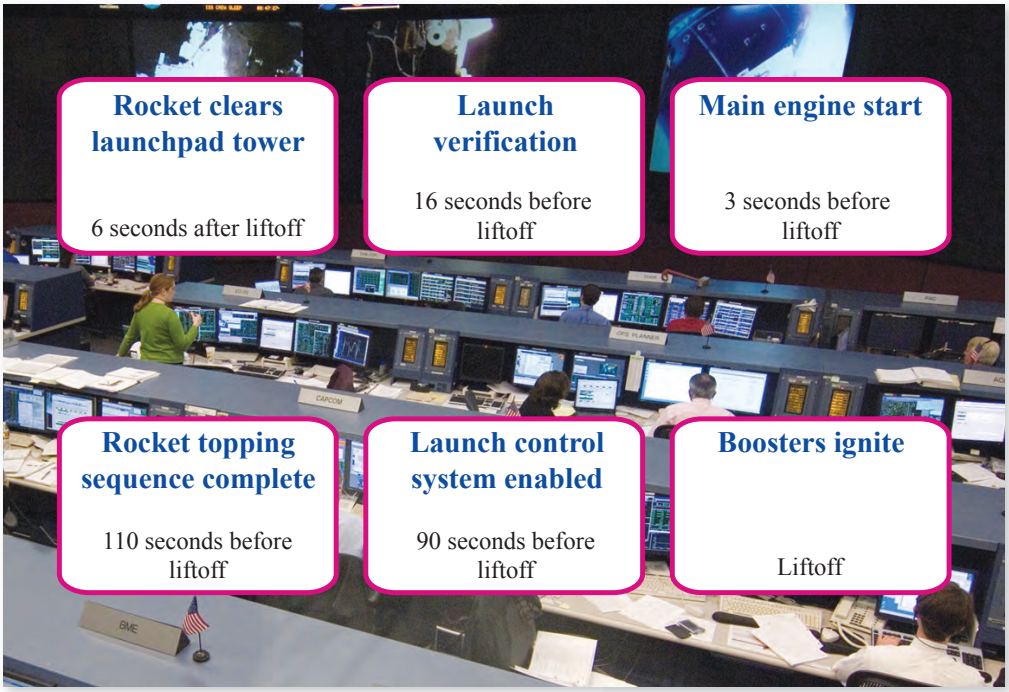
Essential Question How can you use a number line to order real-life events?

1 ACTIVITY: Seconds to Takeoff

Work with a partner. You are listening to a command center before the liftoff of a rocket.

You hear the following:

> **"T minus 10 seconds . . . go for main engine start . . . T minus 9 . . . 8 . . . 7 . . . 6 . . . 5 . . . 4 . . . 3 . . . 2 . . . 1 . . . we have liftoff."**



a. Draw a number line. Then locate the events shown above at appropriate points on the number line.

b. Which event occurs at zero on your number line? Explain.

c. Which of the events occurs first? Which of the events occurs last? How do you know?

d. List the events in the order they occurred.

Integers

In this lesson, you will

- use a number line to compare positive and negative integers.
- use a number line to order positive and negative integers for real-life situations.

Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP5 Use Appropriate Tools Strategically:** It is important for students to create and then represent integers on a number line. A physical model that can be folded at 0 helps students to develop an understanding of the symmetric nature of integers and serves as an informal introduction to absolute value later in the chapter.

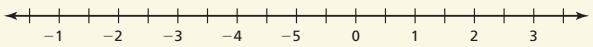
Motivate

- 3-2-1-Blast Off! Ask the NASA space trivia questions and have students write their answers on white boards or raise their hands.
  - "Who was the first person in space?" Major Yuri Gagarin—Russian
  - "Who was the first American woman in space?" Sally Ride; June 18, 1983
  - "Who made the first U.S. spacewalk?" Ed White; June 3, 1965; Gemini 4
  - "Where is the Sea of Tranquility?" moon; site of the first lunar landing
  - "What is the name of the largest volcano in the solar system?" Olympus Mons; located on Mars; bigger than any mountain on Earth

Activity Notes

Activity 1

- Have students draw a number line displaying both positive and negative numbers. Suggest that students not label every number between −31 and 0. Every other number or every third number should be labeled.
- **Common Error:** It is very common for students to place the negative numbers starting with −1 to the left as shown:



- Explain to students what it means to *locate* an event on the number line. They should put a dot on the number line and a letter above it that corresponds to one of the six events. Let rocket clears launchpad tower = A, launch verification = B, and so on.
- **MP6 Attend to Precision:** Discuss with students that even though negative numbers are *not* read as "minus 3," it is common language at a space launch to use the phrase "minus 3" instead of "negative 3."
- Discuss the order of the events. Those that are *before* launch would be to the left of 0.
- **Extension:** If you have a long piece of rope, you could have students *act out* this problem. Two students stretch the rope across the front of the classroom. Six students *locate themselves* on the rope number line. You can hang a large paper clip to identify zero. Then students place themselves in approximately the correct location.
- **Teaching Tip:** The number line doesn't have to have integers labeled. The number line could have 0 labeled and then tic marks with the events labeled.

Common Core State Standards

6.NS.6c Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers . . ..
6.NS.7a Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
6.NS.7b Write, interpret, and explain statements of order for rational numbers in real-world contexts.

Previous Learning

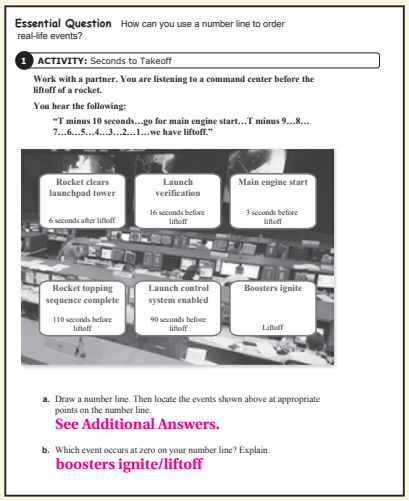
Students should know how to compare whole numbers and graph integers.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

6.2 Record and Practice Journal



Essential Question How can you use a number line to order real-life events?

1 ACTIVITY: Seconds to Takeoff

Work with a partner. You are listening to a command center before the liftoff of a rocket.

You hear the following:

"T minus 10 seconds…go for main engine start…T minus 9…8…7…6…5…4…3…2…1…we have liftoff."

Rocket clears launchpad tower — 6 seconds after liftoff
Launch verification — 16 seconds before liftoff
Main engine start — 3 seconds before liftoff
Rocket topping sequence complete — 110 seconds before liftoff
Launch control system enabled — 90 seconds before liftoff
Boosters ignite — Liftoff

a. Draw a number line. Then locate the events shown above at appropriate points on the number line.
See Additional Answers.

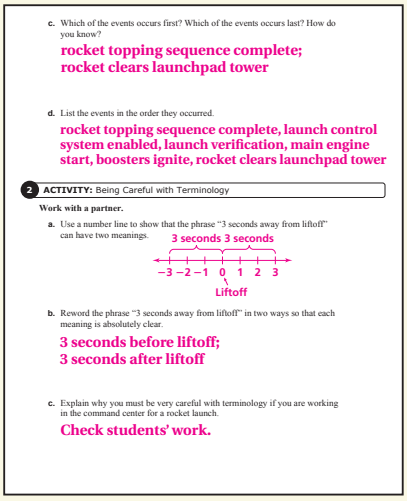
b. Which event occurs at zero on your number line? Explain.
boosters ignite/liftoff

English Language Learners

Vocabulary

Integers may be a new term for English learners. Review vocabulary often with students. For example, you may say, "Name a negative integer less than negative two." Encourage your students to reply in a complete sentence, "A negative integer less than negative two is negative five."

6.2 Record and Practice Journal



c. Which of the events occurs first? Which of the events occurs last? How do you know?

rocket topping sequence complete; rocket clears launchpad tower

d. List the events in the order they occurred.

rocket topping sequence complete, launch control system enabled, launch verification, main engine start, boosters ignite, rocket clears launchpad tower

2 ACTIVITY: Being Careful with Terminology

Work with a partner.

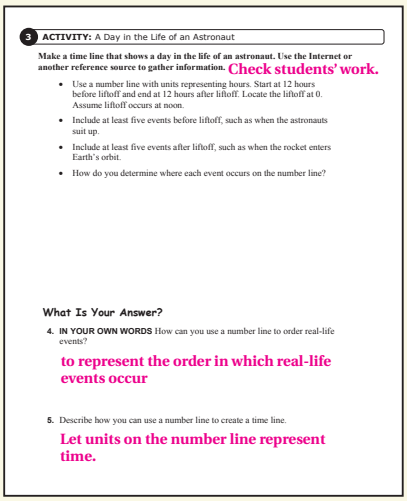
a. Use a number line to show that the phrase "3 seconds away from liftoff" can have two meanings.

b. Reword the phrase "3 seconds away from liftoff" in two ways so that each meaning is absolutely clear.

3 seconds before liftoff;
3 seconds after liftoff

c. Explain why you must be very careful with terminology if you are working in the command center for a rocket launch.

Check students' work.



3 ACTIVITY: A Day in the Life of an Astronaut

Make a time line that shows a day in the life of an astronaut. Use the Internet or another reference source to gather information. Check students' work.

- Use a number line with units representing hours. Start at 12 hours before liftoff and end at 12 hours after liftoff. Locate the liftoff at 0. Assume liftoff occurs at noon.
- Include at least five events before liftoff, such as when the astronauts suit up.
- Include at least five events after liftoff, such as when the rocket enters Earth's orbit.
- How do you determine where each event occurs on the number line?

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use a number line to order real-life events?

to represent the order in which real-life events occur

5. Describe how you can use a number line to create a time line.

Let units on the number line represent time.

Laurie's Notes

Activity 2

- The term *opposite* was introduced in the previous lesson and is revisited here in the context of pre- and post-launch events.
- This activity introduces the idea that every number, other than 0, has an opposite. In other words, the number line is symmetric. Starting at 0, there is a number 4 units to the left and 4 units to the right; there is a number 7 units to the left and 7 units to the right.
- Listen to students' terminology while they discuss this problem.

Activity 3

- **Research:** This is an activity that is most easily accomplished in a library, computer lab, or as a homework assignment.
- Discuss with students the need to make their units clear on the number line. Even though the liftoff may not occur at noon, for the purposes of this example, it is the centering point. So, pre-launch events happen in the morning (A.M.) and post-launch events happen in the afternoon and evening (P.M.).

What Is Your Answer?

- **Think-Pair-Share:** Students should read each question independently and then work in pairs to answer the questions. When they have answered the questions, the pair should compare their answers with another group and discuss any discrepancies.

Closure

- **Exit Ticket:** Make a number line to represent three events prior to math class and three events after math class. The events do not need to take place at school. Answers will vary.

2 ACTIVITY: Being Careful with Terminology

Work with a partner.

a. Use a number line to show that the phrase "3 seconds away from liftoff" can have two meanings.

b. Reword the phrase "3 seconds away from liftoff" in two ways so that each meaning is absolutely clear.

c. Explain why you must be very careful with terminology if you are working in the command center for a rocket launch.

3 ACTIVITY: A Day in the Life of an Astronaut

Math Practice

Recognize the Usefulness of Tools

Which sources would give you the most accurate information? How do you know you can trust the information you find?

Make a time line that shows a day in the life of an astronaut. Use the Internet or another reference source to gather information.

- Use a number line with units representing hours. Start at 12 hours before liftoff and end at 12 hours after liftoff. Locate the liftoff at 0. Assume liftoff occurs at noon.
- Include at least five events before liftoff, such as when the astronauts suit up.
- Include at least five events after liftoff, such as when the rocket enters Earth's orbit.
- How do you determine where each event occurs on the number line?

What Is Your Answer?

4. IN YOUR OWN WORDS How can you use a number line to order real-life events?

5. Describe how you can use a number line to create a time line.

Practice Use what you learned about number lines to complete Exercises 4–7 on page 258.

6.2 Lesson

Check It Out
Lesson Tutorials
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On a horizontal number line, numbers to the left are less than numbers to the right. Numbers to the right are greater than numbers to the left.

EXAMPLE 1 Comparing Integers on a Horizontal Number Line

Compare 2 and −6.

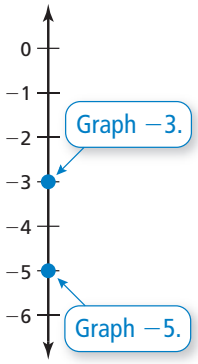
2 is to the right of −6. So, $2 > -6$.

On a vertical number line, numbers below are less than numbers above. Numbers above are greater than numbers below.

EXAMPLE 2 Comparing Integers on a Vertical Number Line

Compare −5 and −3.

−5 is below −3. So, $-5 < -3$.



On Your Own

Now You're Ready
Exercises 4–11

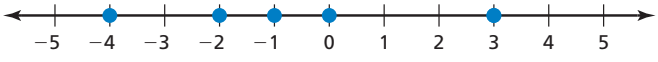
Copy and complete the statement using < or >.

1. 0 ☐ −4 **2.** −5 ☐ 5 **3.** −8 ☐ −7

EXAMPLE 3 Ordering Integers

Order −4, 3, 0, −1, −2 from least to greatest.

Graph each integer on a number line.



Write the integers as they appear on the number line from left to right.

So, the order from least to greatest is −4, −2, −1, 0, 3.

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students explored a real-life application of integers and gained an intuitive understanding of comparing integers. (MP5, MP6)
- **Today:** Students will compare and order integers.

Motivate

- Place a piece of masking tape on the floor to represent a number line. Identify the location of 0 by having a student stand at zero.
- Have two more students stand at two other integers. Have the students change their locations several times to represent the following scenarios: two positive integers, one positive integer and one negative integer, and both negative integers.

Lesson Notes

Example 1

- Draw a number line and graph -6 and 2.
- Discuss with students that 0 does not have to be in the middle of the number line. Sometimes number lines do not show 0.
- **Big Idea:** The further to the left on the number line, the less the number becomes. The further to the right on the number line, the greater the number becomes.
- **Extension:** Ask the students how far -6 and 2 are from 0. -6 is 6 units from 0, 2 is 2 units from 0.

Example 2

- **MP4 Model with Mathematics:** There is an intentional effort in this lesson to display number lines in two orientations—horizontal and vertical. These two models will help students with their graphing in the coordinate plane. They also make connections to common contexts, such as a thermometer.
- Draw a vertical number line and graph -5 and -3.
- A vertical number line should remind students of the thermometer in Activity 6.1.
- **Extension:** Ask the students how far -5 and -3 are from 0. -5 is 5 units from 0, and -3 is 3 units from 0.

On Your Own

- Check students' answers to Question 3. Comparing two negative numbers is a difficult concept for some students. Encourage them to graph the numbers on a number line.

Example 3

- Draw a number line and graph the integers. Discuss with students the difference between labeling the number line with integers and graphing integers with closed circles.

Goal Today's lesson is on comparing and ordering integers.

Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

Compare -2 and -10. $-10 < -2$

Extra Example 2

Compare -1 and -2. $-1 > -2$

On Your Own

1. >
2. <
3. <

Extra Example 3

Order -3, 9, 0, -7, 1 from least to greatest. $-7, -3, 0, 1, 9$

Differentiated Instruction

Auditory

Have a discussion about positive and negative integers. Talk about examples of when we use negative numbers in everyday life. Some examples include temperature and elevation. Have students discuss what a negative value indicates for each example.

Extra Example 4

A number is between -7 and 3. What is the least possible integer value of this number? -6

Extra Example 5

In Example 5, Wallops Island recorded a new record low last night. The new record low is greater than the record low in Roanoke. What integers can represent the new record low on Wallops Island? $-10, -9, -8, -7, -6, -5, -4, -3, -2, -1$

On Your Own

4. $-3, -2, -1, 1, 3$

5. $-8, -7, 1, 4, 6$

6. -7

7. $-9, -8, -7, -6, -5, -4$

Laurie's Notes

Example 4

- Drawing a number line, and plotting the possible answers, is a helpful aid for students.
- Discuss with students that the phrase "between -8 and 0" does not include the integers -8 and 0.
- **Common Error:** Students will often say -7 is the greatest integer between -8 and 0. They are still uncertain why moving to the left on the number line gives numbers that are getting smaller (less than).

Example 5

- Students should notice that all of the temperatures are negative, with the exception of Wallops Island.
- **Extension:** Have students research the coldest and warmest recorded temperatures for their town or city.

On Your Own

- Check to see that students have read Question 7 carefully.

Closure

- Explain why $2 > -42$. 2 is to the right of -42 on a number line.

EXAMPLE 4 Reasoning with Integers

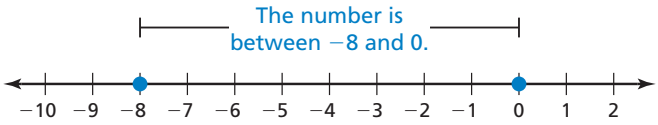
A number is greater than −8 and less than 0. What is the greatest possible integer value of this number?

Ⓐ −10 Ⓑ −7 Ⓒ −1 Ⓓ 2

Study Tip

In Example 4, you can eliminate Choices A and D because −10 is to the left of −8 and 2 is to the right of 0.

The number is greater than −8 and less than 0. So, the number must be to the right of −8 and to the left of 0 on a horizontal number line.

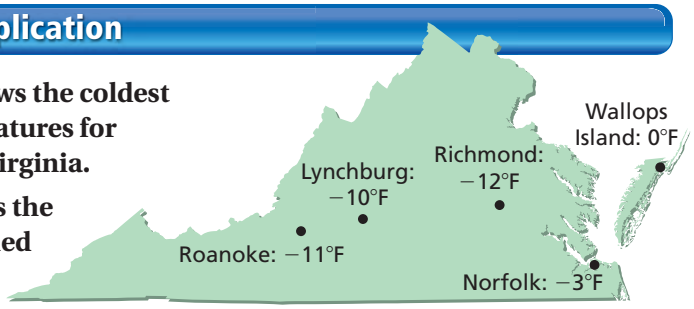


The greatest possible integer value between −8 and 0 is the integer farthest to the right on the number line between these values, which is −1.

∴ So, the correct answer is Ⓒ.

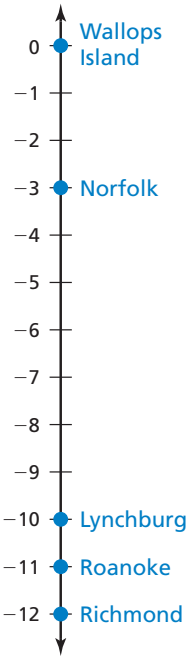
EXAMPLE 5 Real-Life Application

The diagram shows the coldest recorded temperatures for several cities in Virginia.



a. Which city has the coldest recorded temperature?

Graph each integer on a vertical number line.



∴ −12 is the lowest on the number line. So, Richmond has the coldest recorded temperature.

b. Has a negative Fahrenheit temperature ever been recorded on Wallops Island? Explain.

∴ The coldest recorded temperature on Wallops Island is 0°F, which is greater than every negative temperature. So, a negative temperature has never been recorded on Wallops Island.

Now You're Ready

Exercises 14–19 and 22

On Your Own

Order the integers from least to greatest.

4. −2, −3, 3, 1, −1 **5.** 4, −7, −8, 6, 1

6. In Example 4, what is the least possible integer value of the number?

7. In Example 5, Norfolk recorded a new record low last night. The new record low is greater than the record low in Lynchburg. What integers can represent the new record low in Norfolk?

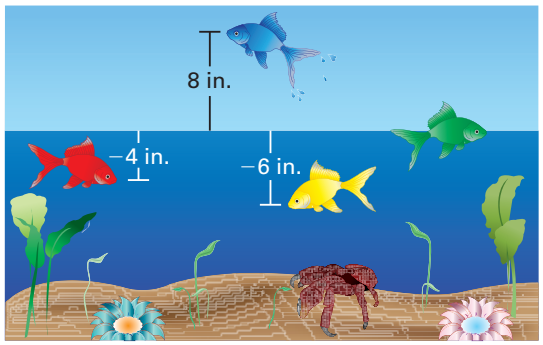
6.2 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **WRITING** Explain how to use a number line to compare two integers.

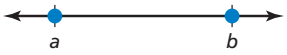
2. **REASONING** The positions of four fish are shown.



   a. Use red, blue, yellow, and green dots to graph the positions of the fish on a horizontal number line and a vertical number line.

   b. Explain how to use the number lines from part (a) to order the positions from least to greatest.

3. **NUMBER SENSE** a and b are negative integers. Compare a and b. Explain your reasoning.



Practice and Problem Solving

Copy and complete the statement using < or >.

1 2 **4.** $3 \square 0$ **5.** $-2 \square 0$ **6.** $6 \square -6$ **7.** $3 \square -4$

8. $-1 \square 4$ **9.** $-7 \square -8$ **10.** $-3 \square -2$ **11.** $-5 \square -10$

ERROR ANALYSIS **Describe and correct the error in comparing the negative numbers.**

12.

✗ Compare −3 and −1.
3 > 1. So, −3 > −1.

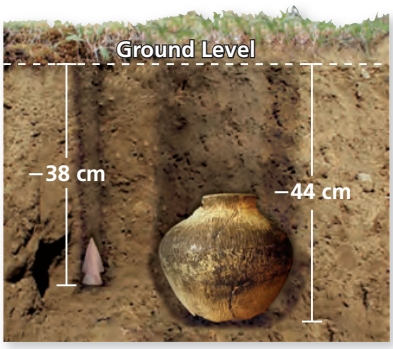
13.

✗ Compare −7 and −3.
Because −7 < −3, −7 is to the right of −3 on a number line.

Order the integers from least to greatest.

3 **14.** 0, −1, 2, 3, −3 **15.** −4, −2, −3, 2, 1 **16.** −2, 3, −3, −4, 4

17. −7, 2, 6, −4, 3 **18.** 10, −10, 30, −30, −50 **19.** −5, 15, −10, −20, 25



20. **ARCHAEOLOGY** An archaeologist discovers the two artifacts shown.

   a. What integer represents ground level?

   b. A dinosaur bone is found 42 centimeters below ground level. Is it deeper than both of the artifacts?

21. **TEMPERATURE** The freezing temperature of nitrogen is −210°C, and the freezing temperature of oxygen is −223°C. Which temperature is colder?

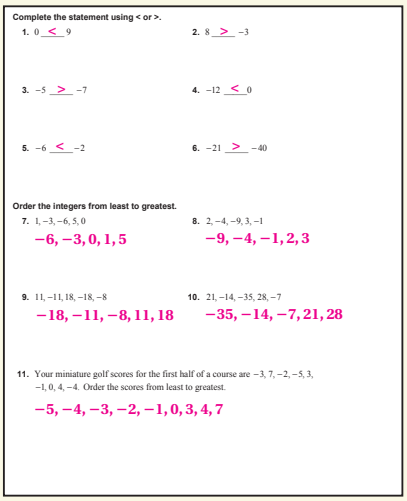
Assignment Guide and Homework Check

Level	Day 1 Activity Assignment	Day 2 Lesson Assignment	Homework Check
Basic	4–7, 29–33	1–3, 8, 11–23 odd	8, 13, 17, 19, 21
Average	4–7, 29–33	1–3, 8–12 even, 13–21 odd, 22–26	10, 17, 19, 21, 25
Advanced	4–7, 29–33	1–3, 10, 12, 16, 18, 20–28	10, 18, 24, 26, 27

Common Errors

- **Exercises 14–19** Students may ignore the signs on the integers and order them incorrectly. Encourage them to use a number line to help them order the integers.

6.2 Record and Practice Journal



Complete the statement using < or >.

1. 0 < 9
2. 8 > −3
3. −5 > −7
4. −12 < 0
5. −6 < −2
6. −21 > −40

Order the integers from least to greatest.

7. 1, −3, −6, 5, 0
   −6, −3, 0, 1, 5
8. 2, −4, −9, 3, −1
   −9, −4, −1, 2, 3
9. 11, −11, 18, −18, −8
   −18, −11, −8, 11, 18
10. 21, −14, −35, 28, −7
    −35, −14, −7, 21, 28
11. Your miniature golf scores for the first half of a course are −3, 7, −2, −5, 3, −1, 0, 4, −4. Order the scores from least to greatest.
    −5, −4, −3, −2, −1, 0, 3, 4, 7

Vocabulary and Concept Check

1. On a number line, numbers to the left are less than numbers to the right. Numbers to the right are greater than numbers to the left.
2. See Additional Answers.
3. The value of a is less than the value b because a is to the left of b.

Practice and Problem Solving

4. >
5. <
6. >
7. >
8. <
9. >
10. <
11. >
12. The student compares 3 and 1, not −3 and −1. So, $-3 < -1$.
13. The explanation about where the integers are located on a number line is incorrect; $-7 < -3$; So, −7 is to the left of −3 on a number line.
14. −3, −1, 0, 2, 3
15. −4, −3, −2, 1, 2
16. −4, −3, −2, 3, 4
17. −7, −4, 2, 3, 6
18. −50, −30, −10, 10, 30
19. −20, −10, −5, 15, 25
20. **a.** 0

    b. no
21. oxygen
22. least: −9; greatest: −3
23. always; The opposite of a positive integer is a negative integer. Positive integers are greater than negative integers.

Practice and Problem Solving

24. never; If an integer is less than its opposite, it must be a negative integer, which is never greater than 0.

25. **a.** Florida, Louisiana, Arkansas, Tennessee, California

b. California, Louisiana, Florida, Arkansas, Tennessee

c. An elevation of 0 feet represents sea level.

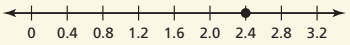
26. See *Taking Math Deeper*.

27. no; In order for the median to be below 0°F, at least 6 of the temperatures must be below 0°F.

28. See Additional Answers.

Fair Game Review

29.



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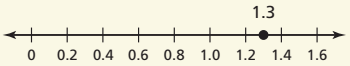

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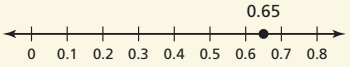

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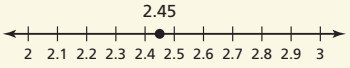


33. B

Mini-Assessment

Copy and complete the statement.

1. -5 ☐ 5 $-5 < 5$

2. -4 ☐ -3 $-4 < -3$

Order the integers from least to greatest.

3. $-8, 4, -1, -2, 1$ $-8, -2, -1, 1, 4$

4. $0, 5, -3, -5, -9$ $-9, -5, -3, 0, 5$

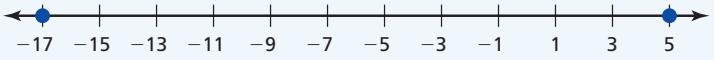
5. In miniature golf, the person with the least score wins. Jeremy has a score of -5 and Mike has a score of -6. Who is the winner? Mike

Taking Math Deeper

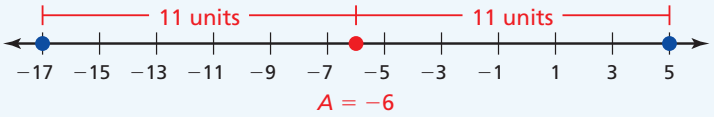
Exercise 26

This exercise is a nice example of a nonstandard problem. It can be solved algebraically or graphically.

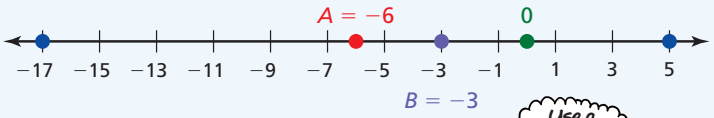
1 Draw a number line that shows -17 and 5.



2 Find the halfway point.



3 Find the point halfway between A and 0.



So, $A = -6$ and $B = -3$.

Project

Research deep sea diving. Find out how deep divers can go and the intervals needed for the ascent back to the surface.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Start the next section

22. REASONING A number is between -2 and -10. What is the least possible integer value of this number? What is the greatest possible integer value of this number?

Tell whether the statement is *always*, *sometimes*, or *never* true. Explain.

23. A positive integer is greater than its opposite.

24. An integer is less than its opposite and greater than 0.

25. ELEVATION The table shows the highest and lowest elevations for five states.

State	Highest Elevation (feet)	Lowest Elevation (feet)
Arkansas	2,753	55
California	14,494	−282
Florida	345	0
Louisiana	535	−8
Tennessee	6,643	178

a. Order the states by their highest elevations, from least to greatest.

b. Order the states by their lowest elevations, from least to greatest.

c. What does the lowest elevation for Florida represent?

26. NUMBER LINE Point A is on a number line halfway between -17 and 5. Point B is halfway between point A and 0. What integer does point B represent?

27. TEMPERATURE Eleven Fahrenheit temperatures are shown on a map during a weather report. When the temperatures are ordered from least to greatest, the middle temperature is below 0°F. Do you know exactly how many of the temperatures are represented by negative numbers? Explain.

28. Puzzle Nine students choose integers. Here are seven of them:

$$5, -8, 10, -1, -12, -20, \text{and } 1.$$

a. When all nine integers are ordered from least to greatest, the middle integer is 1. Describe the integers chosen by the other two students.

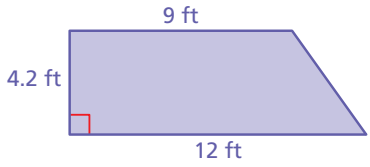
b. When all nine integers are ordered from least to greatest, the middle integer is –3. Describe the integers chosen by the other two students.

Fair Game Review What you learned in previous grades & lessons

Graph the decimal on a number line. *(Skills Review Handbook)*

29. 2.4 **30.** 1.3 **31.** 0.65 **32.** 2.45

33. MULTIPLE CHOICE What is the area of the trapezoid? *(Section 4.3)*



(A) 6.3 ft^2 (B) 44.1 ft^2

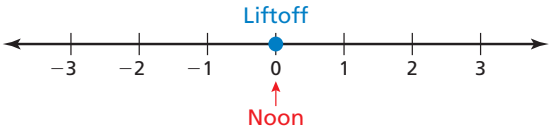
(C) 50.4 ft^2 (D) 88.2 ft^2

6.3 Fractions and Decimals on the Number Line

Essential Question How can you use a number line to compare positive and negative fractions and decimals?

1 ACTIVITY: Locating Fractions on a Number Line

On your time line for "A Day in the Life of an Astronaut" from Activity 3 in Section 6.2, include the following events. Represent each using a fraction or a mixed number.



a. Radio Transmission: 10:30 A.M.

b. Space Walk: 7:30 P.M.

c. Physical Exam: 4:45 A.M.

d. Photograph Taken: 3:15 A.M.

e. Float in the Cabin: 6:20 P.M.

f. Eat Dinner: 8:40 P.M.

Fractions and Decimals

In this lesson, you will

- understand positive and negative numbers and use them to describe real-life situations.
- graph numbers on a number line.

Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP6 Attend to Precision:** Mathematically proficient students are able to communicate precisely to others. In open-ended questions that have more than one possible answer, expect students to be attentive to all students offering solutions. The process students use to arrive at an answer may differ, and students should listen critically to the explanations offered.

Motivate

- Draw a number line on the board from −8 to 8. There should be sufficient space to locate fractions. Under the 0 write *Noon* as is shown in the text.

? Ask students a series of questions about events in their day. Tell students that zero represents noon.

  - "Where is 9 A.M. located?" −3 "How many hours is 9 A.M. from noon?" 3 "What were you doing 3 hours ago?" Answers will vary.
  - "What time is an hour and a half before noon? 10:30 A.M. "To what fractional number does this correspond?" $-1\frac{1}{2}$
  - Continue to ask similar questions about different times.

- **Common Error:** Students may change between decimal hours and hours and minutes. Students incorrectly say 3 hours 15 minutes is 3.15 hours. Remind students that there are 60 minutes in an hour and $3\frac{15}{60}$ is equal to $3\frac{1}{4}$ or 3.25.

- **Common Error:** Students may locate negative mixed numbers incorrectly. Remind students that for negative numbers they start at 0 and move left.

Activity Notes

Activity 1

? Ask questions to check students' understanding.

- "What type of numbers are the A.M. events?" negative numbers; "the P.M. events?" positive numbers
- "Are there any events that are the same distance from liftoff?" yes; physical exam and space walk "Where are the events located on the number line?" $-7\frac{1}{2}$ and $7\frac{1}{2}$ "What do you know about the absolute value of these two numbers?" They are the same.
- "Explain how you wrote 6:20 P.M. as a mixed number and how you located it on the number line." Listen for an explanation that includes $\frac{20}{60} = \frac{1}{3}$ hour and therefore, $\frac{1}{3}$ beyond 6 on the number line.

Common Core State Standards

6.NS.5 . . . use positive and negative numbers to represent quantities in real-world contexts, ...
6.NS.6a Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, ...
6.NS.6c Find and position . . . rational numbers on a horizontal or vertical number line diagram; . . ..
6.NS.7a Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
6.NS.7b Write, interpret, and explain statements of order for rational numbers in real-world contexts.

Previous Learning

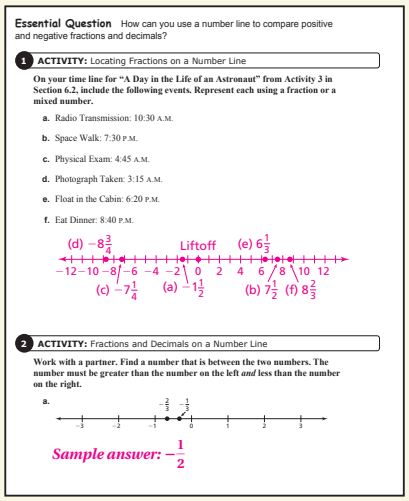
Students should know how to convert between mixed numbers and improper fractions.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

6.3 Record and Practice Journal



Essential Question How can you use a number line to compare positive and negative fractions and decimals?

1 ACTIVITY: Locating Fractions on a Number Line

On your time line for "A Day in the Life of an Astronaut" from Activity 3 in Section 6.2, include the following events. Represent each using a fraction or a mixed number.

a. Radio Transmission: 10:30 A.M.
b. Space Walk: 7:30 P.M.
c. Physical Exam: 4:45 A.M.
d. Photograph Taken: 3:15 A.M.
e. Float in the Cabin: 6:20 P.M.
f. Eat Dinner: 8:40 P.M.

(d) $-8\frac{3}{4}$ Liftoff (e) $6\frac{1}{3}$
−12 −10 −8 −6 −4 −2 0 2 4 6 8 10 12
(c) $-7\frac{1}{4}$ (a) $-1\frac{1}{2}$ (b) $7\frac{1}{2}$ (f) $8\frac{2}{3}$

2 ACTIVITY: Fractions and Decimals on a Number Line

Work with a partner. Find a number that is between the two numbers. The number must be greater than the number on the left *and* less than the number on the right.

a.

Sample answer: $-\frac{1}{2}$

Differentiated Instruction

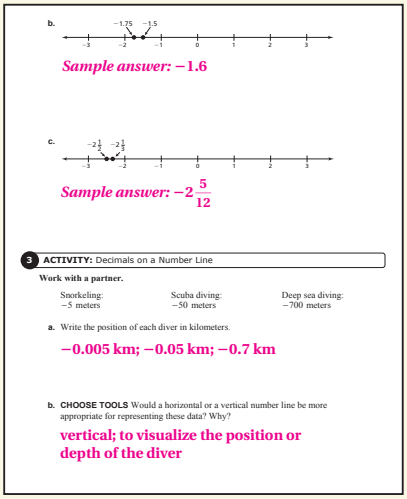
Vocabulary

Write a table of opposites on the board. Encourage students to add to the list.

Word	Opposite
little	big
forward	backward

Ask the English learners to write the words in their native language in another column and to share them with the class. Explain to the class that in mathematics, every nonzero number has an opposite. Every pair of opposites consists of a positive number and a negative number. Ask students to name some pairs of opposite numbers. Write opposite numbers in the table and the words that represent them.

6.3 Record and Practice Journal



b.

Sample answer: -1.6

c.

Sample answer: $-2\frac{5}{12}$

3 ACTIVITY: Decimals on a Number Line

Work with a partner.

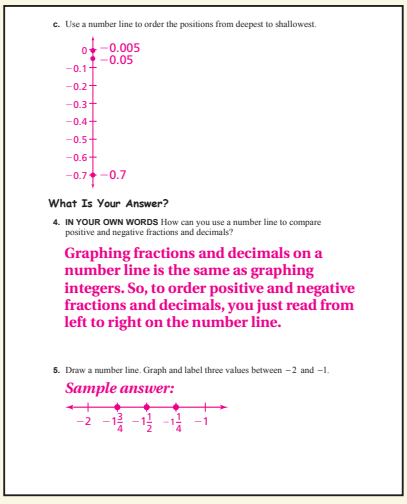
Snorkeling: -5 meters; Scuba diving: -50 meters; Deep sea diving: -700 meters

a. Write the position of each diver in kilometers.

-0.005 km; -0.05 km; -0.7 km

b. **CHOOSE TOOLS** Would a horizontal or a vertical number line be more appropriate for representing these data? Why?

vertical; to visualize the position or depth of the diver



c. Use a number line to order the positions from deepest to shallowest.

0, -0.005, -0.05, -0.1, -0.2, -0.3, -0.4, -0.5, -0.6, -0.7, -0.7

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use a number line to compare positive and negative fractions and decimals?

Graphing fractions and decimals on a number line is the same as graphing integers. So, to order positive and negative fractions and decimals, you just read from left to right on the number line.

5. Draw a number line. Graph and label three values between -2 and -1.

Sample answer:

-2, $-1\frac{3}{4}$, $-1\frac{1}{2}$, $-1\frac{1}{4}$, -1

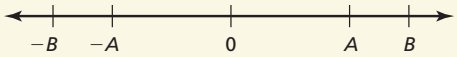
Laurie's Notes

Activity 2

Prepare students for this activity by asking a few questions related to the activity.

- "Which is greater, $1\frac{1}{2}$ or $1\frac{2}{3}$? How do you know?" Students might reason that using a common denominator you have $1\frac{3}{6}$ and $1\frac{4}{6}$.
- "Can you name a number between $1\frac{3}{6}$ and $1\frac{4}{6}$? Explain." Listen for equivalent fractions, probably $1\frac{6}{12}$ and $1\frac{8}{12}$. One fraction between these is $1\frac{7}{12}$.

- Answers will vary for these problems so it is important to give several students the opportunity to share their answers and their thinking.
- **Common Error:** When locating negative fractions, students often move left to right instead of right to left. For example, students incorrectly locate $-1\frac{3}{4}$ closer to -1 than -2. Help students remember to start at -1 and then move $\frac{3}{4}$ of the way towards -2.
- **Big Idea:** If two numbers A and B are positive and $A < B$, then $-B < -A$. Discuss this idea on a number line using numbers from the three problems just completed.



Activity 3

- **MP6:** Mathematically proficient students are careful in specifying units of measure.
- Review meters and kilometers with students. Try to relate the units to common lengths first, such as, the classroom is 6 meters wide; the distance from school to [insert a location] is 6 kilometers. Ask, "How many meters in 6 kilometers?" 6000 "How many kilometers in 6 meters?" 0.006
- Caution students to read carefully. The information is given in meters and the position is to be written in kilometers.

"How should the number line be labeled?" -1 to 0 (in kilometers) will include all three positions.

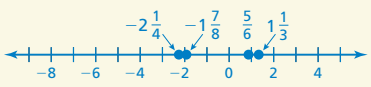
- The vertical number line model is a good visual for the context of this problem.

What Is Your Answer?

- If time permits, have several students share their answers to Question 5.

Closure

- Draw a number line and locate the following fractions: $-2\frac{1}{4}$, $1\frac{1}{3}$, $-1\frac{7}{8}$, $\frac{5}{6}$



2 ACTIVITY: Fractions and Decimals on a Number Line

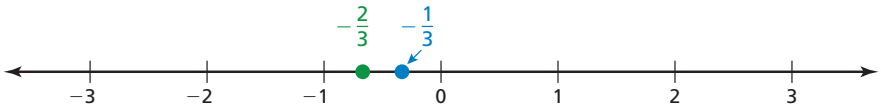
Math Practice

Make a Plan

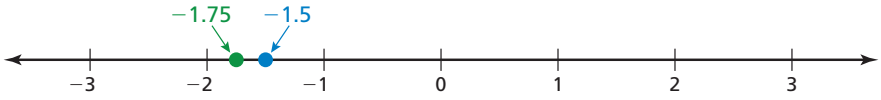
How can you find a number between two given numbers?

Work with a partner. Find a number that is between the two numbers. The number must be greater than the green number *and* less than the blue number.

a.



b.



c.

3 ACTIVITY: Decimals on a Number Line

Work with a partner.

Snorkeling: −5 meters

Scuba diving: −50 meters

Deep-sea diving: −700 meters

a. Write the position of each diver in kilometers.

b. **CHOOSE TOOLS** Would a horizontal or a vertical number line be more appropriate for representing these data? Why?

c. Use a number line to order the positions from deepest to shallowest.

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use a number line to compare positive and negative fractions and decimals?

5. Draw a number line. Graph and label three values between −2 and −1.

Practice Use what you learned about fractions and decimals on a number line to complete Exercises 4 and 5 on page 264.

6.3 Lesson

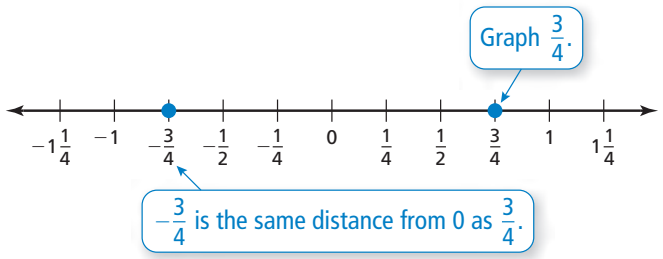
Check It Out
Lesson Tutorials
BigIdeasMath.com

In Section 6.1, you learned that integers can be negative. Fractions and decimals can also be negative.

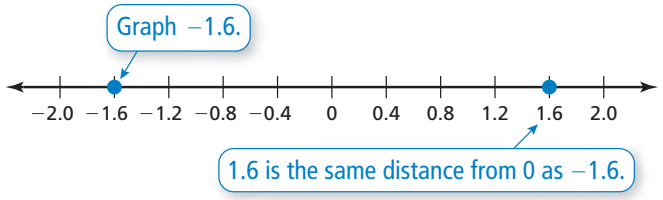
EXAMPLE 1 Graphing Negative Fractions and Decimals

Graph each number and its opposite.

a. $\frac{3}{4}$



b. -1.6



On Your Own

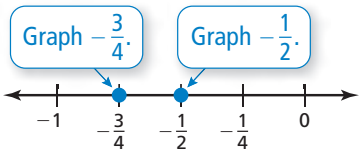
Now You're Ready
Exercises 6–9

Graph the number and its opposite.

1. $2\frac{1}{2}$ **2.** $-\frac{4}{5}$ **3.** -3.5 **4.** 5.25

EXAMPLE 2 Comparing Fractions and Mixed Numbers

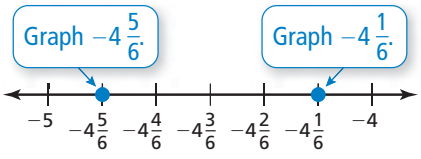
a. Compare $-\frac{1}{2}$ and $-\frac{3}{4}$.



$-\frac{1}{2}$ is to the right of $-\frac{3}{4}$.

$\therefore$ So, $-\frac{1}{2} > -\frac{3}{4}$.

b. Compare $-4\frac{5}{6}$ and $-4\frac{1}{6}$.



$-4\frac{5}{6}$ is to the left of $-4\frac{1}{6}$.

$\therefore$ So, $-4\frac{5}{6} < -4\frac{1}{6}$.

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students developed an understanding of how to locate and compare positive and negative fractions and decimals on a number line. (MP6)
- **Today:** Students will use a number line to compare rational numbers.

Motivate

- Place a piece of masking tape on the floor for a number line.
- Have volunteers hold index cards with 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, or 1 written on them and stand at the correct places on the number line.
- Say, "Reading left to right, $0 < \frac{1}{4} < \frac{1}{2} < \frac{3}{4} < 1$."
- Use a marker to write a negative sign in front of all the numbers except 0.
- ? "Are the numbers still in the correct order as we look at the number line? Explain." No; 0 and -1 need to swap and $-\frac{1}{4}$ and $-\frac{3}{4}$ need to swap.

Discuss

- **Big Idea:** The big idea for students to remember is that the farther to the right a number is on the number line, the greater it is. Likewise, the farther to the left a number is on the number line, the less it is.

Lesson Notes

Example 1

- ? "In part (a), how far is each number from 0?" $\frac{3}{4}$ unit
- ? "In part (b), how far is each number from 0?" 1.6 units
- **MP6 Attend to Precision:** Students may incorrectly believe that the opposite of a number is negative. If the original number is negative, its opposite is positive. Vocabulary: opposite doesn't mean negative.

On Your Own

- **Neighbor Check:** Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.

Example 2

- In part (a), remind students that for negative numbers, you work from right to left: 0, $-\frac{1}{4}$, $-\frac{1}{2}$, $-\frac{3}{4}$, -1.
- In part (b), remind students that for negative numbers, you work from right to left: -4, $-4\frac{1}{6}$, $-4\frac{2}{6}$, and so on.
- ? "Which number is farther from 0, $-4\frac{5}{6}$ or $-4\frac{1}{6}$?" $-4\frac{5}{6}$

Goal Today's lesson is comparing rational numbers on a number line.

Technology for the Teacher

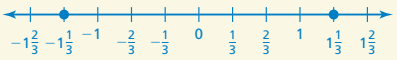
Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

Graph each number and its opposite.

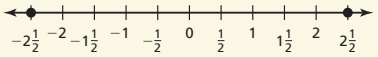
a. $1\frac{1}{3}$



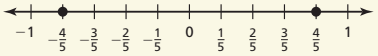
b. -0.7

On Your Own

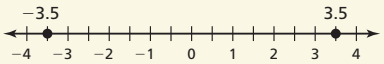
1.



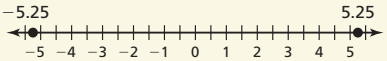
2.



3.



4.



Extra Example 2

a. Compare $-\frac{5}{8}$ and $-\frac{3}{8}$.

$-\frac{5}{8} < -\frac{3}{8}$

b. Compare $-2\frac{2}{3}$ and $-3\frac{1}{3}$.

$-2\frac{2}{3} > -3\frac{1}{3}$

Extra Example 3

Compare 0.3 and -1.2. $0.3 > -1.2$

Extra Example 4

Which is the least, $-1\frac{3}{4}$, $-1\frac{9}{16}$, or $-1\frac{5}{8}$?

$-1\frac{3}{4}$

On Your Own

5. $-\frac{4}{7} < -\frac{1}{7}$
6. $-1\frac{2}{3} > -1\frac{5}{6}$
7. $-0.5 < 0.3$
8. A temperature change of $-3\frac{2}{5}$°F per minute is the farthest to the left on the number line. So, the temperature dropped fastest on this day.

English Language Learners

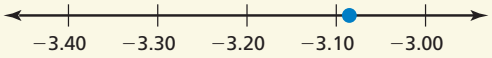
Vocabulary

Let English language learners know that the pronunciation of rational numbers that end in –th such as fourths and fifths is sometimes difficult for native English speakers.

Laurie's Notes

Example 3

- **MP6:** Use correct language in reading the decimals: negative 3 and 8 hundredths, and negative 3 and 8 tenths.
- **Teaching Tip:** To help students compare decimals with different place values, write trailing zeros as shown:



It is not obvious to all students that -3.1 is equivalent to -3.10, or in words, negative 3 and 1 tenth is equivalent to negative 3 and 10 hundredths. The trailing zeros help students recognize where -3.08 is in relation to -3.1 (or -3.10).

Example 4

- Even though the numbers can be compared without graphing on the number line, it is good reinforcement of estimating the location of the numbers.

? "Refer to the diagram. Between what two tick marks is $-\frac{5}{8}$ located? $-2\frac{1}{5}$? $-3\frac{1}{10}$?" $-\frac{3}{4}$ and $-\frac{1}{2}$; $-2\frac{1}{4}$ and -2; $-3\frac{1}{4}$ and -3

- Explain that even though $-3\frac{1}{10}$ is the least of the three numbers, it has the greatest absolute value and therefore represents the greatest drop in temperature.
- Absolute value has not been defined formally, yet students intuitively understand that because $3\frac{1}{10} > 2\frac{1}{5} > \frac{5}{8}$, when you have the opposite of each of these numbers, $-3\frac{1}{10} < -2\frac{1}{5} < -\frac{5}{8}$.

On Your Own

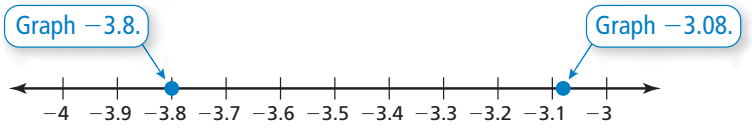
- **Think-Pair-Share:** Students should read each question independently and then work in pairs to answer the questions. When they have answered the questions, the pair should compare their answers with another group and discuss any discrepancies.

Closure

- **Exit Ticket:** Which is greater, $-1\frac{2}{5}$ or $-1\frac{3}{4}$? Explain. $-1\frac{2}{5}$; It is farther to the right on the number line.

EXAMPLE 3 Comparing Decimals

Compare −3.08 and −3.8.

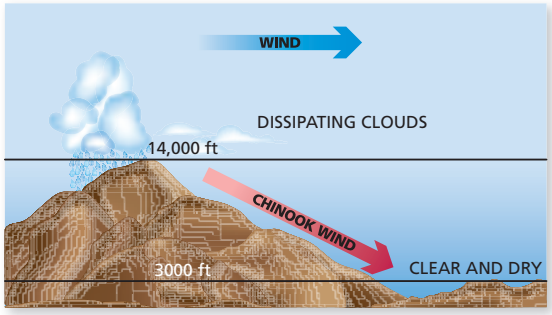


−3.08 is to the right of −3.8.

∴ So, −3.08 > −3.8.

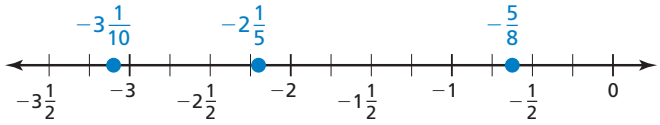
EXAMPLE 4 Real-Life Application

A *Chinook wind* is a warm mountain wind that can cause rapid temperature changes. The table shows three of the greatest temperature drops ever recorded after a Chinook wind occurred. On which date did the temperature drop the fastest? Explain.



Date	Temperature Change
January 10, 1911	$-3\frac{1}{10}$°F per minute
November 10, 1911	$-\frac{5}{8}$°F per minute
January 22, 1943	$-2\frac{1}{5}$°F per minute

Graph the numbers on a number line.



$-3\frac{1}{10}$ is farthest to the left.

∴ So, the temperature dropped the fastest on January 10, 1911.

On Your Own

Now You're Ready
Exercises 10–18 and 20–23

Copy and complete the statement using < or >.

5. $-\frac{4}{7}$ ☐ $-\frac{1}{7}$ **6.** $-1\frac{2}{3}$ ☐ $-1\frac{5}{6}$ **7.** -0.5 ☐ 0.3

8. WHAT IF? In Example 4, a temperature change of $-3\frac{2}{5}$°F per minute is recorded. How does this temperature change compare with the other temperature changes? Explain.

6.3 Exercises

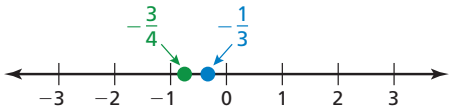
Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **NUMBER SENSE** Which statement is *not* true?
   - **a.** On a number line, $-2\frac{1}{6}$ is to the left of $-2\frac{2}{3}$.
   - **b.** $-2\frac{2}{3}$ is less than $-2\frac{1}{6}$.
   - **c.** $-2\frac{1}{6}$ is greater than $-2\frac{2}{3}$.
   - **d.** On a number line, $-2\frac{2}{3}$ is to the left of $-2\frac{1}{6}$.
2. **NUMBER SENSE** Is a negative decimal *always*, *sometimes*, or *never* equal to a positive decimal? Explain.
3. **NUMBER SENSE** On a number line, is -2.06 or -2.6 farther to the left?

Practice and Problem Solving

Find a fraction or mixed number that is between the two numbers.

4.


5.


Graph the number and its opposite.

1 **6.** $\frac{2}{3}$ **7.** $-2\frac{1}{4}$ **8.** -3.8 **9.** 2.15

Copy and complete the statement using < or >.

2 3 **10.** $-3\frac{1}{3}$ ☐ $-3\frac{2}{3}$ **11.** $-\frac{1}{2}$ ☐ $-\frac{1}{6}$ **12.** $-\frac{3}{4}$ ☐ $\frac{5}{8}$

13. $-2\frac{2}{3}$ ☐ $-2\frac{1}{2}$ **14.** $-1\frac{5}{6}$ ☐ $-1\frac{3}{4}$ **15.** -4.6 ☐ -4.8

16. -0.12 ☐ -0.05 **17.** 2.41 ☐ -3.16 **18.** -3.524 ☐ -3.542

19. SAND DOLLARS In rough water, a small sand dollar burrows $-\frac{1}{2}$ centimeter into the sand. A larger sand dollar burrows $-1\frac{1}{4}$ centimeters into the sand. Which sand dollar burrowed farther?

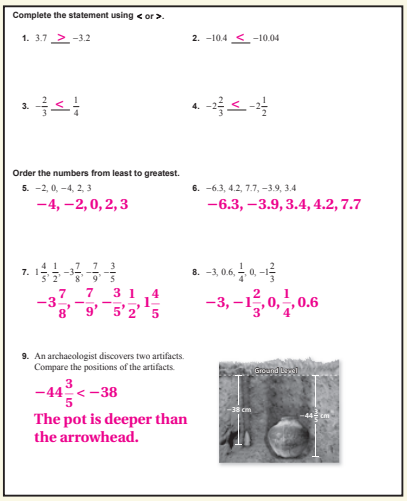
Assignment Guide and Homework Check

Level	Day 1 Activity Assignment	Day 2 Lesson Assignment	Homework Check
Basic	4, 5, 28–32	1–3, 7–23 odd, 24	7, 11, 15, 19, 21
Average	4, 5, 28–32	1–3, 8–16 even, 17, 19, 21–25	8, 12, 16, 19, 22
Advanced	4, 5, 28–32	1–3, 8, 14–27	8, 14, 18, 19, 22

Common Errors

- **Exercises 10–18, 20–23** Students may ignore the sign of the number and order the numbers incorrectly. Encourage them to use a number line to determine which number is farthest to the right.
- **Exercises 11–14, 20, 21** Students may place the fractions in the wrong order. Encourage them to find a common denominator before ordering.

6.3 Record and Practice Journal



Complete the statement using < or >.

1. $3.7 > -3.2$
2. $-10.4 < -10.04$
3. $-\frac{2}{3} < \frac{1}{4}$
4. $-2\frac{2}{3} < -2\frac{1}{2}$

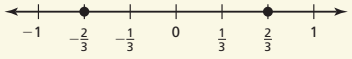
Order the numbers from least to greatest.

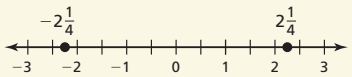
5. $-2, 0, -4, 2, 3$
   $-4, -2, 0, 2, 3$
6. $-6.3, 4.2, 7.7, -3.9, 3.4$
   $-6.3, -3.9, 3.4, 4.2, 7.7$
7. $1\frac{4}{5}, \frac{1}{2}, -3\frac{7}{8}, -\frac{7}{9}, -\frac{3}{5}$
   $-3\frac{7}{8}, -\frac{7}{9}, -\frac{3}{5}, \frac{1}{2}, 1\frac{4}{5}$
8. $-3, 0.6, \frac{1}{4}, 0, -1\frac{2}{3}$
   $-3, -1\frac{2}{3}, 0, \frac{1}{4}, 0.6$
9. An archaeologist discovers two artifacts. Compare the positions of the artifacts.
   $-44\frac{3}{5} < -38$
   The pot is deeper than the arrowhead.

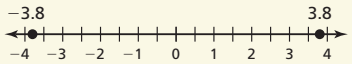
Vocabulary and Concept Check

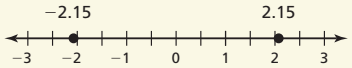
1. a
2. never; A negative decimal is to the left of 0 on a number line and a positive decimal is to the right of 0 on a number line.
3. -2.6

Practice and Problem Solving

4. *Sample answer:* $-\frac{1}{2}$
5. *Sample answer:* $-2\frac{1}{4}$
6. 

7. 

8. 

9. 

10. $>$
11. $<$
12. $<$
13. $<$
14. $<$
15. $>$
16. $<$
17. $>$
18. $>$
19. the larger sand dollar
20. $-3, -2\frac{1}{2}, -2\frac{2}{5}, -2\frac{3}{10}, -2$
21. $-1, -\frac{3}{4}, -\frac{5}{8}, -\frac{1}{20}, 0$
22. $-2, -1.8, -1.75, 0, 1.3$
23. $-5, -4.9, -4.35, -4.3, -4$
24. Sirius

Practice and Problem Solving

25. See *Taking Math Deeper.*

26. **a.** 0

b. Strings 1, 4, and 6

c. String 6

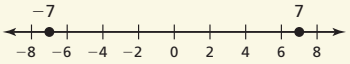
d. String 1

e. String 6

27. 1, 2, and any integer less than −3

Fair Game Review

28.



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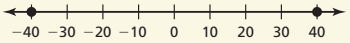

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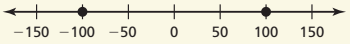
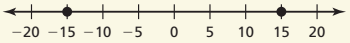


31.



32. D

Mini-Assessment

Which number is greater?

1. 2.3, −1.4 2.3

2. $-\frac{4}{5}, \frac{6}{7}$ $\frac{6}{7}$

3. $-1\frac{1}{2}, -1\frac{1}{3}$ $-1\frac{1}{3}$

4. −0.06, −0.10 −0.06

5. In a pond, a school of bass is $-2\frac{1}{3}$ feet from the surface of the pond. A school of trout is $-2\frac{3}{4}$ feet from the surface of the pond. Which school of fish is farther from the surface of the pond? school of trout

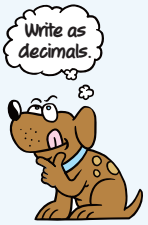
Taking Math Deeper

Exercise 25

It would be difficult to compare all of the water levels in fraction form. (The least common denominator is 300.) So, for this problem it is probably better to rewrite each of the water levels in decimal form.

1 Rewrite the given facts.

Sunday	−0.12
Monday	−0.35
Tuesday	−0.54
Wednesday	−0.65
Thursday	−0.64
Friday	−0.53
Saturday	−0.333 . . .



2 Is the level lower or higher?

Sunday	−0.12	
Monday	−0.35	Lower
Tuesday	−0.54	Lower
Wednesday	−0.65	Lower
Thursday	−0.64	Higher
Friday	−0.53	Higher
Saturday	−0.333. . .	Higher

3 On which days is the level higher than the day before? On which days is it lower?

Higher on: Thursday, Friday, Saturday
Lower on: Monday, Tuesday, Wednesday

Project

Research the changing of the tides. What makes them change? How frequently do they change? Where in the world do they have the most dramatic changes in the tides?

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Start the next section

Order the numbers from least to greatest.

20. $-2\frac{3}{10}, -2\frac{2}{5}, -2, -2\frac{1}{2}, -3$

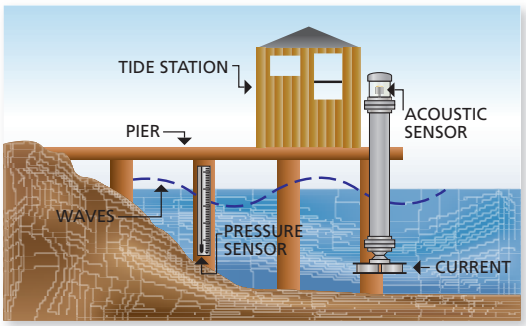
21. $-\frac{1}{20}, -\frac{5}{8}, 0, -1, -\frac{3}{4}$

22. $1.3, -2, -1.8, 0, -1.75$

23. $-4, -4.35, -4.9, -5, -4.3$

24. STARS The *apparent magnitude* of a star measures how bright the star appears as seen from Earth. The brighter the star, the lesser the number. Which star is the brightest?

Star	Alpha Centauri	Antares	Canopus	Deneb	Sirius
Apparent Magnitude	−0.27	0.96	−0.72	1.25	−1.46



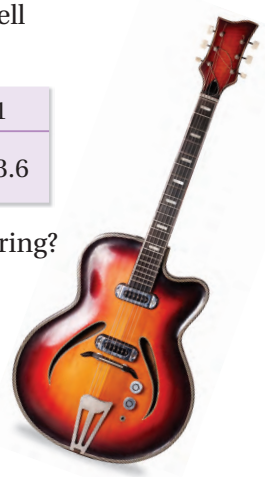
25. LOW TIDE The daily water level is recorded for seven straight days at a tide station on the Big Marco River in Florida. On which days is the water level higher than on the previous day? On which days is it lower?

Day	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
Water Level of the Day (feet)	$-\frac{3}{25}$	$-\frac{7}{20}$	$-\frac{27}{50}$	$-\frac{13}{20}$	$-\frac{16}{25}$	$-\frac{53}{100}$	$-\frac{1}{3}$

26. PROBLEM SOLVING A guitar tuner allows you to tune a guitar string to its correct pitch. The units on a tuner are measured in *cents*. The units tell you how far the string tone is above or below the correct pitch.

STEM VIDEO

Guitar String	6	5	4	3	2	1
Number of Cents Away from the Correct Pitch	−0.3	1.6	−2.3	2.8	2.4	−3.6



a. What number on the tuner represents a correctly tuned guitar string?

b. Which strings have a pitch below the correct pitch?

c. Which string has a pitch closest to its correct pitch?

d. Which string has a pitch farthest from its correct pitch?

e. The tuner is rated to be accurate to within 0.5 cent of the true pitch. Which string could possibly be correct?

27. Number Sense What integer values of x make the statement $-\frac{3}{x} < -\frac{x}{3}$ true?

Fair Game Review What you learned in previous grades & lessons

Graph the integer and its opposite. *(Section 6.1)*

28. -7 **29.** 40 **30.** 100 **31.** -15

32. MULTIPLE CHOICE You pay \$48 for 8 pounds of chicken. Which is an equivalent rate? *(Section 5.3)*

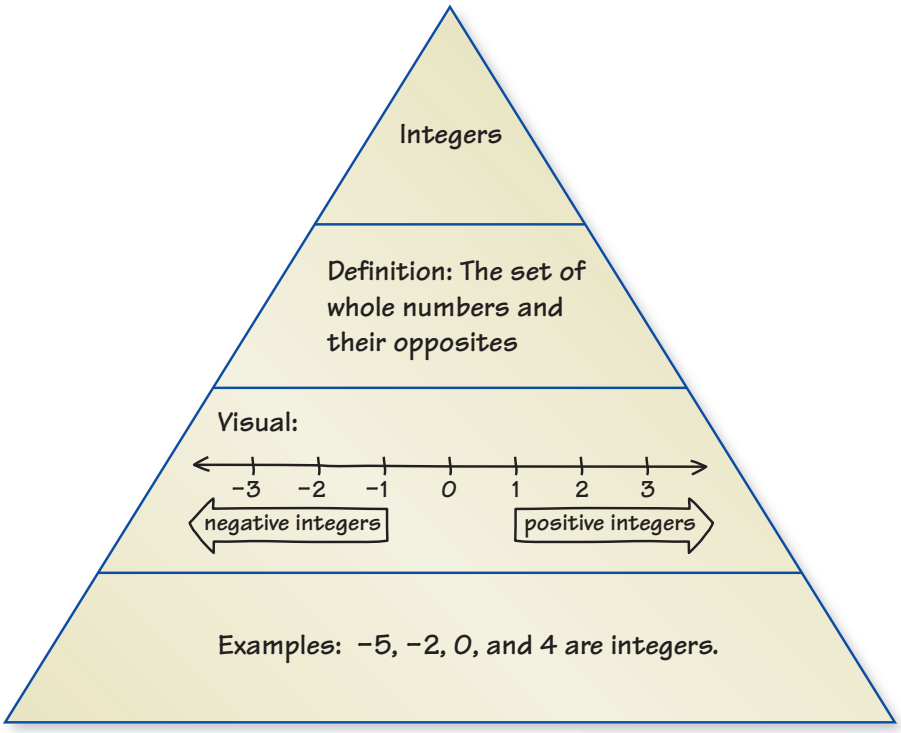
(A) \$44 for 4 pounds (B) \$28 for 4 pounds

(C) \$15 for 3 pounds (D) \$30 for 5 pounds

6 Study Help

Check It Out
Graphic Organizer
BigIdeasMath.com

You can use a **summary triangle** to explain a concept. Here is an example of a summary triangle for integers.



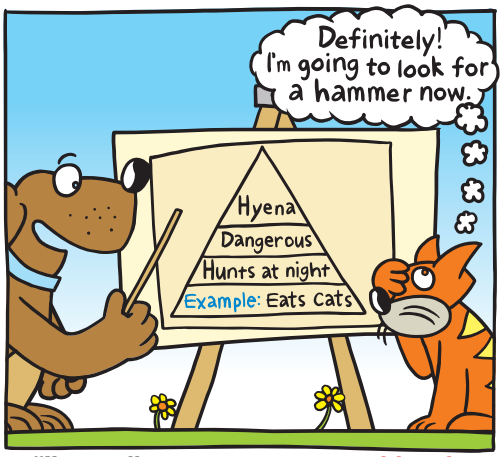
On Your Own

Make summary triangles to help you study these topics.

1. positive integers
2. negative integers
3. opposites

After you complete this chapter, make summary triangles for the following topics.

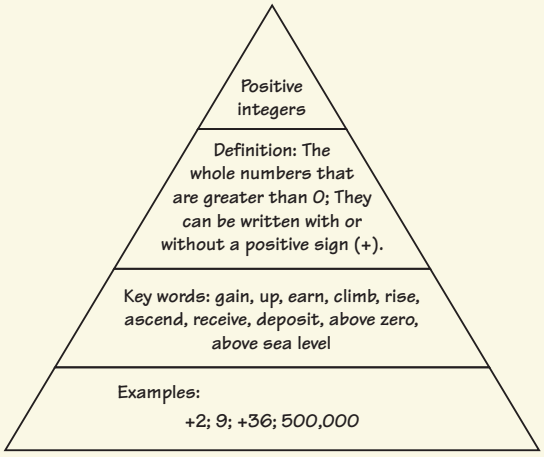
4. absolute value
5. coordinate plane
6. origin
7. quadrants



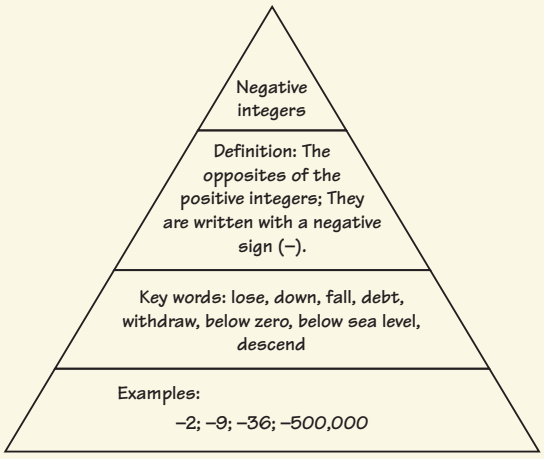
"I'm posting my new summary triangle on my daily blog. Do you think it will get me more hits?"

Sample Answers

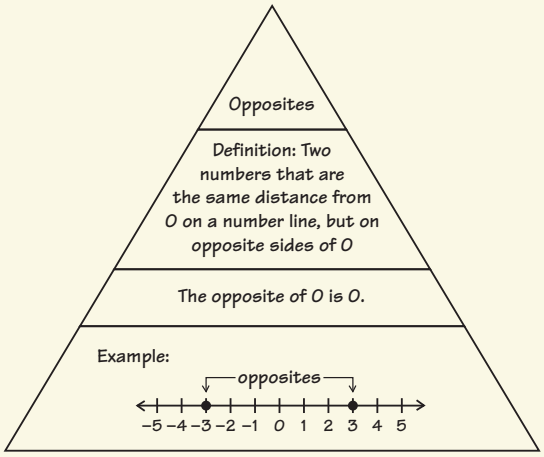
1.



2.



3.



List of Organizers

Available at *BigIdeasMath.com*

Comparison Chart
Concept Circle
Definition (Idea) and Example Chart
Example and Non-Example Chart
Formula Triangle
Four Square
Information Frame
Information Wheel
Notetaking Organizer
Process Diagram
Summary Triangle
Word Magnet
Y Chart

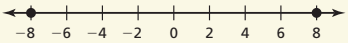
About this Organizer

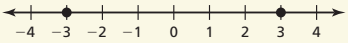
A **Summary Triangle** can be used to explain a concept. Typically, the summary triangle is divided into 3 or 4 parts. In the top part, students write the concept being explained. In the middle part(s), students write any procedure, explanation, description, definition, theorem, and/or formula(s). In the bottom part, students write an example to illustrate the concept. A summary triangle can be used as an assessment tool, in which blanks are left for students to complete. Also, students can place their summary triangles on note cards to use as a quick study reference.

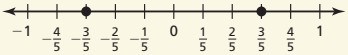
Technology for the Teacher

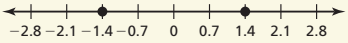
Editable Graphic Organizer

Answers

1. 2
2. -15
3. 

4. 

5. $<$
6. $>$
7. $-5, -2, -1, 0, 3$
8. $-8, -6, 3, 5, 7$
9. 

10. 

11. $<$
12. $>$
13. -195
14. Neptune; Its temperature lies farthest to the left on a number line.
15. $-0.45, -0.42, 0.26, 0.37$

Alternative Quiz Ideas

100% Quiz	Math Log
Error Notebook	Notebook Quiz
Group Quiz	Partner Quiz
Homework Quiz	Pass the Paper

100% Quiz

This is a quiz where students are given the answers and then they have to explain and justify each answer.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Lesson Tutorials *BigIdeasMath.com*	Resources by Chapter • Enrichment and Extension • Technology Connection Game Closet at *BigIdeasMath.com* Start the next section

Technology for the Teacher

Online Assessment
Assessment Book
ExamView® Assessment Suite

6.1–6.3 Quiz

Check It Out
Progress Check
BigIdeasMath.com

Write a positive or negative integer that represents the situation. *(Section 6.1)*

1. The price of a stock goes up $2.

2. You descend 15 feet.

Graph the integer and its opposite. *(Section 6.1)*

3. 8

4. −3

Copy and complete the statement using < or >. *(Section 6.2)*

5. −5 ☐ 0

6. −7 ☐ −9

Order the integers from least to greatest. *(Section 6.2)*

7. 3, −2, 0, −1, −5

8. −6, 5, 3, −8, 7

Graph the number and its opposite. *(Section 6.3)*

9. $\frac{3}{5}$

10. −1.4

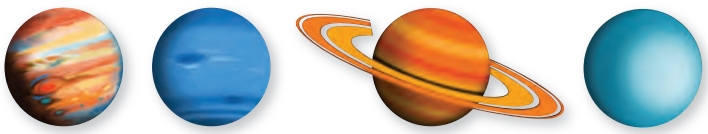
Copy and complete the statement using < or >. *(Section 6.3)*

11. $-2\frac{2}{5}$ ☐ $-2\frac{1}{4}$

12. −3.28 ☐ −3.72

13. ROLLER COASTER At the top of a roller coaster hill, you are 210 feet above ground. At the bottom of the hill, you are 15 feet above ground. Write an integer that represents the change in height from the top to the bottom. *(Section 6.1)*

14. PLANETS The table shows the average surface temperatures of four planets. Which planet is the coldest? Explain. *(Section 6.2)*



Planet	Jupiter	Neptune	Saturn	Uranus
Temperature (°C)	−150	−220	−180	−214

15. STOCK The table shows the changes in the value of a stock over several days. Order the numbers from least to greatest. *(Section 6.3)*

Day	Change (dollars)
1	−0.42
2	0.26
3	−0.45
4	0.37

6.4 Absolute Value

Essential Question How can you describe how far an object is from sea level?

1 ACTIVITY: Sea Level

Work with a partner. Write an integer that represents the elevation of each object. How far is each object from sea level? Explain your reasoning.

a. Boeing 747

b. Seaplane

c. Bald eagle

d. Leatherback turtle

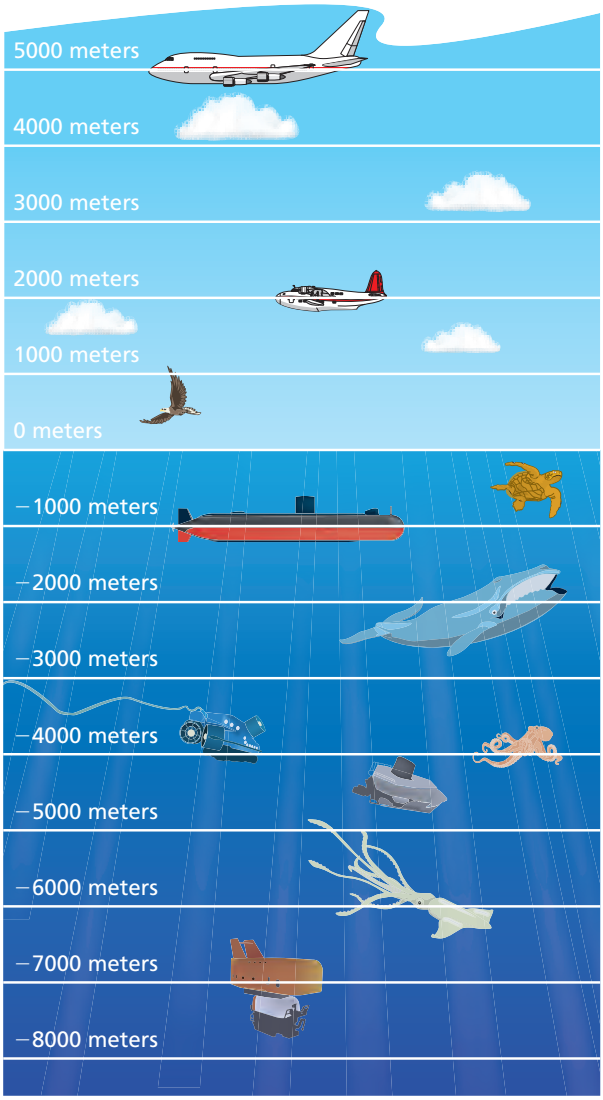
e. U.S.S. Dolphin

f. Whale

g. Jason Jr.

h. Alvin

i. Kaiko



Absolute Value

In this lesson, you will

- find the absolute value of numbers.
- use absolute value to compare numbers in real-life situations.

Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP6 Attend to Precision:** Absolute value is introduced in this section. Students may incorrectly say, "Absolute value just makes the number positive." Be sure that students correctly refer to the absolute value of a number as the distance the number is from 0.

Motivate

- **FYI:** Share information with students about NASA aquanauts.
  - Aquanauts live in an undersea habitat to simulate conditions in space. The extensive training, being submerged in one of the world's most hostile environments, is designed to prepare humans and technology for the rigors of life in a vacuum. Aquanauts say living under the sea is the best way to train for space and to study the effects of isolated missions.
  - Aquanaut crews live for days or weeks at the bottom of the ocean. Aquarius, an underwater ocean laboratory, descends about 60 feet to the sea floor. The aquanauts assist scientists with research into coral health and conduct excursions that mimic spacewalks. These excursions are critical to the International Space Station.

Activity Notes

Activity 1

? "What does 500 feet below sea level mean?" The surface of the water is sea level. So, 500 feet below sea level means 500 feet below the surface of the water.

- Remind students that if distances are below sea level, the numbers will be negative.
- After students have finished, discuss their explanations.
- You could create a vertical model, using masking tape (placed on the wall) or string (attached to the ceiling), and index cards with the names of the objects written on the cards. Deciding on an appropriate scale is a good exercise for students. Have a student place the cards in approximately the correct positions and then ask general questions.

? "Which objects are closest to sea level?" bald eagle and leatherback turtle "Which objects are closest together?" leatherback turtle and U.S.S. Dolphin "As objects descend into the water, how do their elevations change?" Listen for language that suggests that the numbers are increasing, but they are negative.

Common Core State Standards

6.NS.7c Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.

6.NS.7d Distinguish comparisons of absolute value from statements about order.

Previous Learning

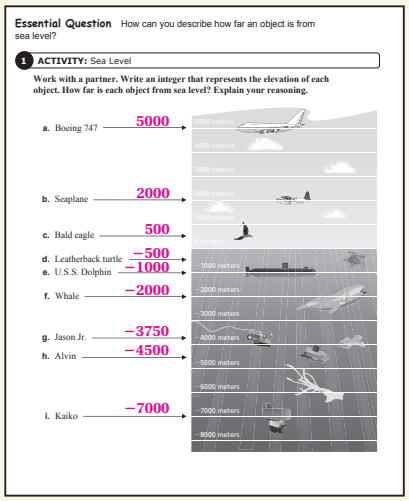
Students should know how to compare and order integers.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

6.4 Record and Practice Journal



Essential Question How can you describe how far an object is from sea level?

1 ACTIVITY: Sea Level

Work with a partner. Write an integer that represents the elevation of each object. How far is each object from sea level? Explain your reasoning.

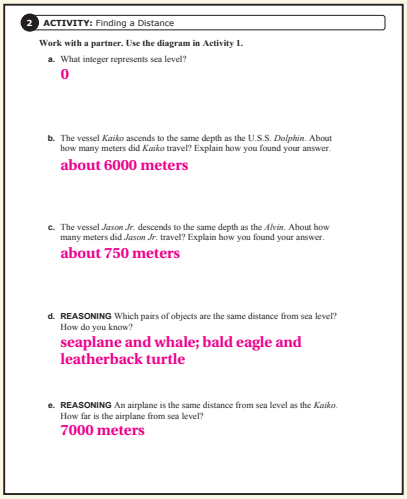
a. Boeing 747 — 5000
b. Seaplane — 2000
c. Bald eagle — 500
d. Leatherback turtle — −500
e. U.S.S. Dolphin — −1000
f. Whale — −2000
g. Jason Jr. — −3750
h. Alvin — −4500
i. Kaiko — −7000

English Language Learners

Vocabulary

In Activity 1, make sure that students understand that the words on the left of the figure are the names of the objects, and that they are determining the elevation of each.

6.4 Record and Practice Journal



2 ACTIVITY: Finding a Distance

Work with a partner. Use the diagram in Activity 1.

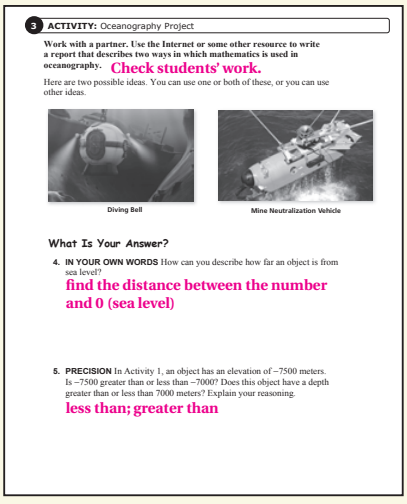
a. What integer represents sea level?
0

b. The vessel *Kaiko* ascends to the same depth as the U.S.S. *Dolphin*. About how many meters did *Kaiko* travel? Explain how you found your answer.
about 6000 meters

c. The vessel *Jason Jr.* descends to the same depth as the *Alvin*. About how many meters did *Jason Jr.* travel? Explain how you found your answer.
about 750 meters

d. **REASONING** Which pairs of objects are the same distance from sea level? How do you know?
seaplane and whale; bald eagle and leatherback turtle

e. **REASONING** An airplane is the same distance from sea level as the *Kaiko*. How far is the airplane from sea level?
7000 meters



3 ACTIVITY: Oceanography Project

Work with a partner. Use the Internet or some other resource to write a report that describes two ways in which mathematics is used in oceanography. **Check students' work.**

Here are two possible ideas. You can use one or both of these, or you can use other ideas.

Diving Bell

Mine Neutralization Vehicle

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you describe how far an object is from sea level?
find the distance between the number and 0 (sea level)

5. **PRECISION** In Activity 1, an object has an elevation of −7500 meters. Is −7500 greater than or less than −7000? Does this object have a depth greater than or less than 7000 meters? Explain your reasoning.
less than; greater than

Laurie's Notes

Activity 2

- Students have a vertical number line to use as a model in answering the questions about how far a vessel traveled.
- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Students will have different ways in which they explain how far a vessel traveled. Elicit different methods from students. Knowing that their answer is correct, students are more apt to share their reasoning with classmates.
- **Extension:** You could tell students the end location and how far it moved, and then ask where the vessel started. Note, there are two answers—one above and one below the ending position.

Activity 3

- Ask questions to determine students' knowledge of oceanography. The main connection you want them to make to mathematics is the use of negative numbers to represent depths below sea level.
- Explain that oceanography is the study of Earth's oceans. Oceanographers study everything about the ocean, including marine animals, currents and waves, and the ocean floor.
- Discuss the photos shown.
- Students will need access to a library or the Internet to find information on oceanography.

What Is Your Answer?

- **Think-Pair-Share:** Students should read the question independently and then work in pairs to answer the question. When they have answered the question, the pair should compare their answer with another group and discuss any discrepancies.

Closure

- **Exit Ticket:** Fill in the blank with < or >. Explain your answer.

-15 ___>___ -20

2 ACTIVITY: Finding a Distance

Work with a partner. Use the diagram in Activity 1.

a. What integer represents sea level?

b. The vessel *Kaiko* ascends to the same depth as the U.S.S. *Dolphin*. About how many meters did *Kaiko* travel? Explain how you found your answer.

c. The vessel *Jason Jr.* descends to the same depth as the *Alvin*. About how many meters did *Jason Jr.* travel? Explain how you found your answer.

d. **REASONING** Which pairs of objects are the same distance from sea level? How do you know?

e. **REASONING** An airplane is the same distance from sea level as the *Kaiko*. How far is the airplane from sea level?

3 ACTIVITY: Oceanography Project

Work with a partner. Use the Internet or some other resource to write a report that describes two ways in which mathematics is used in oceanography.

Here are two possible ideas. You can use one or both of these, or you can use other ideas.

Math Practice

Use Technology to Explore

How can you find more information on oceanography? What information is useful to your report?

Diving Bell

Mine Neutralization Vehicle

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you describe how far an object is from sea level?

5. **PRECISION** In Activity 1, an object has an elevation of −7500 meters. Is −7500 greater than or less than −7000? Does this object have a depth greater than or less than 7000 meters? Explain your reasoning.

Practice Use what you learned about elevation and sea level to complete Exercises 4–6 on page 272.

6.4 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

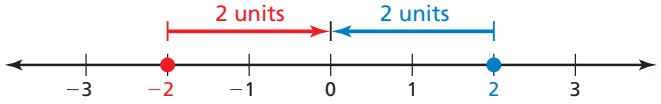
Key Vocabulary
absolute value, *p. 270*

Key Idea

Absolute Value

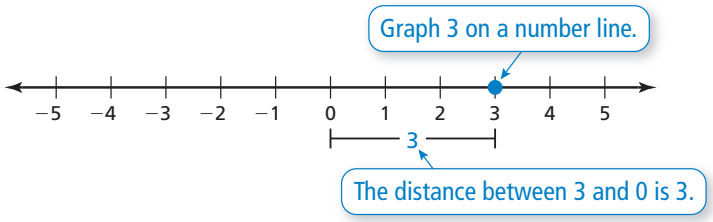
Words The **absolute value** of a number is the distance between the number and 0 on a number line. The absolute value of a number a is written as $|a|$.

Numbers $|-2| = 2$ $|2| = 2$



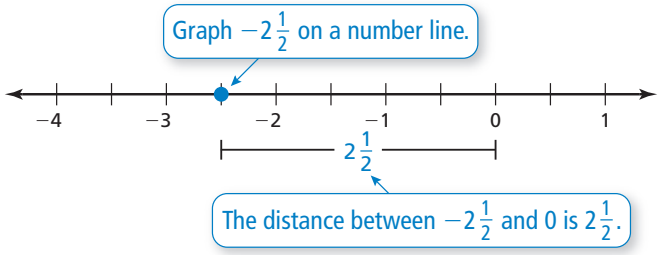
EXAMPLE 1 Finding Absolute Value

a. Find the absolute value of 3.



∴ So, $|3| = 3$.

b. Find the absolute value of $-2\frac{1}{2}$.



∴ So, $\left|-2\frac{1}{2}\right| = 2\frac{1}{2}$.

On Your Own

Now You're Ready
Exercises 7–14

Find the absolute value.

1. $|8|$
2. $|-6|$
3. $|0|$
4. $\left|\frac{1}{4}\right|$
5. $\left|-7\frac{1}{3}\right|$
6. $|-12.9|$

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students developed an understanding of absolute value. (MP3, MP6)
- **Today:** Students will find absolute values of rational numbers.

Motivate

- Have two students (A and B) stand at the front of the room holding a piece of string between them. You hold a piece of paper with the number 0 written on it. Share that the distance between the two students is 10 units.

? Position yourself at various points along the line so that A and B have various values, such as A = 3, 5, or 8 and B = −7, −5, or −2, respectively.

- "If A is _____ [3, 5, 8], what number does B represent?" −7, −5, −2
- "Who is closer to me (meaning 0)?" A; Neither; B
- "How far away from me is each person?" Answers will vary depending on the numbers chosen.

? Without the string, ask students "What number or numbers are 4 units from 0?" 4 and −4

- Discuss positive and negative directions from 0 on a number line. For instance, it is possible to be 5 units from 0 in either direction.

Lesson Notes

Key Idea

- Write the Key Idea on the board.
- $|a|$ is read, "the absolute value of a."
- Stress that the **absolute value** of a number is the *distance* from 0. Distance is a positive number or 0. Although directed distances are addressed in science, the concept of distance here is that it is a positive number or 0.

Example 1

- Work through each example.
- **Common Misconception:** Students may say, "Absolute values are always positive." This statement can cause problems when students find $|0|$, which is 0.

? After part (b) ask, "What is the absolute value of −2.4?" 2.4

Words of Wisdom

- Point out that when you write the notation for the absolute value, it means *take the absolute value of the number inside the symbols.*

On Your Own

- **Common Error:** Students may incorrectly find that the absolute value of a negative number is negative.

Goal Today's lesson is finding the **absolute value** of a number.

Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

a. Find the absolute value of −10. 10
b. Find the absolute value of 8.3. 8.3

On Your Own

1. 8
2. 6
3. 0
4. $\frac{1}{4}$
5. $7\frac{1}{3}$
6. 12.9

Extra Example 2

Compare $|-10|$ and 4. $|-10| > 4$

On Your Own

7. >

8. <

9. <

10. =

Extra Example 3

The table shows the elevations of a bird and a fish. Is the bird or the fish closer to sea level? fish

Animal	Elevation (ft)
Bird	10
Fish	−9

On Your Own

11. the seagull; Because $|56| < |-65|$, the seagull is closer to 0 feet, or sea level.

Differentiated Instruction

Auditory

If students have difficulty remembering how to find the absolute value of a number, give them a mnemonic device: *I am absolutely positive that absolute value is positive.* Make sure, however, that students understand the one exception, that the absolute value of zero is zero.

Laurie's Notes

Example 2

- Explain that one way to compare numbers is to graph the numbers on a number line. The farther to the right a number is, the greater it is. The farther to the left a number is, the less it is.

? "What is $|-5|$?" 5

- **Common Error:** Students may graph $|-5|$ at −5. Explain that you find the absolute value of the number first and then graph it.
- Note that it would also be correct to conclude that $|-5| > 2$.

On Your Own

- Draw number lines on the board and ask volunteers to solve the problems by graphing.
- **MP6 Attend to Precision:** To practice re-writing inequalities, for each of the problems you could have students write two equivalent inequalities. For Problem 7, they would write $|-4| > -2$ and $-2 < |-4|$.

Example 3

- Discuss the number line. Make sure students observe that, for instance, −22 is below −20. It is not uncommon for students to graph −22 above −20 because 22 is above 20.
- Work through each part of the problem.

On Your Own

- **Think-Pair-Share:** Students should read the question independently and then work in pairs to answer the question. When they have answered the question, the pair should compare their answer with another group and discuss any discrepancies.

Closure

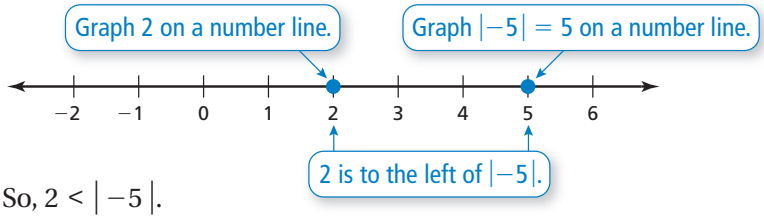
- **Exit Ticket:** Write these numbers in order from least to greatest.

$$|-12|,\ -8,\ -10,\ |6|,\ |-4|$$

$-10, -8, 4, 6, 12$

EXAMPLE 2 Comparing Values

Compare 2 and $|-5|$.



∴ So, $2 < |-5|$.

On Your Own

Now You're Ready
Exercises 17–22

Copy and complete the statement using <, >, or =.

7. $|-4|$ ☐ -2

8. -5 ☐ $|5|$

9. $|9|$ ☐ 10

10. 3.9 ☐ $|-3.9|$

EXAMPLE 3 Real-Life Application

Animal	Elevation (ft)
Shark	−4
Sea lion	5
Seagull	56
Shrimp	−65
Turtle	−22

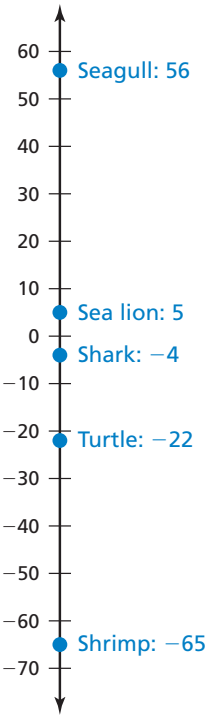
The table shows the elevations of several animals.

a. Which animal is the deepest? Explain.

Graph each elevation.

The lowest elevation represents the animal that is the deepest. The integer that is lowest on the number line is −65.

∴ So, the shrimp is the deepest.



b. Is the shark or the sea lion closer to sea level?

Because sea level is at 0 feet, use absolute values.

Shark: $|-4| = 4$ **Sea lion:** $|5| = 5$

∴ Because 4 is less than 5, the shark is closer to sea level than the sea lion.

On Your Own

11. Is the seagull or the shrimp closer to sea level? Explain your reasoning.

6.4 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** Explain how to find the absolute value of an integer.
2. **REASONING** Which integer is greater, -50 or 25? Which has the greater absolute value? Explain.
3. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

How far is -3 from 0?

What integer is 3 units to the left of 0?

What is the absolute value of -3?

What is the distance between -3 and 0?

Practice and Problem Solving

Use a vertical number line to graph the location of each object. Then tell which object is farther from sea level.

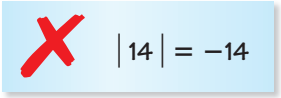
4. Scuba diver: -15 m
   Dolphin: -22 m
5. Seagull: 12 m
   School of fish: -4 m
6. Shark: -40 m
   Flag on a ship: 32 m

Find the absolute value.

1 7. $|-2|$
8. $|23|$
9. $|-8.35|$
10. $\left|\frac{1}{6}\right|$
11. $\left|-3\frac{2}{5}\right|$
12. $|11|$
13. $|14.06|$
14. $|-68|$

15. **REASONING** Write two integers that have an absolute value of 10.

16. **ERROR ANALYSIS** Describe and correct the error in finding the absolute value.



Copy and complete the statement using <, >, or =.

2 17. $6 \square |-8|$
18. $|-3| \square 3$
19. $|-5.5| \square |-3.1|$
20. $\frac{3}{4} \square \left|-\frac{2}{5}\right|$
21. $|-6.8| \square |8.25|$
22. $-12 \square |12|$

23. **CAVES** Three scientists explore a cave. Which scientist is farthest underground?

Scientist A: -48 ft
Scientist B: -62 ft
Scientist C: -53 ft

MATCHING Match the account balance with the debt that it represents. Explain your reasoning.

24. account balance $= -\$25$
25. account balance $< -\$25$
26. account balance $> -\$25$

A. debt $> \$25$
B. debt $= \$25$
C. debt $< \$25$

Assignment Guide and Homework Check

Level	Day 1 Activity Assignment	Day 2 Lesson Assignment	Homework Check
Basic	4–6, 40–44	1–3, 7–15 odd, 16, 17–23 odd	11, 13, 19, 23
Average	4–6, 40–44	1–3, 8–14 even, 15, 16–22 even, 23–31 odd	14, 20, 23, 27
Advanced	4–6, 40–44	1–3, 12–22 even, 26–38 even, 39	14, 20, 28, 32, 34

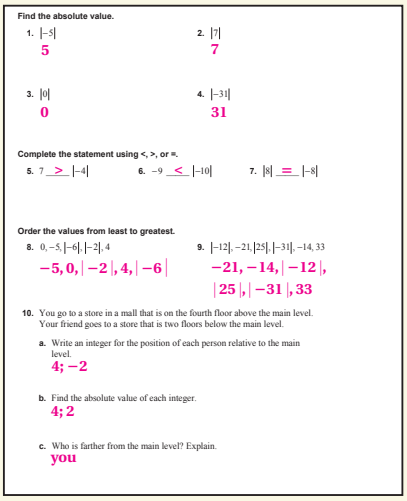
For Your Information

- **Exercise 34** The Kelvin temperature scale does not use a degree symbol.
- **Exercise 38** Spaces and punctuation can be adjusted in a palindrome.

Common Errors

- **Exercises 7–14, 37** Students may think the absolute value of a number is its opposite. For example, they may think $|8| = -8$. Use a number line to remind students that absolute value is a number's distance from zero, so it is always a positive number or zero.
- **Exercises 17–22, 27–30** Students may ignore the absolute value bars when comparing or ordering values. Encourage them to graph the values on a number line, as shown in Example 3, to help them correctly compare or order values.
- **Exercise 33** Students may treat the absolute value bars as parentheses and write $-|-1| = 1$. Remind them of the definition of absolute value, and that they should find the absolute value first.

6.4 Record and Practice Journal



Find the absolute value.

1. $|-5|$ **5**
2. $|7|$ **7**
3. $|0|$ **0**
4. $|-31|$ **31**

Complete the statement using <, >, or =.

5. 7 **>** $|-4|$
6. -9 **<** $|-10|$
7. $|8|$ **=** $|-8|$

Order the values from least to greatest.

8. $0, -5, |-6|, |-2|, 4$ **$-5, 0, |-2|, 4, |-6|$**
9. $|-12|, -21, |25|, |-31|, -14, 33$ **$-21, -14, |-12|, |25|, |-31|, 33$**

10. You go to a store in a mall that is on the fourth floor above the main level. Your friend goes to a store that is two floors below the main level.
    a. Write an integer for the position of each person relative to the main level. **4; −2**
    b. Find the absolute value of each integer. **4; 2**
    c. Who is farther from the main level? Explain. **you**

Vocabulary and Concept Check

1. Find the distance between the number and 0 on a number line.
2. 25; -50; 25 is to the right of -50 on a number line, so $25 > -50$. $|-50|$ is farther from 0 on a number line than $|25|$, so $|-50|$ has the greater absolute value.
3. What integer is 3 units to the left of 0?; -3; 3

Practice and Problem Solving

4–6. See Additional Answers.

7. 2 **8.** 23

9. 8.35 **10.** $\frac{1}{6}$

11. $3\frac{2}{5}$ **12.** 11

13. 14.06 **14.** 68

15. $-10, 10$

16. The absolute value of a number cannot be negative. $|14| = 14$

17. < **18.** =

19. > **20.** >

21. < **22.** <

23. Scientist B

24. B; You owe \$25, so debt = \$25.

25. A; You owe more than \$25, so debt > \$25.

26. C; You owe less than \$25, so debt < \$25.

27. $-2, 0, |-1|, |4|, 5$

28. $-4, -3, |-3|, |-4|, |5|$

29. $-11, 0, |3|, |-6|, 9, 10$

30. $-20, -19, -18, |-18|, |-22|, |30|$

Practice and Problem Solving

31. 0 **32.** −6

33. −1

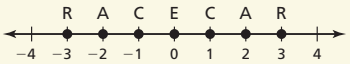
34. See *Taking Math Deeper*.

35. sometimes; If the number is negative then its absolute value is greater, but if the number is positive or zero then it is equal to its absolute value.

36. always; The absolute value is the positive distance from zero on a number line.

37. never; The absolute value of a positive number is the number itself.

38. **a.**



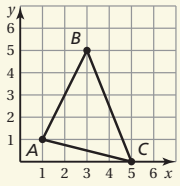
b. racecar; yes

c. *Sample answer:* Madam, I'm Adam.

39. *Sample answer:* $x = -2$, $y = -3$

Fair Game Review

40.



41–44. See Additional Answers.

Mini-Assessment

Find the absolute value.

1. $|-83|$ 83 **2.** $|38|$ 38

Copy and complete the statement using <, >, or =.

3. -2 ☐ $|-2|$ < **4.** 8 ☐ $|-8|$ =

5. The freezing point of airplane fuel is −53°C and the freezing point of candle wax is 55°C. Which is closer to the freezing point of water, 0°C?
the freezing point of airplane fuel

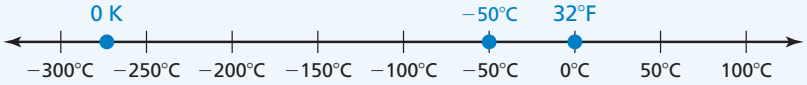
Taking Math Deeper

Exercise 34

1 The Kelvin temperature scale and the Kelvin are named after the British physicist William Thomson, Lord Kelvin (1824–1907). The Kelvin scale is a temperature scale referenced to absolute zero, the absence of all energy. Scientists believe that it is not possible to reach absolute zero. In 2003, researchers at the Massachusetts Institute of Technology produced a gas that was within one-billionth of a degree of absolute zero.

Note that the Kelvin is not referred to as a "degree," and it is not written with a degree symbol.

2 **a.** Use the graph in the exercise to determine that 32°F = 0°C and 0 K = −273°C.



−50°C is closer to 0 K than 32°F.

3 **b.** One thing that the Kelvin scale and absolute values have in common is that neither can have *negative* numbers. For both, the least possible number is 0.

In fact, the Kelvin temperature scale is often called the *absolute temperature scale*.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Start the next section

Order the values from least to greatest.

27. $5,\ 0,\ |-1|,\ |4|,\ -2$

28. $|-3|,\ |5|,\ -3,\ -4,\ |-4|$

29. $10,\ |-6|,\ 9,\ |3|,\ -11,\ 0$

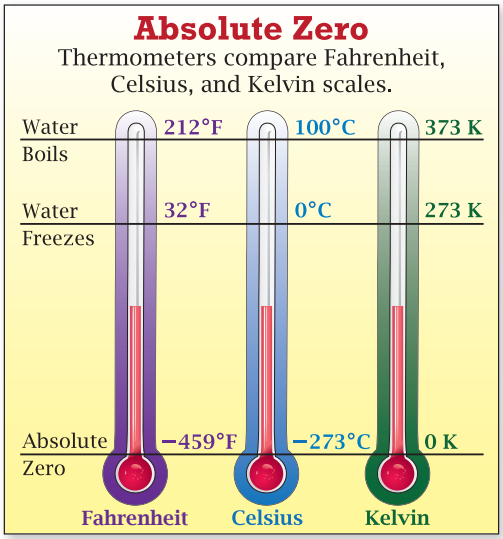
30. $-18,\ |30|,\ -19,\ |-22|,\ -20,\ |-18|$

Simplify the expression.

31. $|0|$

32. $-|6|$

33. $-|-1|$



34. ABSOLUTE ZERO The coldest possible temperature is called *absolute zero*. It is represented by 0 K on the Kelvin temperature scale.

a. Which temperature is closer to 0 K: 32°F or −50°C?

b. What do absolute values and temperatures on the Kelvin scale have in common?

Tell whether the statement is *always*, *sometimes*, or *never* true. Explain.

35. The absolute value of a number is greater than the number.

36. The absolute value of a negative number is positive.

37. The absolute value of a positive number is its opposite.

38. PALINDROME A *palindrome* is a word or sentence that reads the same forward as it does backward.

a. Graph and label the following points on a number line: $A = -2$, $C = -1$, $E = 0$, $R = -3$. Then graph and label the absolute value of each point on the *same* number line.

b. What word do the letters spell? Is this a palindrome?

c. Make up your own palindrome.

39. Critical Thinking Find values of x and y so that $|x| < |y|$ and $x > y$.

Fair Game Review What you learned in previous grades & lessons

Draw the polygon with the given vertices in a coordinate plane. *(Section 4.4)*

40. $A(1, 1), B(3, 5), C(5, 0)$

41. $D(0, 6), E(2, 1), F(6, 3)$

42. $P(2, 1), Q(4, 4), R(8, 4), S(6, 1)$

43. $W(1, 6), X(9, 6), Y(9, 1), Z(4, 1)$

44. MULTIPLE CHOICE Which expression represents "6 less than the product of 4 and a number x"? *(Section 3.2)*

Ⓐ $(6 - 4)x$ Ⓑ $6 - 4x$ Ⓒ $\frac{6}{4x}$ Ⓓ $4x - 6$

6.5 The Coordinate Plane

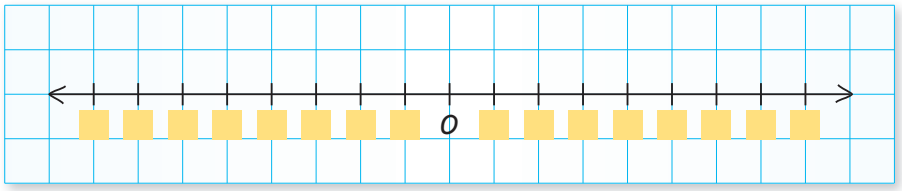
Essential Question How can you graph and locate points that contain negative numbers in a coordinate plane?

You have already graphed points and polygons in one part of the coordinate plane. In Activity 1, you will form the *entire* coordinate plane.

1 ACTIVITY: Forming the Entire Coordinate Plane

Work with a partner.

a. In the middle of a sheet of grid paper, construct a horizontal number line as shown. Label the tick marks. On a different sheet of grid paper, construct and label a similar vertical number line.



b. Cut out the vertical number line and tape it on top of the horizontal number line so that the zeros overlap. Make sure the number lines are perpendicular to one another. How many regions did you form by doing this?

c. **REASONING** What ordered pair represents the point where the number lines intersect? Why do you think this point is called the *origin*? Explain.

2 ACTIVITY: Describing Points in the Coordinate Plane

Work with a partner. Use your perpendicular number lines from Activity 1.

a. Plot and label (3, 2) on your coordinate plane. Shade this region in your coordinate plane. What do you notice about the integers along the number lines that surround (3, 2)?

b. Can you plot a point in your coordinate plane so that it is surrounded by negative numbers on the axes? If so, where is this point? Use a different color to shade this region in your coordinate plane.

c. What do you notice about the integers along the number lines for points in the regions that are not shaded?

d. **STRUCTURE** Describe how you would plot $(-3, -2)$. How is plotting this point similar to plotting (3, 2)? Plot $(-3, -2)$ in your coordinate plane.

e. **REASONING** Where in your coordinate plane do you plot $(2, -4)$? Where do you plot $(-2, 4)$? Explain your reasoning.

Coordinate Plane

In this lesson, you will

- describe the locations of points in the coordinate plane.
- plot points in the coordinate plane given ordered pairs.
- find distances between points in the coordinate plane.

Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP7 Look for and Make Use of Structure:** Mathematically proficient students make use of the patterns when plotting ordered pairs. Students recognize the relationship between plotting (a, b) and $(-a, -b)$.

Motivate

- **Kinesthetic Class Activity:** Create a coordinate grid on your classroom floor or school foyer. Use two strips of masking tape laid perpendicular to one another. Use a thick marker to mark the axes, leaving 1–1.5 feet between each label. Write each of the following points on a piece of paper large enough for the class to see: (3, 2), (−3, 2), (−3, −2), (3, −2)
- Ask four students to volunteer. Have each student start at the origin and face towards the positive *y*-axis. Give the following directions:
  Student 1: Move 3 spaces to the right and forward 2 spaces.
  Student 2: Move 3 spaces to the left and forward 2 spaces.
  Student 3: Move 3 spaces to the left and back 2 spaces.
  Student 4: Move 3 spaces to the right and back 2 spaces.

? "What figure would be formed if you connected each student in order with a piece of string?" a rectangle

Discuss

- Students should recall plotting ordered pairs in the first quadrant. This lesson expands the coordinate plane to include negative coordinates.
- Work with horizontal and vertical number lines should help students extend their understanding to plotting points in all four quadrants.

Activity Notes

Activity 1

- Circulate to be sure that the number lines are labeled correctly. In particular, check for symmetry of opposites about 0.
- Students can use a protractor or the corner of a piece of paper to check that their number lines are perpendicular.

? "Describe the coordinate grid." The number lines represent the axes of the coordinate plane: the horizontal number is the *x*-axis and the vertical number line is the *y*-axis.

Activity 2

- The goal of this activity is for students to recognize the sign of the coordinates in each quadrant, referred to as regions in this activity.

? "What do all of the coordinates of the ordered pairs have in common for the shaded region in part (a)?" all positive "in part (b)?" all negative

- **MP7:** In part (d), students should recognize that plotting $(-3, -2)$ is similar to plotting (3, 2) except the directions are opposite. Instead of 3 units right it is 3 units left. Instead of 2 units up it is 2 units down.

Common Core State Standards

6.NS.6b Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
6.NS.6c Find and position . . . pairs of integers and other rational numbers on a coordinate plane.
6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Previous Learning

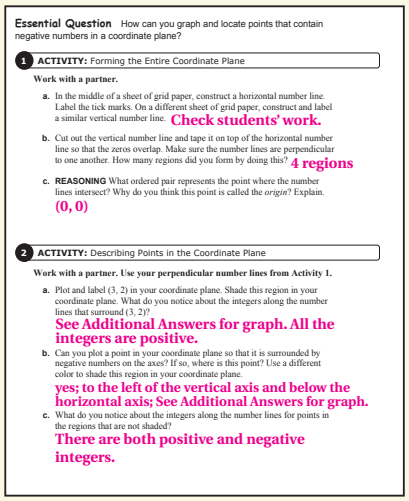
Students should be able to graph integers on a number line.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

6.5 Record and Practice Journal



Essential Question How can you graph and locate points that contain negative numbers in a coordinate plane?

1 ACTIVITY: Forming the Entire Coordinate Plane

Work with a partner.

a. In the middle of a sheet of grid paper, construct a horizontal number line. Label the tick marks. On a different sheet of grid paper, construct and label a similar vertical number line. **Check students' work.**

b. Cut out the vertical number line and tape it on top of the horizontal number line so that the zeros overlap. Make sure the number lines are perpendicular to one another. How many regions did you form by doing this? **4 regions**

c. **REASONING** What ordered pair represents the point where the number lines intersect? Why do you think this point is called the *origin*? Explain. **(0, 0)**

2 ACTIVITY: Describing Points in the Coordinate Plane

Work with a partner. Use your perpendicular number lines from Activity 1.

a. Plot and label (3, 2) in your coordinate plane. Shade this region in your coordinate plane. What do you notice about the integers along the number lines that surround (3, 2)? **See Additional Answers for graph. All the integers are positive.**

b. Can you plot a point in your coordinate plane so that it is surrounded by negative numbers on the axes? If so, where is this point? Use a different color to shade this region in your coordinate plane. **yes; to the left of the vertical axis and below the horizontal axis; See Additional Answers for graph.**

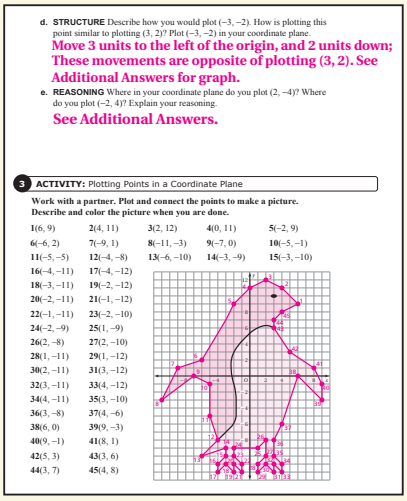
c. What do you notice about the integers along the number lines for points in the regions that are not shaded? **There are both positive and negative integers.**

Differentiated Instruction

Kinesthetic

For kinesthetic learners who have difficulty plotting points in the coordinate plane, suggest they use a finger for tracing. Have students place a finger at the origin and trace left or right along the *x*-axis to the first coordinate, then trace up or down to the second coordinate. Students should also practice writing the coordinates of a plotted point. Guide students with questions such as, "Should you move left or right? How far? Should you move up or down? How far?"

6.5 Record and Practice Journal



d. **STRUCTURE** Describe how you would plot (−3, −2). How is plotting this point similar to plotting (3, 2)? Plot (−3, −2) in your coordinate plane.

Move 3 units to the left of the origin, and 2 units down; These movements are opposite of plotting (3, 2). See Additional Answers for graph.

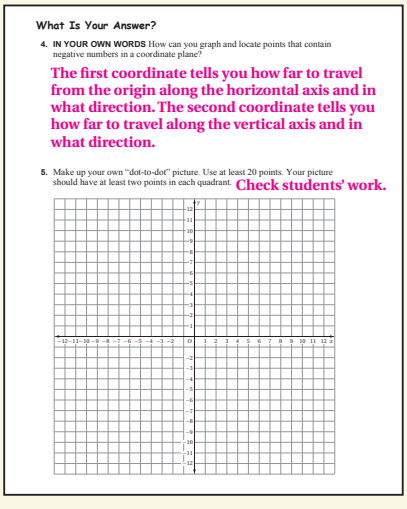
e. **REASONING** Where in your coordinate plane do you plot (2, −4)? Where do you plot (−2, 4)? Explain your reasoning.

See Additional Answers.

3 ACTIVITY: Plotting Points in a Coordinate Plane

Work with a partner. Plot and connect the points to make a picture. Describe and color the picture when you are done.

1(6, 9)	2(4, 11)	3(2, 12)	4(0, 11)	5(−2, 9)
6(−6, 2)	7(−9, 1)	8(−11, −3)	9(−7, 0)	10(−5, −1)
11(−5, −5)	12(−4, −8)	13(−6, −10)	14(−3, −9)	15(−3, −10)
16(−4, −11)	17(−4, −12)			
18(−3, −11)	19(−2, −12)			
20(−2, −11)	21(−1, −12)			
22(−1, −11)	23(−2, −10)			
24(−2, −9)	25(1, −9)			
26(2, −8)	27(2, −10)			
28(1, −11)	29(1, −12)			
30(2, −11)	31(3, −12)			
32(3, −11)	33(4, −12)			
34(4, −11)	35(3, −10)			
36(3, −8)	37(4, −6)			
38(6, 0)	39(9, −3)			
40(9, −1)	41(8, 1)			
42(5, 3)	43(3, 6)			
44(3, 7)	45(4, 8)			



What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you graph and locate points that contain negative numbers in a coordinate plane?

The first coordinate tells you how far to travel from the origin along the horizontal axis and in what direction. The second coordinate tells you how far to travel along the vertical axis and in what direction.

5. Make up your own "dot-to-dot" picture. Use at least 20 points. Your picture should have at least two points in each quadrant. **Check students' work.**

Laurie's Notes

Activity 3

- Stress that the order in which you plot the points is important.
  - The *x*-coordinate is always first. It tells us how far to go horizontally, and in which direction.
  - The *y*-coordinate is always second. It tells us how far to go vertically, and in which direction.
- Leave a summary on the board for students to reference as they work through the example.
- Students should work with partners and check their work.

Words of Wisdom

- Discuss difficulties students had in completing their pictures. Common problems/errors include carelessness, interchanging *x* and *y*, and moving in the opposite direction.
- Another error may occur when students are plotting a point. They may start at the last plotted point, rather than starting at the origin.

What Is Your Answer?

- Students should understand that each point in the coordinate plane has a unique name. The *x*- and *y*-coordinates specify an exact spot, similar to latitude and longitude on maps. Students who finish early could start a dot-to-dot design of their own.

? **Extension:** Think of some real-life situations that use coordinates to locate points. *Sample answers:* maps often use letters and numbers; stadiums give section-row-seat numbers, games (like Battleship) use numbers and letters; some maps use longitude and latitude

Closure

- "Today you learned about plotting points. You did an activity and summary that should help you remember and reinforce this concept."

3 ACTIVITY: Plotting Points in a Coordinate Plane

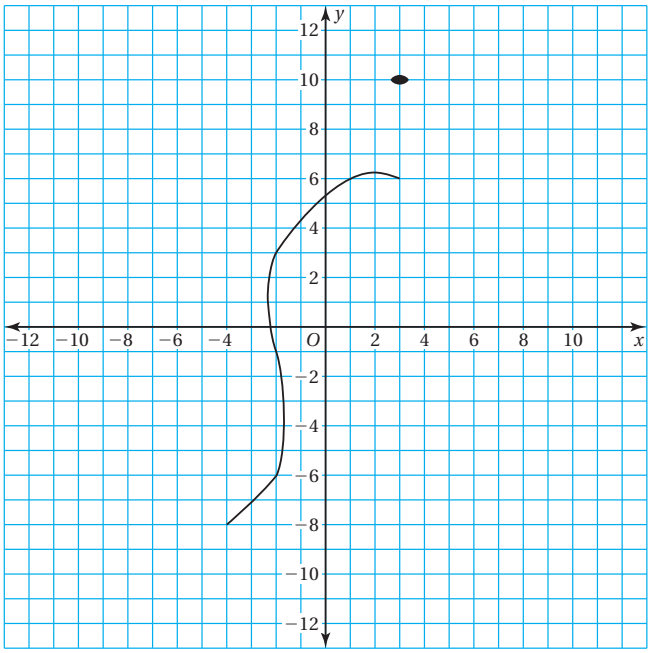
Math Practice

Check Progress

How can you check your progress to make sure you are accurately drawing the picture?

Work with a partner. Plot and connect the points to make a picture. Describe and color the picture when you are done.

1(6, 9)	**2**(4, 11)	**3**(2, 12)	**4**(0, 11)	**5**(−2, 9)
6(−6, 2)	**7**(−9, 1)	**8**(−11, −3)	**9**(−7, 0)	**10**(−5, −1)
11(−5, −5)	**12**(−4, −8)	**13**(−6, −10)	**14**(−3, −9)	**15**(−3, −10)
16(−4, −11)	**17**(−4, −12)	**18**(−3, −11)	**19**(−2, −12)	**20**(−2, −11)
21(−1, −12)	**22**(−1, −11)	**23**(−2, −10)	**24**(−2, −9)	**25**(1, −9)
26(2, −8)	**27**(2, −10)	**28**(1, −11)	**29**(1, −12)	**30**(2, −11)
31(3, −12)	**32**(3, −11)	**33**(4, −12)	**34**(4, −11)	**35**(3, −10)
36(3, −8)	**37**(4, −6)	**38**(6, 0)	**39**(9, −3)	**40**(9, −1)
41(8, 1)	**42**(5, 3)	**43**(3, 6)	**44**(3, 7)	**45**(4, 8)



What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you graph and locate points that contain negative numbers in a coordinate plane?

5. Make up your own "dot-to-dot" picture. Use at least 20 points. Your picture should have at least two points in each region of the coordinate plane.

Practice Use what you learned about the coordinate plane to complete Exercise 4 on page 279.

6.5 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

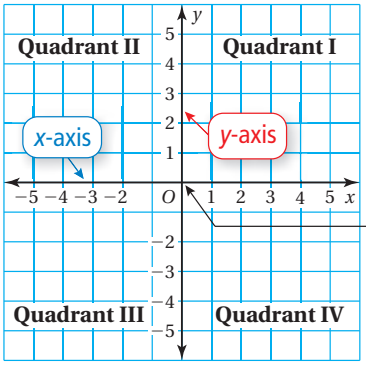
Key Vocabulary
coordinate plane, *p. 276*
origin, *p. 276*
quadrants, *p. 276*

Previously, you plotted points with positive coordinates. Now you will plot points with positive and negative coordinates.

Key Idea

The Coordinate Plane

A **coordinate plane** is formed by the intersection of a horizontal number line and a vertical number line. The number lines intersect at the **origin** and separate the coordinate plane into four regions called **quadrants**.



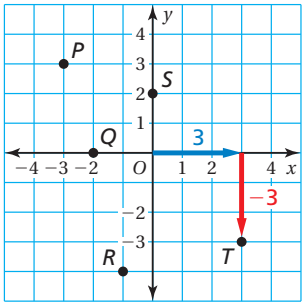
An *ordered pair* is used to locate a point in a coordinate plane.

ordered pair: (4, −2)
x-coordinate
y-coordinate

EXAMPLE 1 Identifying an Ordered Pair

Which ordered pair corresponds to point *T*?

Ⓐ (−3, −3) Ⓑ (−3, 3)
Ⓒ (3, −3) Ⓓ (3, 3)



Point *T* is 3 units to the right of the origin and 3 units down. So, the *x*-coordinate is 3 and the *y*-coordinate is −3.

∴ The ordered pair (3, −3) corresponds to point *T*. The correct answer is Ⓒ.

On Your Own

Now You're Ready
Exercises 5–14

Use the graph in Example 1 to write an ordered pair corresponding to the point.

1. Point *P*
2. Point *Q*
3. Point *R*
4. Point *S*

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students should be comfortable with plotting ordered pairs from the dot-to-dot activity they completed yesterday. (MP7)
- **Today:** The vocabulary and concepts are presented more formally.

Motivate

- Share a fictitious story with your students. "Last night when I was out shopping I got hungry, so I decided to get a snack from the vending machine. I decided that I was going to get crackers. To select them, I had to enter the row (a letter from A to E) and a column (a number from 1 to 6). I entered B-4 instead of C-4 and ended up with popcorn instead of crackers. But, I was hungry, so I still ate it!"

? "How is the vending machine like what you did yesterday?" Students should see that the crackers were in a particular spot associated with the ordered pair (C, 4). When a different ordered pair is entered, you plot a different point. (You get an unexpected item, the popcorn.)

Lesson Notes

Key Idea

- It is important to have a model of the coordinate *grid* versus only a model of scaled axes. The grid is essential in helping the students understand that a point plotted by moving in two directions (horizontal and vertical). You may want to project a coordinate grid, if possible, on the wall or board.
- Use the model of the coordinate grid to identify key vocabulary: coordinate plane, origin, quadrants, *x*-axis, *y*-axis, and ordered pair.
- **Connections:** To help students remember which way is horizontal, hold your arms out horizontally and relate them to the horizon. Origin means "where something starts." If your students have played the game of *four square*, relate the quadrants to the four square court.
- Stress that the ordered pairs (2, 4) and (4, 2) are not the same. The order matters and the ordered pairs are always (*x*, *y*). If it helps, tell students that the order of the coordinates is in alphabetical order, *x* and then *y*.

Example 1

- The colored arrows on the diagram will help students plot in the *x*-direction first, followed by the *y*-direction.

On Your Own

- Ask if there are any questions. Then have students work independently to write the ordered pairs for the remaining points, *P*, *Q*, *R*, and *S*. Ask volunteers to present their answers to the class.

Goal Today's lesson is identifying and plotting ordered pairs in a **coordinate plane**.

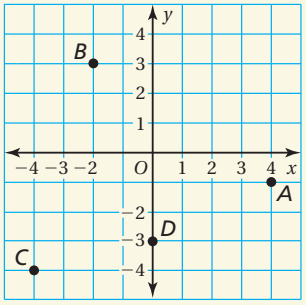
Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

Write the ordered pair represented by point *A*, point *B*, point *C*, and point *D*.



A(4, −1), *B*(−2, 3), *C*(−4, −4), *D*(0, −3)

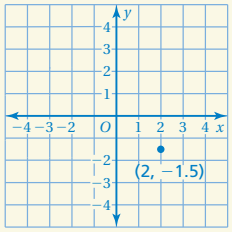
On Your Own

1. (−3, 3) **2.** (−2, 0)

3. (−1, −4) **4.** (0, 2)

Extra Example 2

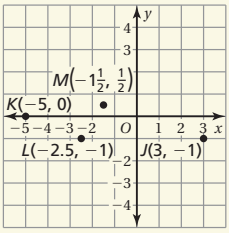
Plot (2, −1.5) in a coordinate plane. Describe the location of the point.



The point is in Quadrant IV.

On Your Own

5–8.



5. Quadrant IV
6. x-axis
7. Quadrant III
8. Quadrant II

Extra Example 3

An interior designer maps out a room design using a coordinate plane in which each unit represents 1 yard. The corners of the room are located at (−3, 2), (5, 2), (5, −3), and (−3, −3). What are the dimensions of the room?

8 yards long and 5 yards wide

On Your Own

9. 1 meter

English Language Learners

Visual

English learners may confuse the words *horizontal* and *vertical*. To help students distinguish between the words, draw a picture of the ocean with a ship on the horizon. Connect the word horizontal to the word horizon.

Laurie's Notes

Example 2

Ask questions about how to position the point in part (a):

- "What is the x-coordinate?" −2 "Which direction do you move from (0, 0)?" Left 2 units
- "What is the y-coordinate?" 3 "Which direction do you move from (0, 0)?" Up 3 units

- **Common Error:** Students often plot points on the axes incorrectly. Make sure the plotted point in part (b) is (0, −3.5)
- Explain to students that ordered pairs are in one of the four quadrants, on an axis, or at the origin. The ordered pair in part (b) is on an axis, not in a quadrant.

On Your Own

- **Common Error:** Students may move the wrong direction, or mix up the x- and y-coordinates and plot the wrong point.

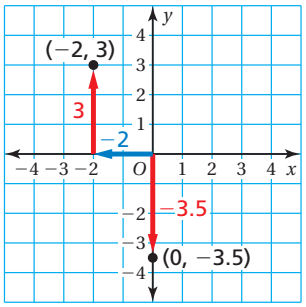
Example 3

- **Connection:** Point out that the absolute values for the length come from the x-coordinates of the points. The absolute values for the width come from the y-coordinates of the points. The absolute values represent the distance from 0 for that coordinate, i.e. the distance from the axis.
- Note that you can also use the points (−4, 1) and (2, 1) to find the length and the points (−4, 5) and (−4, 1) to find the width.
- Explain why you add the absolute values to find the length (points are in different quadrants), but subtract to find the width (points are in the same quadrant).
- Explain to students that even though two of the ordered pairs are in Quadrant II where x-coordinates are negative, the dimensions of the rectangle are positive.

On Your Own

- **Neighbor Check:** Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.

EXAMPLE 2 Plotting Ordered Pairs



Plot (a) (−2, 3) and (b) (0, −3.5) in a coordinate plane. Describe the location of each point.

a. Start at the origin. Move 2 units left and 3 units up. Then plot the point.

∴ The point is in Quadrant II.

b. Start at the origin. Move 3.5 units down. Then plot the point.

∴ The point is on the *y*-axis.

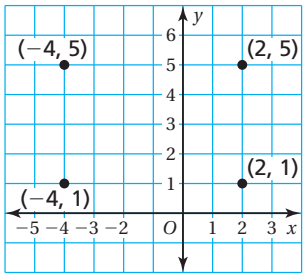
On Your Own

Now You're Ready
Exercises 15–22

Plot the ordered pair in a coordinate plane. Describe the location of the point.

5. (3, −1) **6.** (−5, 0) **7.** (−2.5, −1) **8.** $\left(-1\frac{1}{2}, \frac{1}{2}\right)$

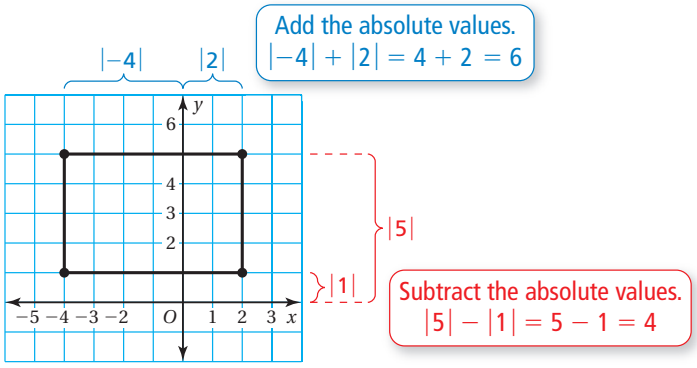
EXAMPLE 3 Finding Distances in the Coordinate Plane



An *archaeologist* divides an area using a coordinate plane in which each unit represents 1 meter. The corners of a secret chamber are shown in the graph. What are the dimensions of the secret chamber?

The length of the chamber is the distance between (−4, 5) and (2, 5). The width of the chamber is the distance between (2, 5) and (2, 1).

You can use absolute values to find the distances between the points.



Reading

An **archaeologist** studies ancient ruins and objects to learn about people and cultures.

∴ The secret chamber is 6 meters long and 4 meters wide.

On Your Own

Now You're Ready
Exercises 25–30

9. In Example 3, the archaeologist finds a gold coin at (−1, 4), a silver coin at (−4, 2), and pottery at (−4, 4). How much closer is the pottery to the silver coin than to the gold coin?

You can use line graphs to display data that is collected over a period of time. Graphing and connecting the ordered pairs can show patterns or trends in the data. This type of line graph is also called a *time series graph*.

EXAMPLE 4 Real-Life Application

A blizzard hits a town at midnight. The table shows the hourly temperatures from midnight to 8:00 A.M.

Hours after Midnight, x	0	1	2	3	4	5	6	7	8
Temperature, y	7°F	5°F	3°F	0°F	−1°F	−4°F	−5°F	−2°F	2°F

a. Display the data in a line graph.

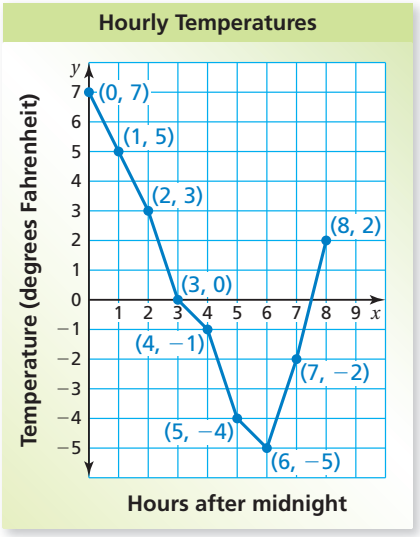
Write the ordered pairs.

(0, 7) (1, 5) (2, 3)

(3, 0) (4, –1) (5, −4)

(6, −5) (7, −2) (8, 2)

Plot and label the ordered pairs. Then connect the ordered pairs with line segments.



b. Make three observations from the graph.

Three possible observations follow:

- The hourly temperatures decrease from midnight to 6:00 A.M.
- The hourly temperatures increase from 6:00 A.M. to 8:00 A.M.
- The greatest decrease in hourly temperatures from one hour to the next is 3°F. This happens twice: from 2:00 A.M. to 3:00 A.M. and from 4:00 A.M. to 5:00 A.M.

Study Tip

The observations given in Example 4(b) are sample answers. You can make many other correct observations.

On Your Own

10. In Example 4, the blizzard hits another town at noon. The table shows the hourly temperatures from noon to 6:00 P.M.

Hours after Noon	0	1	2	3	4	5	6
Temperature	6°F	7°F	5°F	1°F	1°F	0°F	−3°F

a. Display the data in a line graph.

b. Make three observations from the graph.

Laurie's Notes

Example 4

- Ask students to describe the types of graphs and plots with which they are familiar.
- Define a *time series graph.* a line graph that connects ordered pairs to show patterns and trends in data over a period of time
- "What does the x-coordinate represent?" number of hours since midnight
- "What does the y-coordinate represent?" temperature
- Write the data from the table as ordered pairs.
- "What does (5, −4) represent?" At 5 A.M., the temperature was −4°F.
- "The x-coordinates are all positive, and the y-coordinates are positive and negative. What quadrants will the graph be in?" Quadrants I and IV
- Draw and label the axes so that the context is known.
- Plot the ordered pairs. Connect the ordered pairs with line segments. The segments allow the trend in the data to be more obvious.
- **MP3 Construct Viable Arguments and Critique the Reasoning of Others** and **MP6 Attend to Precision:** Ask students to make observations about the graph. Students should offer adequate detail in their observations. For instance, it would not be sufficient to simply say, "The graph goes down then up." Observations should give details and reference the context. An improved observation would be, "The temperatures decreased from midnight to 6 A.M., and then they increased for the next two hours until 8 A.M.
- After you have modeled making an observation that is detailed, give students time to brainstorm with their neighbors about other observations.
- **Extension:** Record additional data for temperatures prior to midnight. Ask students what changes are needed to the graph.

On Your Own

- **Think-Pair-Share:** Students should read the question independently and then work in pairs to answer the question. When they have answered the question, the pair should compare their answer with another group and discuss any discrepancies.

Closure

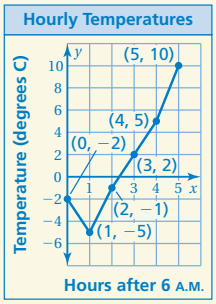
- Graph a rectangle with coordinates in Quadrants III and IV. Name the coordinates.

Extra Example 4

The table shows the hourly temperatures on a winter morning from 6 A.M. to 11 A.M.

Hours after 6 A.M., x	0	1	2
Temperature, y	−2°C	−5°C	−1°C
Hours after 6 A.M., x	3	4	5
Temperature, y	2°C	5°C	10°C

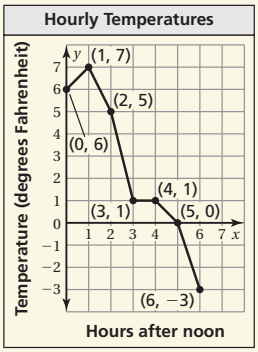
a. Display the data in a line graph.



b. Make three observations from the graph. *Sample answers:* The hourly temperatures decrease from 6 A.M. to 7 A.M.; The hourly temperatures increase from 7 A.M. to 11 A.M.; The greatest increase in temperature from one hour to the next is 5°C from 10 A.M. to 11 A.M.

On Your Own

10. a.



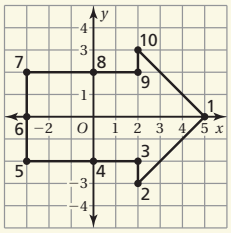
b. *Sample answers:* The greatest hourly temperature occurs at 1 P.M.; The greatest decrease in temperature from one hour to the next is 4°F; Most of the hourly temperatures are less than the previous hourly temperature.

Vocabulary and Concept Check

1. 4
2. *y*-axis
3. (2, −3); (2, −3) is in Quadrant IV. The other three points are in Quadrant II.

Practice and Problem Solving

4.



5. (3, 1)

6. (−3, −2)

7. (−2, 4)

8. (1, 2)

9. (2, −2)

10. (0, −4)

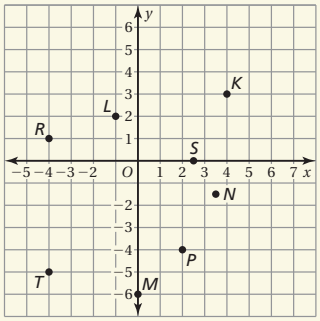
11. (−4, 2)

12. (−4, −4)

13. (4, 0)

14. (4, −4)

15–22.



15. Quadrant I

16. Quadrant II

17. *y*-axis

18. Quadrant IV

19. Quadrant IV

20. Quadrant II

21. *x*-axis

22. Quadrant III

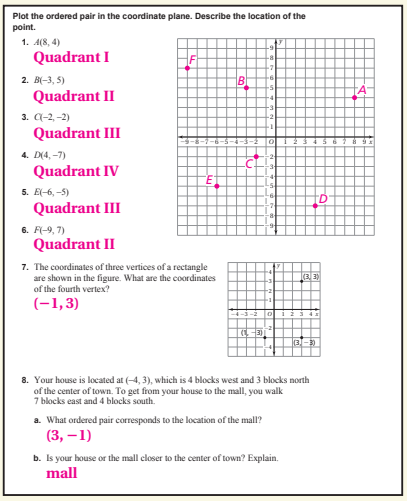
Assignment Guide and Homework Check

Level	Day 1 Activity Assignment	Day 2 Lesson Assignment	Homework Check
Basic	4, 55–59	1–3, 5–13 odd, 19–25 odd, 26, 31	7, 13, 21, 25
Average	4, 55–59	1–3, 11, 13, 19–23 odd, 28–36	13, 21, 28, 32, 35
Advanced	4, 55–59	1–3, 22–54 even	22, 24, 28, 34, 36

Common Errors

- **Exercises 5–14** Students may write the *y*-coordinate first and then the *x*-coordinate for the ordered pair. Tell them that *x* comes before *y* in the alphabet, so the *x*-coordinate must come before the *y*-coordinate in the ordered pair.
- **Exercises 15–22** Students may plot the *x*-coordinate vertically instead of horizontally and the *y*-coordinate horizontally instead of vertically. Remind them that *x* is horizontal and *y* is vertical.

6.5 Record and Practice Journal



Plot the ordered pair in the coordinate plane. Describe the location of the point.

1. *A*(8, 4) Quadrant I
2. *B*(−3, 5) Quadrant II
3. *C*(−2, −2) Quadrant III
4. *D*(4, −7) Quadrant IV
5. *E*(−6, −5) Quadrant III
6. *F*(−9, 7) Quadrant II
7. The coordinates of three vertices of a rectangle are shown in the figure. What are the coordinates of the fourth vertex? (−1, 3)
8. Your house is located at (−4, 3), which is 4 blocks west and 3 blocks north of the center of town. To get from your house to the mall, you walk 7 blocks east and 4 blocks south.
   a. What ordered pair corresponds to the location of the mall? (3, −1)
   b. Is your house or the mall closer to the center of town? Explain. mall

6.5 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** How many quadrants are in a coordinate plane?
2. **VOCABULARY** Is the point $(0, -7)$ on the x-axis or the y-axis?
3. **WHICH ONE DOESN'T BELONG?** Which point does *not* belong with the other three? Explain your reasoning.

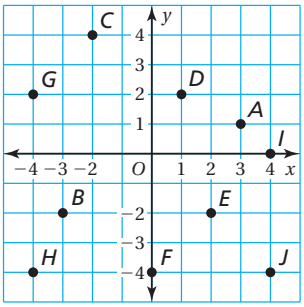
$(-2, 1)$ $(-4, 5)$ $(2, -3)$ $(-1, 3)$

Practice and Problem Solving

4. Plot and connect the points to make a picture.

1 $(5, 0)$ **2** $(2, -3)$ **3** $(2, -2)$ **4** $(0, -2)$ **5** $(-3, -2)$
6 $(-3, 0)$ **7** $(-3, 2)$ **8** $(0, 2)$ **9** $(2, 2)$ **10** $(2, 3)$

Write an ordered pair corresponding to the point.



1 **5.** Point A **6.** Point B
7. Point C **8.** Point D
9. Point E **10.** Point F
11. Point G **12.** Point H
13. Point I **14.** Point J

Plot the ordered pair in a coordinate plane. Describe the location of the point.

2 **15.** $K(4, 3)$ **16.** $L(-1, 2)$ **17.** $M(0, -6)$ **18.** $N(3.5, -1.5)$
19. $P(2, -4)$ **20.** $R(-4, 1)$ **21.** $S\left(2\frac{1}{2}, 0\right)$ **22.** $T(-4, -5)$

ERROR ANALYSIS **Describe and correct the error in the solution.**

23.

To plot (4, 5), start at (0, 0) and move 5 units right and 4 units up.

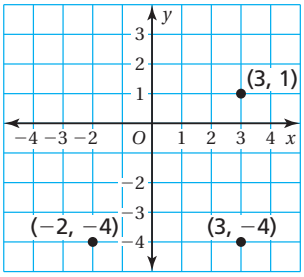
24.

To plot (−6, 3), start at (0, 0) and move 6 units right and 3 units down.

Plot the points and find the distance between the points.

3 **25.** $(2, -3), (6, -3)$ **26.** $(4, 2), (4, -1)$
27. $(-1, 1), (-1, 7)$ **28.** $(-5, -2), (4, -2)$
29. $(-3, 4), (5, 4)$ **30.** $(-2, -4), (-2, 1)$

31. REASONING The coordinates of three vertices of a square are shown in the figure. What are the coordinates of the fourth vertex?



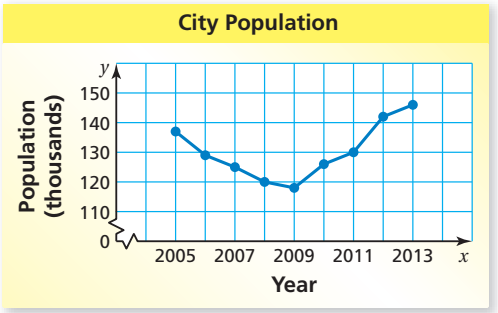
Draw the figure with the given vertices in a coordinate plane. Find the perimeter and the area of the figure.

32. $D(1, 1), E(1, -2), F(-2, -2), G(-2, 1)$

33. $P(-2, 3), Q(5, 3), R(5, -1), S(-2, -1)$

34. $W(-3, 2), X(2, 2), Y(2, -7), Z(-3, -7)$

35. POPULATION The line graph shows the population of a city from 2005 to 2013.



a. Estimate the population of the city in 2012.

b. Between which two years did the population increase the most?

c. Estimate the total change in population from 2005 to 2013.

36. MODELING The table shows the total miles run through 18 weeks for a marathon training program.

Week	1	2	3	4	5	6	7	8	9
Total Miles	22	46	72	96	124	151	181	211	244

Week	10	11	12	13	14	15	16	17	18
Total Miles	279	317	357	397	437	473	506	530	544

a. Create a table for the distance run during each week of training.

b. Display the data from part (a) in a line graph.

c. Make three observations from the graph.

d. Explain the pattern shown in the graph.

37. PROFITS The table shows the profits of a company from 2007 to 2013.

Years since 2000, x	7	8	9	10	11	12	13
Profit (millions of dollars), y	0.6	−0.2	−1.2	1.2	0.8	1	−0.6

a. Display the data in a line graph.

b. Make three observations from the graph.

c. What was the total profit from 2007 to 2013?

d. How could you include profits from the years 1990 to 2006 on your graph? Explain.

Describe the possible location(s) of the point (x, y).

38. $x > 0, y > 0$

39. $x < 0, y < 0$

40. $x > 0, y < 0$

41. $x > 0$

42. $y < 0$

43. $x = 0, y = 0$

Common Errors

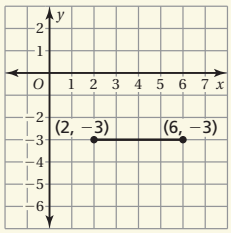
- **Exercises 32–34** Students may plot the x-coordinate vertically instead of horizontally and the y-coordinate horizontally instead of vertically. Remind them that x is horizontal and y is vertical.
- **Exercise 44** Students may not think about the origin, which is the reason the statement is *sometimes* true. Tell students to think about the x-axis as a number line and ask if there is any place where x would be 0.

Practice and Problem Solving

23. The numbers are reversed. To plot (4, 5), start at (0, 0) and move 4 units right and 5 units up.

24. The directions are reversed. To plot (−6, 3), start at (0, 0) and move 6 units left and 3 units up.

25.



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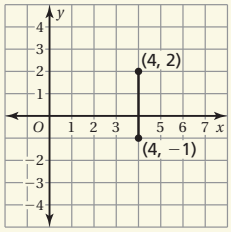
26.



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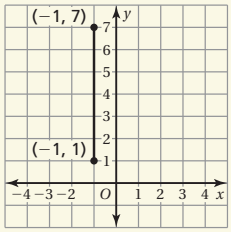
27.



6

28–44. See Additional Answers.

English Language Learners

Vocabulary

Tell students that different words may be used to describe a reflection in a coordinate plane. For example, a figure is a reflection *in* the x-axis, *about* the x-axis, *across* the x-axis, or *over* the x-axis. The same words can be used to describe a reflection in the y-axis as well.

Practice and Problem Solving

45. never; All points in Quadrant III have negative y-coordinates.

46. always; The x-coordinate of a point in Quadrant II is negative, and so is the y-coordinate of a point in Quadrant IV.

47–51. See *Taking Math Deeper.*

52. (2, 2)

53. *Sample answer:* (−6, 3), (−2, 3), (−2, −9), (2, −9)

54. a. (7, −3)

b. school; (2, −1) is closer to (0, 0) than (7, −3).

c. 70%

Fair Game Review

55. $y - 4$

56. $18b$

57. $x + 9$

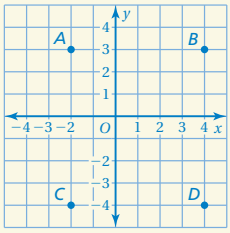
58. $\frac{w}{3}$

59. C

Mini-Assessment

The points $A(-2, 3)$, $B(4, 3)$, $C(-2, -4)$, and $D(4, -4)$ represent the vertices of a garden.

1. Plot the ordered pairs in a coordinate plane.



2. What shape does the garden form? a rectangle

3. Describe the location of each point. *A:* Quadrant II, *B:* Quadrant I, *C:* Quadrant III, *D:* Quadrant IV

Taking Math Deeper

Exercises 47–51

This problem shows the common practice of placing a coordinate plane over a real-life diagram or map. The decision for the location of the origin is arbitrary. The scale may also be arbitrary.

1 Use the coordinate grid to answer the questions.

(48) Flamingo Café is on positive y-axis.

(47) Reptiles is at (2, 1).

(49) Parking only in Quadrants III and IV.

(50) Many answers. Two are (5, −1) and (5, −2).

(51) Safari Africa is closest to (−8, −3).

2 The National Zoo in Washington, D.C., is 163 acres (over 7 million square feet). It was created by Congress in 1889. The zoo includes about 400 different species, including giant pandas, which are a symbol of the zoo's conservation efforts.

Project

Use a grid to draw a map of your school or town. Include important buildings or rooms, parks, ball fields, playground equipment, and other key landmarks in your school or town.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Start the next section

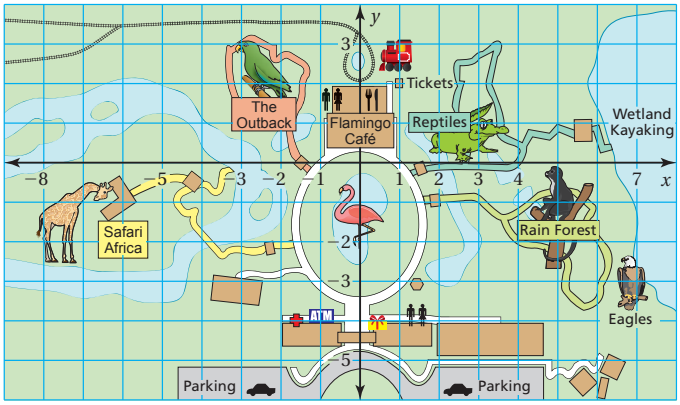
Tell whether the statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

44. The x-coordinate of a point on the x-axis is zero.

45. The y-coordinates of points in Quadrant III are positive.

46. The x-coordinate of a point in Quadrant II has the same sign as the y-coordinate of a point in Quadrant IV.

ZOO In Exercises 47–51, use the map of the zoo.



47. Which exhibit is located at (2, 1)?

48. Name an attraction on the positive y-axis.

49. Is parking available in Quadrant II? If not, name a quadrant in which you can park.

50. Write two different ordered pairs that represent the location of the Rain Forest.

51. Which exhibit is closest to $(-8, -3)$?

52. NUMBER SENSE Name the ordered pair that is 5 units right and 2 units down from $(-3, 4)$.

53. OPEN-ENDED The vertices of triangle ABC are $A(-6, -3)$ and $B(2, -3)$. List four possible coordinates of the third vertex so that the triangle has an area of 24 square units.

54. Reasoning Your school is located at $(2, -1)$, which is 2 blocks east and 1 block south of the center of town. To get from your house to the school, you walk 5 blocks west and 2 blocks north.

a. What ordered pair corresponds to the location of your house?

b. Is your house or your school closer to the center of town? Explain.

c. You can only walk along streets that are north and south or streets that are east and west. You are at the center of town and decide to take the shortest path home that passes by the school. When you are at the school, what percent of the walk home remains?

Fair Game Review What you learned in previous grades & lessons

Write the phrase as an expression. *(Section 3.2)*

55. 4 less than a number y

56. the product of 18 and a number b

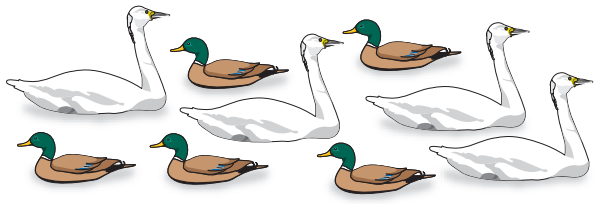
57. a number x increased by 9

58. a number w divided by 3

59. MULTIPLE CHOICE What is the ratio of ducks to swans? *(Section 5.1)*

Ⓐ 4 : 9 Ⓑ 4 : 5

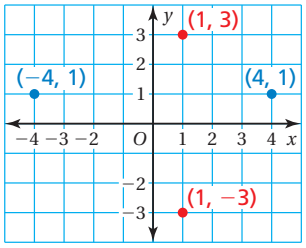
Ⓒ 5 : 4 Ⓓ 5 : 9



Extension 6.5 Reflecting Points in the Coordinate Plane

Check It Out
Lesson Tutorials
BigIdeasMath.com

You can *reflect* a point in the x-axis, in the y-axis, or in both axes.



The red points are mirror images of each other in the x-axis because the x-coordinates are the same and the y-coordinates are opposites. So, the red points are 3 units from the x-axis in opposite directions. The red points represent a *reflection in the x-axis*.

The blue points are mirror images of each other in the y-axis because the y-coordinates are the same and the x-coordinates are opposites. So, the blue points are 4 units from the y-axis in opposite directions. The blue points represent a *reflection in the y-axis*.

Key Idea

Reflecting a Point in the Coordinate Plane

- To reflect a point in the x-axis, use the same x-coordinate and take the opposite of the y-coordinate.
- To reflect a point in the y-axis, use the same y-coordinate and take the opposite of the x-coordinate.

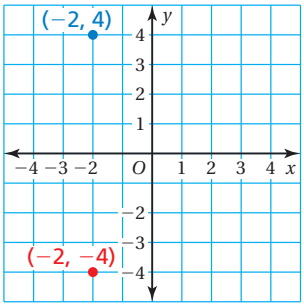
EXAMPLE 1 Reflecting Points in One Axis

a. Reflect $(-2, 4)$ in the x-axis.

Plot $(-2, 4)$.

To reflect $(-2, 4)$ in the x-axis, use the same x-coordinate, -2, and take the opposite of the y-coordinate. The opposite of 4 is -4.

∴ So, the reflection of $(-2, 4)$ in the x-axis is $(-2, -4)$.

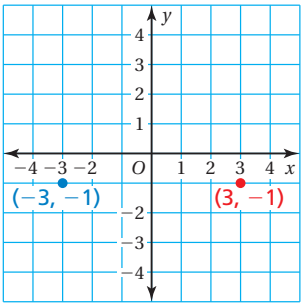


b. Reflect $(-3, -1)$ in the y-axis.

Plot $(-3, -1)$.

To reflect $(-3, -1)$ in the y-axis, use the same y-coordinate, -1, and take the opposite of the x-coordinate. The opposite of -3 is 3.

∴ So, the reflection of $(-3, -1)$ in the y-axis is $(3, -1)$.



Coordinate Plane

In this extension, you will
- understand reflections of points in the coordinate plane.

Laurie's Notes

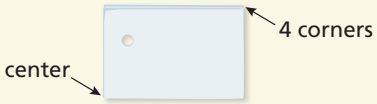
Introduction

Connect

- **Yesterday:** Students learned to plot ordered pairs in the coordinate plane in all four quadrants. (MP3, MP6)
- **Today:** Students will plot ordered pairs that are reflections of each other.

Motivate

- Take a plain piece of paper and draw two dark, perpendicular lines (*x*- and *y*-axes) that intersect near the center of the paper. Fold the paper, creasing on one of the dark lines. Fold again, creasing on the other dark line. You have now folded the paper to create the four quadrants.
- Take a paper punch and tell students that you're going to punch your paper once, going through all layers. Punch in a spot that is not on the diagonal.



? "Can you describe what the paper will look like when I unfold it?" Listen for several ideas. 4 holes, same distance from the creases; Students may use language such as "the holes are across from one another."

- **Extension:** If you can find a paper punch that is not circular, it makes it more interesting! A primary level teacher may have one.

Lesson Notes

Discuss

- Draw a coordinate grid. Plot and label ordered pairs: two points should be reflections in the *x*-axis and two should be reflections in the *y*-axis. Color code as shown.
- Ask about features of a point reflected in the *x*-axis. Students should mention that the points are the same distance from the *x*-axis, the *x*-coordinates are the same, and the *y*-coordinates are opposites.
- Repeat for the two points reflected in the *y*-axis.

Key Idea

- Refer to the coordinate grid drawn as it provides a good visual reference.
- **Common Error:** Students think that when you take the opposite of a number, it becomes negative. If the original number is negative, the opposite of it is positive.

Example 1

? "What quadrant is (−2, 4) in?" Quadrant II "If you reflect (−2, 4) in the *x*-axis, what quadrant will the reflection be in?" Quadrant III

- Because (−2, 4) is 4 units above the *x*-axis its reflection will be 4 units below the *x*-axis. Plot (−2, −4).
- Work through part (b) as shown.

Common Core State Standards

6.NS.6b Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

Goal Today's lesson is plotting ordered pairs that are reflections of each other in the coordinate plane.

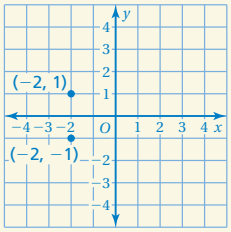
Technology for the Teacher

Dynamic Classroom

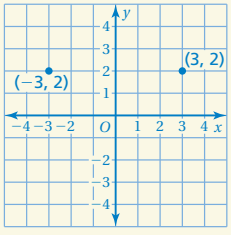
Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

a. Reflect (−2, −1) in the *x*-axis.



b. Reflect (3, 2) in the *y*-axis.

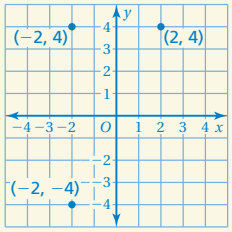


Record and Practice Journal Extension 6.5 Practice

1–11. See Additional Answers.

Extra Example 2

Reflect $(-2, -4)$ in the x-axis followed by the y-axis.



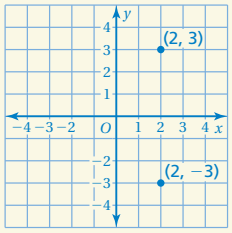
Practice

1. **a.** $(3, -2)$ **b.** $(-3, 2)$
2. **a.** $(-4, -4)$ **b.** $(4, 4)$
3. **a.** $(-5, 6)$ **b.** $(5, -6)$
4. **a.** $(4, 7)$ **b.** $(-4, -7)$
5. **a.** $(0, 1)$ **b.** $(0, -1)$

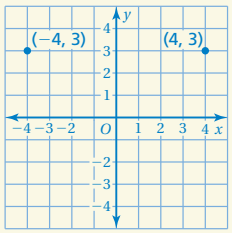
6–16. See Additional Answers.

Mini-Assessment

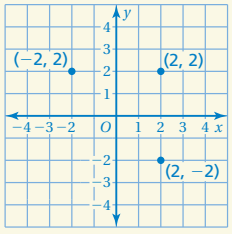
1. Reflect $(2, -3)$ in the x-axis.



2. Reflect $(4, 3)$ in the y-axis.



3. Reflect $(2, -2)$ in the x-axis followed by the y-axis.



Laurie's Notes

Example 2

- **MP8 Look for and Express Regularity in Repeated Reasoning:** When students reflect a point in one axis followed by the other axis, they must repeat their thinking.

? "What quadrant is the point (2, 1) in?" Quadrant I

? "What is the reflection of (2, 1) in the x-axis?" $(2, -1)$

- At this point tell students that they are reflecting $(2, -1)$ in the y-axis and not the original point (2, 1).

? "What is the reflection of $(2, -1)$ in the y-axis?" $(-2, -1)$

- **Extension (MP7 Look for and Make Use of Structure):** "If you reflect (2, 1) first in the y-axis and then in the x-axis, will you end at the same location as in the example? In other words, does the order in which you do the reflections matter?" no; The order doesn't matter because you end up at the same spot.

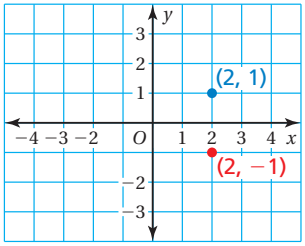
Closure

- Plot $(-3, 4)$.
  - **a.** Reflect $(-3, 4)$ in the x-axis. $(-3, -4)$
  - **b.** Reflect $(-3, 4)$ in the y-axis. $(3, 4)$
  - **c.** If these are three of the vertices of a rectangle, what is the fourth vertex? Explain how you know. $(3, -4)$; The fourth vertex can be found by reflecting $(-3, -4)$ in the y-axis or reflecting $(3, 4)$ in the x-axis.

EXAMPLE 2 Reflecting a Point in Both Axes

Reflect (2, 1) in the *x*-axis followed by the *y*-axis.

Step1: First, plot (2, 1).

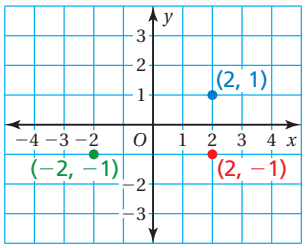


Step 2: Next, reflect (2, 1) in the *x*-axis. Use the same *x*-coordinate, 2, and take the opposite of the *y*-coordinate. The opposite of 1 is −1.

The point (2, 1) reflected in the *x*-axis is (2, −1).

Step 3: Finally, reflect (2, −1) in the *y*-axis. Use the same *y*-coordinate, −1, and take the opposite of the *x*-coordinate. The opposite of 2 is −2.

The point (2, −1) reflected in the *y*-axis is (−2, −1).



∴ So, (2, 1) reflected in the *x*-axis followed by the *y*-axis is (−2, −1).

Common Error

When reflecting a second time, be sure to use the reflected point and not the original point.

Practice

Reflect the point in (a) the *x*-axis and (b) the *y*-axis.

1. (3, 2)
2. (−4, 4)
3. (−5, −6)
4. (4, −7)
5. (0, −1)
6. (−8, 0)
7. (2.5, 4.5)
8. $\left(-5\frac{1}{2}, 3\right)$

Reflect the point in the *x*-axis followed by the *y*-axis.

9. (4, 5)
10. (−1, 7)
11. (−2, −2)
12. (6.5, −10.5)

13. **REASONING** A point is reflected in the *x*-axis. The reflected point is (3, −9). What is the original point? What is the distance between the points?

14. **REASONING** A point is reflected in the *y*-axis. The reflected point is (5.75, 0). What is the original point? What is the distance between the points?

15. **a.** **STRUCTURE** In Exercises 9–12, reflect the point in the *y*-axis followed by the *x*-axis. Do you get the same results? Explain.

b. **LOGIC** Make a conjecture about how to use the coordinates of a point to find its reflection in both axes.

16. **GEOMETRY** The vertices of a triangle are (−1, 3), (−5, 3), and (−5, 7). How would you reflect the triangle in the *x*-axis? in the *y*-axis? Give the coordinates of the reflected triangle for each case.

6.4–6.5 Quiz

Check It Out
Progress Check
BigIdeasMath.com

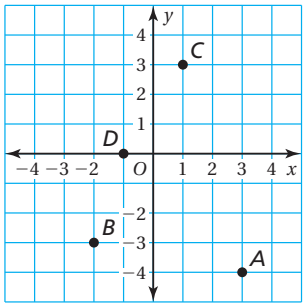
Find the absolute value. *(Section 6.4)*

1. $|-12|$ **2.** $|4|$

Copy and complete the statement using <, >, or =. *(Section 6.4)*

3. $5 \square |-9|$ **4.** $|-11| \square |-10|$

Write an ordered pair corresponding to the point. *(Section 6.5)*



5. Point A

6. Point B

7. Point C

8. Point D

Plot the ordered pair in a coordinate plane. Describe the location of the point. *(Section 6.5)*

9. $Q(2, 5)$ **10.** $R(1, -4)$ **11.** $S(-2.5, 3.5)$ **12.** $T\left(0, -1\frac{1}{2}\right)$

Reflect the point in (a) the x-axis and (b) the y-axis. *(Section 6.5)*

13. $(1, 3)$ **14.** $(-2, 6)$

Reflect the point in the x-axis followed by the y-axis. *(Section 6.5)*

15. $(3, -2)$ **16.** $(-4, -5)$

17. HIKING The table shows the elevations of several checkpoints along a hiking trail. *(Section 6.4)*

Checkpoint	Elevation (feet)
1	110
2	38
3	−24
4	12
5	−142

a. Which checkpoint is farthest from sea level?

b. Which checkpoint is closest to sea level?

c. Is Checkpoint 2 or Checkpoint 3 closer to sea level? Explain.

18. GEOMETRY The points $A(-4, 2)$, $B(1, 2)$, $C(1, -1)$, and $D(-4, -1)$ are the vertices of a figure. *(Section 6.5)*

a. Draw the figure in a coordinate plane.

b. Find the perimeter of the figure.

c. Find the area of the figure.

Alternative Assessment Options

Math Chat Student Reflective Focus Question
Structured Interview Writing Prompt

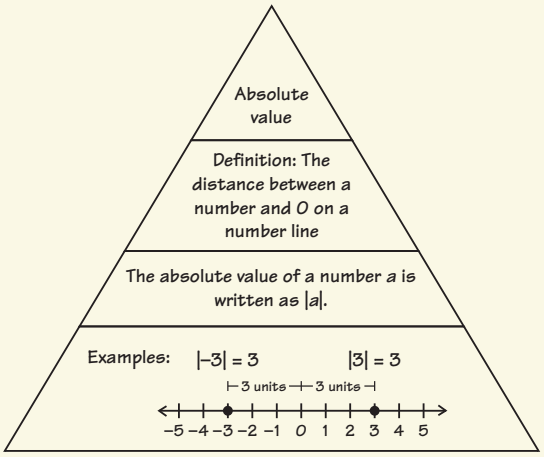
Math Chat

Ask students to use their own words to summarize what they know about comparing and ordering integers, fractions, and decimals on the number line. Be sure that they include examples. Select students at random to present their summary to the class.

Study Help Sample Answers

Remind students to complete Graphic Organizers for the rest of the chapter.

4.



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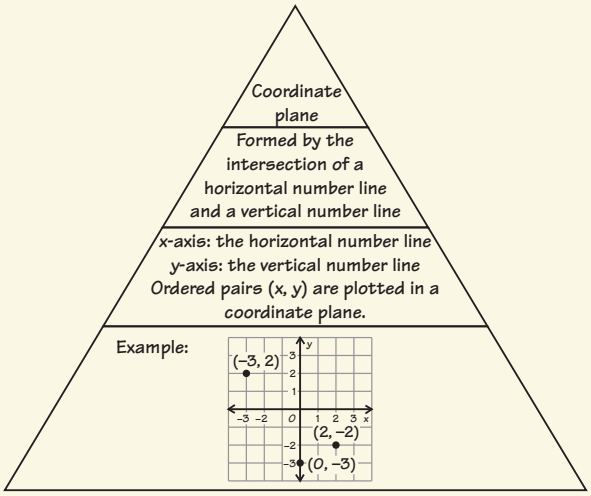


6–7. Available at *BigIdeasMath.com.*

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Lesson Tutorials *BigIdeasMath.com*	Resources by Chapter • Enrichment and Extension • Technology Connection Game Closet at *BigIdeasMath.com* Start the Chapter Review

Answers

1. 12 **2.** 4

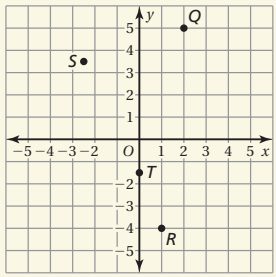
3. $5 < |-9|$

4. $|-11| > |-10|$

5. $(3, -4)$ **6.** $(-2, -3)$

7. $(1, 3)$ **8.** $(-1, 0)$

9–12.



9. Quadrant I

10. Quadrant IV

11. Quadrant II

12. *y*-axis

13. **a.** $(1, -3)$ **b.** $(-1, 3)$

14. **a.** $(-2, -6)$ **b.** $(2, 6)$

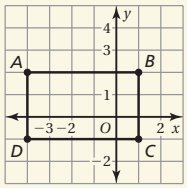
15. $(-3, 2)$ **16.** $(4, 5)$

17. **a.** Checkpoint 5

b. Checkpoint 4

c. Checkpoint 3, because $|-24| < |38|$ and the absolute value of the elevation represents its distance from sea level.

18. **a.**



b. 16 units

c. 15 units^2

Technology for the Teacher

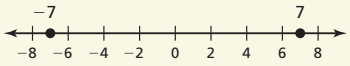
Online Assessment
Assessment Book
ExamView® Assessment Suite

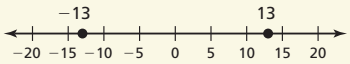
For the Teacher

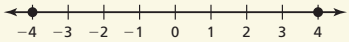
Additional Review Options

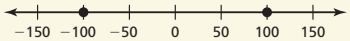
- *BigIdeasMath.com*
- Online Assessment
- Game Closet at *BigIdeasMath.com*
- Vocabulary Help
- Resources by Chapter

Answers

1. -8
2. 12
3. 

4. 

5. 

6. 

7. $-5, -3, -1, 2, 4$
8. $-20, -10, 5, 10, 15$
9. $-12°\text{C}, -7°\text{C}, -3°\text{C}, 0°\text{C}, 8°\text{C}$

Review of Common Errors

Exercises 1 and 2

- Students may use the wrong sign when writing the integer. Tell them to look for key words to help them determine the sign.

Exercises 3–6

- Students may think that *opposites* are only negative numbers. For example, the opposite of -8 is -8 and the opposite of 8 is -8. Remind them that the opposite of a negative number is positive.

Exercises 7–9

- Students may ignore the signs on the integers and order them incorrectly. Encourage students to use a number line to help them order the integers.

Exercises 14–16

- Students may ignore the sign of the number and order the numbers incorrectly. Encourage them to use a number line to determine which number is farthest to the right.

Exercise 15

- Students may place the fractions in the wrong order on the number line. Encourage them to find a common denominator before graphing on the number line.

Exercises 17–20

- Students may think the absolute value of a number is its opposite. For example, they may think $|6| = -6$. Use a number line to remind students that absolute value is a number's distance from zero, so it is always a positive number or zero.

Exercises 24–27

- Students may plot the x-coordinate vertically instead of horizontally and the y-coordinate horizontally instead of vertically. Remind students that x is horizontal and y is vertical.

6 Chapter Review

Check It Out
Vocabulary Help
BigIdeasMath.com

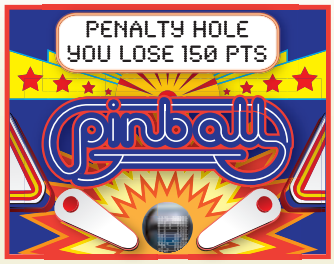
Review Key Vocabulary

positive numbers, *p. 250*
negative numbers, *p. 250*
opposites, *p. 250*
integers, *p. 250*
absolute value, *p. 270*
coordinate plane, *p. 276*
origin, *p. 276*
quadrants, *p. 276*

Review Examples and Exercises

6.1 Integers *(pp. 248–253)*

Write a positive or negative integer to represent losing 150 points in a pinball game.



"Lose" indicates a number less than 0. So, use a negative integer.

∴ -150

Exercises

Write a positive or negative integer that represents the situation.

1. An elevator goes down 8 floors.

2. You earn \$12.

Graph the integer and its opposite.

3. -7

4. 13

5. 4

6. -100

6.2 Comparing and Ordering Integers *(pp. 254–259)*

Order $-3, -4, 2, 0, -1$ from least to greatest.

Graph each integer on a number line.

Write the integers as they appear on the number line from left to right.

∴ So, the order from least to greatest is $-4, -3, -1, 0, 2$.

Exercises

Order the integers from least to greatest.

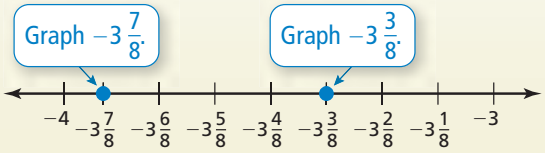
7. $-5, 4, 2, -3, -1$

8. $5, -20, -10, 10, 15$

9. Order the temperatures $-3°C$, $8°C$, $-12°C$, $-7°C$, and $0°C$ from coldest to warmest.

6.3 Fractions and Decimals on the Number Line *(pp. 260–265)*

Compare $-3\frac{7}{8}$ and $-3\frac{3}{8}$.



$-3\frac{7}{8}$ is to the left of $-3\frac{3}{8}$.

∴ So, $-3\frac{7}{8} < -3\frac{3}{8}$.

Exercises

Graph the number and its opposite.

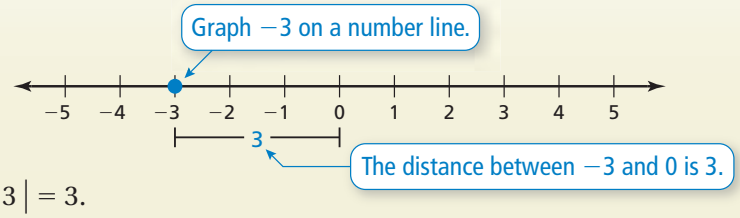
10. $-\frac{2}{5}$ **11.** $1\frac{3}{4}$ **12.** -1.2 **13.** 2.75

Copy and complete the statement using < or >.

14. $-2\frac{1}{6}$ ☐ $-2\frac{5}{6}$ **15.** $-\frac{1}{3}$ ☐ $-\frac{1}{8}$ **16.** -3.27 ☐ -2.68

6.4 Absolute Value *(pp. 268–273)*

Find the absolute value of -3.



∴ So, $|-3| = 3$.

Exercises

Find the absolute value.

17. $|-8|$ **18.** $|13|$ **19.** $\left|3\frac{6}{7}\right|$ **20.** $|-1.34|$

Copy and complete the statement using <, >, or =.

21. $|-2|$ ☐ 2 **22.** $|4.4|$ ☐ $|-2.8|$ **23.** $\left|\frac{1}{6}\right|$ ☐ $\left|-\frac{2}{9}\right|$

Review Game

Number Lines

Materials

- 1 deck of cards with the jacks, queens, kings, aces, and jokers removed
- 2 wooden craft sticks
- paper
- pencil

Players: 3

Directions:

Divide the class into groups of three. Each group draws a number line labeled −10 to 10 in increments of 1. Two students each draw a card and lay a stick on the number represented by the card. Black cards represent positive integers and red cards represent negative integers. The third student is turned so he or she cannot see the two integers being plotted. The third student says "greatest" or "least." If the third student says "greatest," then the student who plotted the greatest integer gets 1 point. If the third student says "least," then the student who plotted the least integer gets 1 point. Continue playing until one of the students reaches 10 points. That student wins. Now the three students can switch roles and play again.

Who Wins?

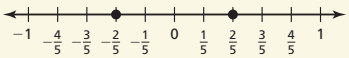
The student that reaches 10 points the most.

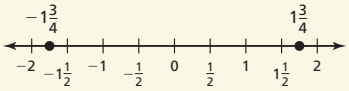
For the Student

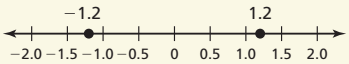
Additional Practice

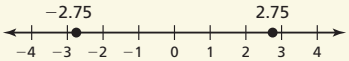
- Lesson Tutorials
- Multi-Language Glossary
- Self-Grading Progress Check
- *BigIdeasMath.com*
  Dynamic Student Edition
  Student Resources

Answers

10. 

11. 

12. 

13. 

14. >

15. <

16. <

17. 8

18. 13

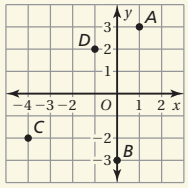
19. $3\frac{6}{7}$

20. 1.34

21. =

22. >

23. <

24–27. 

24. Quadrant I

25. *y*-axis

26. Quadrant III

27. Quadrant II

28. **a.** (4, −1) **b.** (−4, 1)

29. **a.** (−2, −3) **b.** (2, 3)

30. **a.** (2, 5) **b.** (−2, −5)

31. **a.** (−3.5, 2.5)

b. (3.5, −2.5)

32. (−1, −2)

33. (4, −6)

34. (−3, 4)

35. (3, 3)

My Thoughts on the Chapter

What worked. . .

Teacher Tip

Not allowed to write in your teaching edition? Use sticky notes to record your thoughts.

What did not work. . .

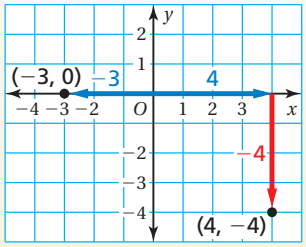
What I would do differently. . .

6.5 The Coordinate Plane *(pp. 274–283)*

a. Plot (−3, 0) and (4, −4) in a coordinate plane. Describe the location of each point.

To plot (−3, 0), start at the origin. Move 3 units left. Then plot the point.

To plot (4, −4), start at the origin. Move 4 units right and 4 units down. Then plot the point.

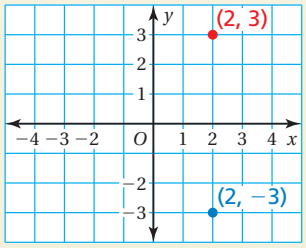


∴ The point (−3, 0) is on the *x*-axis.
The point (4, −4) is in Quadrant IV.

b. Reflect (2, −3) in the *x*-axis.

Plot (2, −3).

To reflect (2, −3) in the *x*-axis, use the same *x*-coordinate, 2, and take the opposite of the *y*-coordinate. The opposite of −3 is 3.

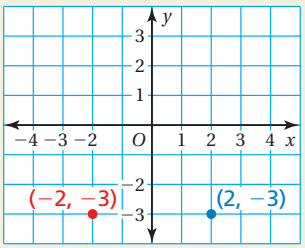


∴ So, the reflection of (2, −3) in the *x*-axis is (2, 3).

c. Reflect (2, −3) in the *y*-axis.

Plot (2, −3).

To reflect (2, −3) in the *y*-axis, use the same *y*-coordinate, −3, and take the opposite of the *x*-coordinate. The opposite of 2 is −2.



∴ So, the reflection of (2, −3) in the *y*-axis is (–2, –3).

Exercises

Plot the ordered pair in a coordinate plane. Describe the location of the point.

24. $A(1, 3)$

25. $B(0, -3)$

26. $C(-4, -2)$

27. $D(-1, 2)$

Reflect the point in (a) the *x*-axis and (b) the *y*-axis.

28. (4, 1)

29. (−2, 3)

30. (2, −5)

31. (−3.5, −2.5)

Reflect the point in the *x*-axis followed by the *y*-axis.

32. (1, 2)

33. (−4, 6)

34. (3, −4)

35. (−3, −3)

6 Chapter Test

Check It Out
Test Practice
BigIdeasMath.com

Order the integers from least to greatest.

1. $0, -2, 3, 1, -4$

2. $-8, -3, 5, 4, -5$

Graph the number and its opposite.

3. 14

4. -40

5. $-1\frac{1}{3}$

6. 1.75

Find the absolute value.

7. $|-7|$

8. $|-11|$

Copy and complete the statement using <, >, or =.

9. $-\frac{2}{3} \square -\frac{3}{5}$

10. $1.55 \square -2.46$

11. $|-6| \square -3$

12. $-2.5 \square |2.5|$

Plot the ordered pair in a coordinate plane. Describe the location of the point.

13. $J(4, 0)$

14. $K(-3, 5)$

15. $L(1.5, -3.5)$

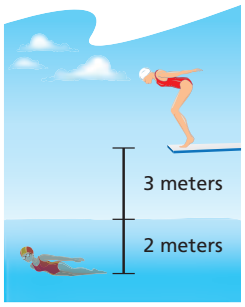
16. $M(-2, -3)$

Reflect the point in the *x*-axis followed by the *y*-axis.

17. $(2, 4)$

18. $(-5, 1)$

19. **POOL** A diver is on a springboard that is 3 meters above the surface of a pool. Another diver is 2 meters below the surface of the pool.



a. Write an integer for the position of each diver relative to the surface of the pool.

b. Find the absolute value of each integer.

c. Who is farther from the surface of the pool?

20. **OPEN-ENDED** Two vertices of a triangle are $F(1, -4)$ and $G(6, -4)$. List two possible coordinates of the third vertex so that the triangle has an area of 20 square units.

21. **MELTING POINT** The table shows the melting points (in degrees Celsius) of several elements. Compare the melting point of mercury to the melting point of each of the other elements.

Element	Mercury	Radon	Bromine	Cesium	Francium
Melting Point (°C)	−38.83	−71	−7.2	28.5	27

Test Item References

Chapter Test Questions	Section to Review	Common Core State Standards
3, 4	6.1	6.NS.5, 6.NS.6a, 6.NS.6c
1, 2	6.2	6.NS.6c, 6.NS.7a, 6.NS.7b
5, 6, 9, 10, 21	6.3	6.NS.5, 6.NS.6a, 6.NS.6c, 6.NS.7a, 6.NS.7b
7, 8, 11, 12, 19	6.4	6.NS.7c, 6.NS.7d
13–18, 20	6.5	6.NS.6b, 6.NS.6c, 6.NS.8

Test-Taking Strategies

Remind students to quickly look over the entire test before they start so that they can budget their time. On this test, it is very important for them to **Stop** and **Think**. When students hurry on a test dealing with signed numbers, they often make 'sign errors.' Encourage them to represent problems with a number line, if appropriate, to ensure that they think through the process.

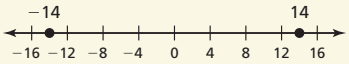
Common Errors

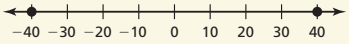
- **Exercises 1, 2, 9–12** Students may ignore the signs on the integers and order them incorrectly. Encourage students to use a number line to help them order the integers.
- **Exercises 3–6** Students may think that *opposites* are only negative numbers. For example, the opposite of -3 is -3 and the opposite of 3 is -3. Remind them that the opposite of a negative number is positive.
- **Exercises 13–16** Students may plot the x-coordinate vertically instead of horizontally and the y-coordinate horizontally instead of vertically. Remind them that x is horizontal and y is vertical.

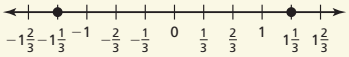
Reteaching and Enrichment Strategies

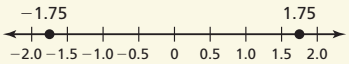
If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials *BigIdeasMath.com* Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Game Closet at *BigIdeasMath.com* Start Cumulative Assessment

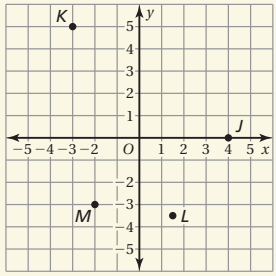
Answers

1. $-4, -2, 0, 1, 3$
2. $-8, -5, -3, 4, 5$
3. 

4. 

5. 

6. 

7. 7
8. 11
9. $<$
10. $>$
11. $>$
12. $<$
13–16. 

13. x-axis
14. Quadrant II
15. Quadrant IV
16. Quadrant III
17. $(-2, -4)$
18. $(5, -1)$
19. **a.** 3; -2
    b. 3; 2
    c. the diver on the springboard
20. *Sample answer:* (1, 4), (6, 4)
21. Mercury has a higher melting point than Radon and a lower melting point than Bromine, Cesium, and Francium.

Technology for the Teacher

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Test-Taking Strategies

Available at *BigIdeasMath.com*

After Answering Easy Questions, Relax
Answer Easy Questions First
Estimate the Answer
Read All Choices before Answering
Read Question before Answering
Solve Directly or Eliminate Choices
Solve Problem before Looking at Choices
Use Intelligent Guessing
Work Backwards

About this Strategy

When taking a multiple choice test, be sure to read each question carefully and thoroughly. It is also very important to read each answer choice carefully. Do not pick the first answer that you think is correct! If two answer choices are the same, eliminate them both. There can only be one correct answer.

Answers

1. C
2. G
3. $\frac{3}{4}$
4. D
5. H

Technology for the Teacher

Performance Tasks
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Item Analysis

1. **A.** The student makes an order of operations error, adding $3c$ to $5b$ and then subtracting this sum from $8a$.
   B. The student substitutes 5 for c and 4 for b.
   C. Correct answer
   D. The student makes two-digit numbers out of the coefficients and the variables, such as 43 instead of 4×3.

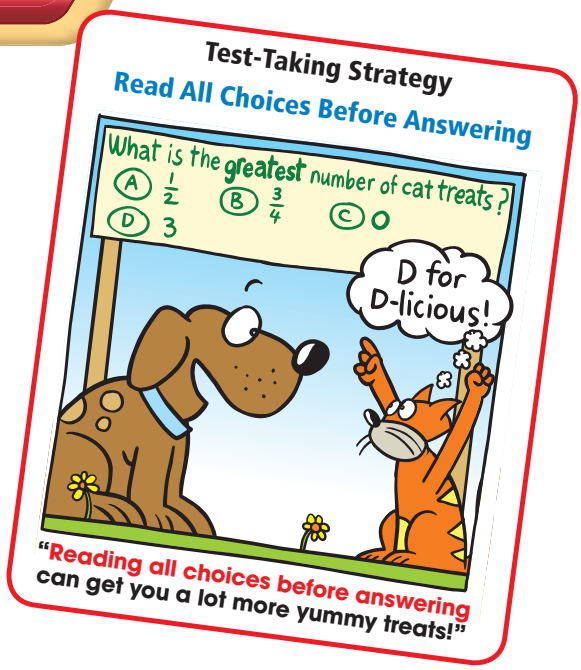
2. **F.** The student incorrectly thinks that points in Quadrant II have negative y-coordinates.
   G. Correct answer
   H. The student incorrectly thinks that points in Quadrant II have negative y-coordinates. The student also reverses the x- and y-coordinates.
   I. The student reverses the x- and y-coordinates.

3. **Gridded Response:** Correct answer: $\frac{3}{4}$
   Common Error: The student divides the greater number by the lesser number, getting an answer of $\frac{4}{3}$ or equivalent.

4. **A.** The student orders the numbers by magnitude.
   B. The student incorrectly thinks $|4| = -4$
   C. The student ignores the absolute value bars.
   D. Correct answer

5. **F.** The student multiplies the numerator and denominator.
   G. The student uses the digits from the numerator and denominator.
   H. Correct answer
   I. The student divides 5 by 4 and converts the quotient into a percent.

6 Cumulative Assessment



1. What is the value of the expression below when $a = 6$, $b = 5$, and $c = 4$?

$$8a - 3c + 5b$$

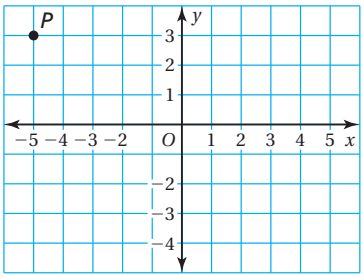
A. 11

B. 53

C. 61

D. 107

2. Point P is plotted in the coordinate plane below.



What are the coordinates of point P?

F. $(-5, -3)$

G. $(-5, 3)$

H. $(-3, -5)$

I. $(3, -5)$

3. What is the value of the expression below?

$$4\frac{1}{8} \div 5\frac{1}{2}$$

4. Which list of numbers is in order from least to greatest?

A. $2, |-3|, |4|, -6$

B. $-6, |4|, 2, |-3|$

C. $-6, |-3|, 2, |4|$

D. $-6, 2, |-3|, |4|$

5. Which percent is equivalent to $\frac{4}{5}$?

F. 20%

G. 45%

H. 80%

I. 125%

6. Which property is illustrated by the statement below?

$$4 + (6 + n) = (4 + 6) + n$$

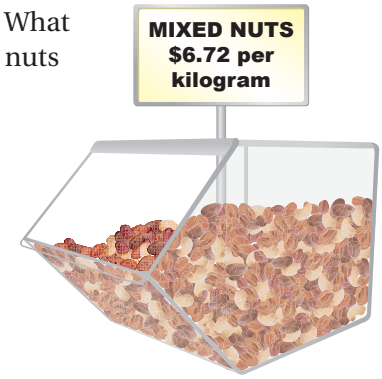
A. Associative Property of Addition

B. Commutative Property of Addition

C. Associative Property of Multiplication

D. Distributive Property

7. You bought 0.875 kilogram of mixed nuts. What was the total cost, in dollars, of the mixed nuts that you bought?



8. On Saturday, you earned $35 mowing lawns. This was x dollars more than you earned on Thursday. Which expression represents the amount, in dollars, you earned mowing lawns on Thursday?

F. $35x$

G. $x + 35$

H. $x - 35$

I. $35 - x$

9. Helene was finding the percent of a number in the box below.

75% of 24 is what number?

$$75\% \text{ of } 24 = 24 \div \frac{3}{4}$$

$$= 32$$

What should Helene do to correct the error that she made?

A. Divide 24 by 75.

B. Divide $\frac{3}{4}$ by 24.

C. Multiply 24 by 75.

D. Multiply 24 by $\frac{3}{4}$.

Item Analysis (continued)

6. **A.** Correct answer

B. The student misidentifies the property being used as the Commutative Property of Addition.

C. The student misidentifies the property being used as the Associative Property of Multiplication.

D. The student misidentifies the property being used as the Distributive Property.

7. **Gridded Response:** Correct answer: 5.88

Common Error: The student incorrectly places the decimal point in the product.

8. **F.** The student misinterprets "more than" as meaning multiplication.

G. The student misinterprets the problem, thinking that you received more on Thursday than Saturday.

H. The student subtracts in the wrong order and writes an expression that represents $35 less than the difference of the amounts earned on Thursday and Saturday.

I. Correct answer

9. **A.** The student divides by 100 times the percent.

B. The student switches the order of division instead of changing to multiplication.

C. The student multiplies by 100 times the percent.

D. Correct answer

Answers

6. A

7. 5.88

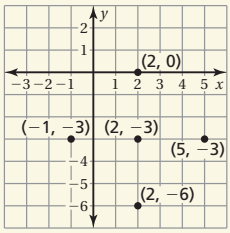
8. I

9. D

Answers

10. F

11. *Part A* and *Part B*



12. A

Item Analysis (continued)

10. **F.** Correct answer

G. The student chooses a benchmark percent that visually approximates the number of squares shaded red.

H. The student thinks that the quantity of 48 squares is the same as 48%.

I. The student chooses a benchmark percent that approximates the number of squares shaded red.

11. **2 points** The student demonstrates a thorough understanding of plotting points in all four quadrants of the coordinate plane. The student correctly plots and labels points at (2, −3), (2, −6), (−1, −3), (2, 0), and (5, −3).

1 point The student demonstrates a partial understanding of plotting points in all four quadrants of the coordinate plane. The student shows some knowledge of how to plot points, but does not successfully plot all points.

0 points The student demonstrates insufficient understanding of plotting points in all four quadrants of the coordinate plane. The student makes many errors in plotting points and/or is unable to correctly draw an *x*-axis and *y*-axis.

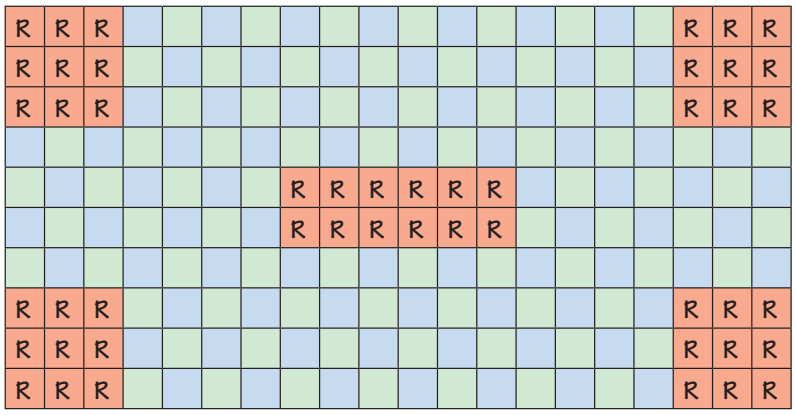
12. **A.** Correct answer

B. The student does not find the reciprocal of $\frac{10}{3}$.

C. The student does not change division to multiplication.

D. The students incorrectly rewrites $3\frac{1}{3}$ as $\frac{7}{3}$ by adding 3, 3, and 1 to get 7.

10. In the mural below, the squares that are painted red are marked with the letter R.



What percent of the mural is painted red?

F. 24%

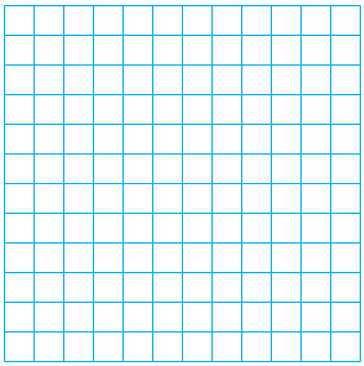
G. 25%

H. 48%

I. 50%

11. Use grid paper to complete the following.

Think
Solve
Explain



Part A Draw an x-axis and a y-axis in the coordinate plane. Then plot and label the point $(2, -3)$.

Part B Plot and label *four* points that are 3 units away from $(2, -3)$.

12. Which expression is equivalent to the expression below?

$$k \div 3\frac{1}{3}$$

A. $k \cdot \frac{3}{10}$

B. $k \cdot \frac{10}{3}$

C. $k \div \frac{3}{10}$

D. $k \div \frac{7}{3}$