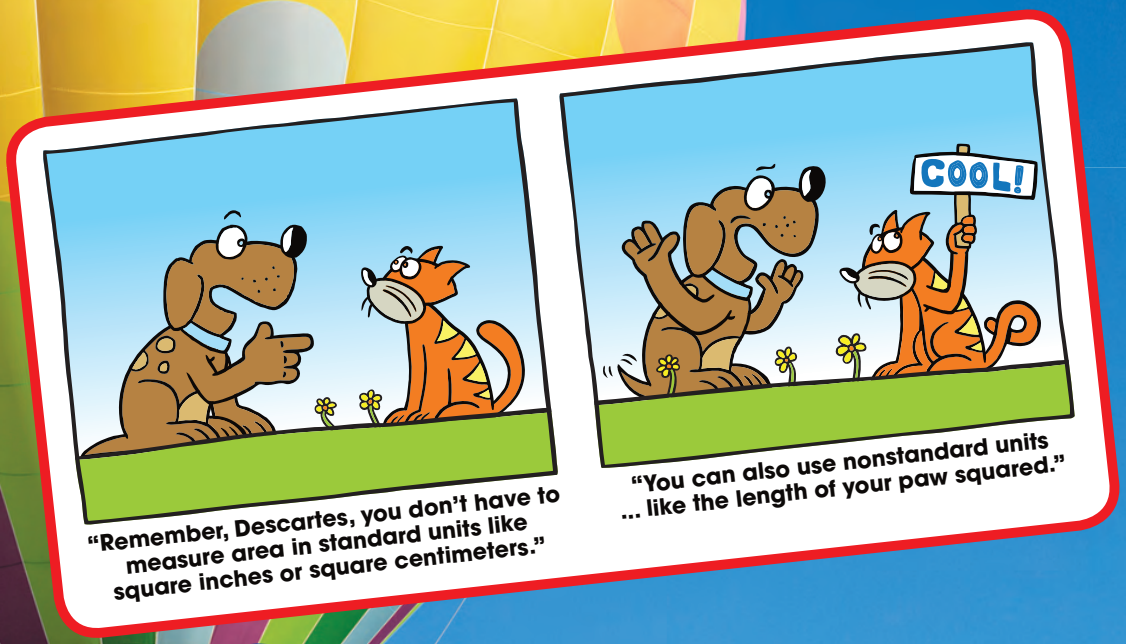
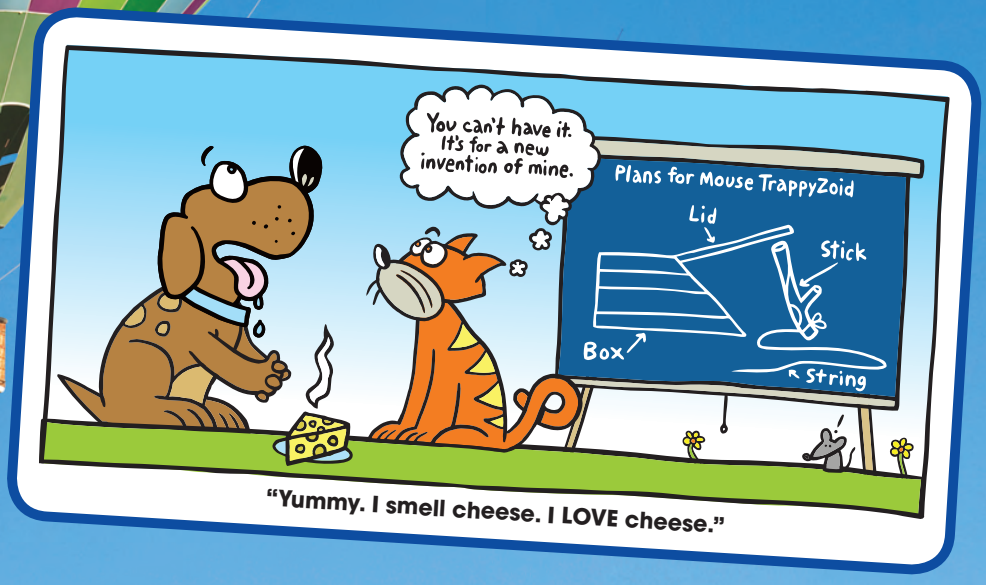
4 Areas of Polygons





Common Core Progression

4th Grade

- Find the area and perimeter of a rectangle using formulas.
- Draw points, line segments, parallel lines, and perpendicular lines.
- Classify two-dimensional figures based on angles, parallel lines, and perpendicular lines.

5th Grade

- Find the area of a rectangle with fractional side lengths.
- Classify two-dimensional figures into categories based on properties.
- Generate numerical patterns given rule, identify the relationship, and form ordered pairs.
- Plot points in the first quadrant of the coordinate plane.

6th Grade

- Find areas of triangles, special quadrilaterals, and polygons.
- Find the distance between points with the same x- or y-coordinate.
- Draw polygons in the coordinate plane given vertices and find lengths of sides.

Pacing Guide for Chapter 4

Chapter Opener	1 Day
Section 1	
Activity	1 Day
Lesson	1 Day
Section 2	
Activity	1 Day
Lesson	1 Day
Study Help / Quiz	1 Day
Section 3	
Activity	1 Day
Lesson	1 Day
Extension	1 Day
Section 4	
Activity	1 Day
Lesson	1 Day
Chapter Review/ Chapter Tests	2 Days
Total Chapter 4	13 Days
Year-to-Date	64 Days

Chapter Summary

Section	Common Core State Standard	
4.1	Learning	6.G.1
4.2	Learning	6.G.1
4.3	Learning	6.G.1 ★
4.4	Learning	6.G.3 ★
★ Teaching is complete. Standard can be assessed.		

Technology for the Teacher

BigIdeasMath.com
Chapter at a Glance
Complete Materials List
Parent Letters: English and Spanish

Common Core State Standards

4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).

Additional Topics for Review

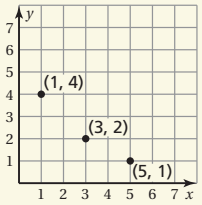
- Parallel and Perpendicular Lines
- Classify Two-Dimensional Figures

Try It Yourself

1. $49\ m^2$ **2.** $180\ yd^2$

3. $5850\ mm^2$

4–6.



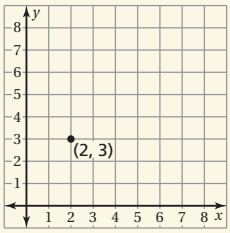
Record and Practice Journal
Fair Game Review

1. $50\ ft^2$ **2.** $9\ cm^2$

3. $16\ in.^2$ **4.** $14\ yd^2$

5. $80\ ft^2$

6.



7–11. See Additional Answers.

Math Background Notes

Vocabulary Review

- Area
- Coordinate Plane
- *x*-axis
- *y*-axis
- Origin
- Ordered Pair
- *x*-coordinate
- *y*-coordinate

Finding Areas of Squares and Rectangles

- Remind students that the exponent means to multiply the base by itself. 15^2 is 15×15 and not 15×2.

? "What type of units do you use to measure area?" square units

? "Why is area measured using square units?" There are two dimensions with multiplication in the formula. cm $\times$ cm = cm^2

- Review how square units are written: cm^2, sq cm, or square centimeters.

Plotting Ordered Pairs

- Use a model of a coordinate grid to identify key vocabulary: coordinate plane, *x*-axis, *y*-axis, origin, ordered pair, *x*-coordinate, *y*-coordinate.
- **Connections:** Hold your arms out horizontally and relate them to the horizon to help students remember which way is horizontal. Origin means "where something starts."
- Students should understand that each point in the coordinate plane has a unique name. The *x*- and *y*-coordinates specify an exact spot, similar to latitude and longitude on a map.
- Stress that the order in which you plot the points is important. The ordered pairs (2, 4) and (4, 2) are not the same.
  - The *x*-coordinate is always first. It tells how far to go horizontally.
  - The *y*-coordinate is always second. It tells how far to go vertically.
- The ordered pairs are always (*x*, *y*). If it helps, tell students that the order of the coordinates is in alphabetical order, *x* and then *y*.

Reteaching and Enrichment Strategies

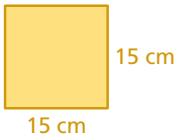
If students need help. . .	If students got it. . .
Record and Practice Journal • Fair Game Review Skills Review Handbook Lesson Tutorials	Game Closet at *BigIdeasMath.com* Start the next section

What You Learned Before

Finding Areas of Squares and Rectangles

Example 1 **Find the area of the square or rectangle.**

a.



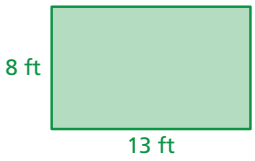
$A = s^2$ Write formula.

$= 15^2$ Substitute.

$= 225$ Simplify.

∴ The area of the square is 225 square centimeters.

b.



$A = \ell w$

$= 13(8)$

$= 104$

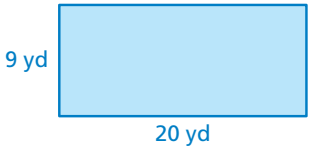
∴ The area of the rectangle is 104 square feet.

Try It Yourself

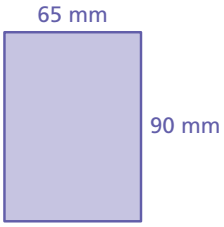
Find the area of the square or rectangle.

1.

2.



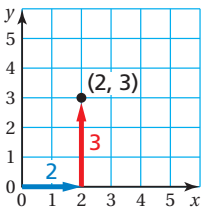
3.



Plotting Ordered Pairs

Example 2 **Plot (2, 3) in a coordinate plane.**

Start at the origin. Move 2 units right and 3 units up. Then plot the point.



Try It Yourself

Plot the ordered pair in a coordinate plane.

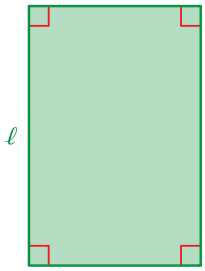
4. (1, 4) **5.** (3, 2) **6.** (5, 1)

4.1 Areas of Parallelograms

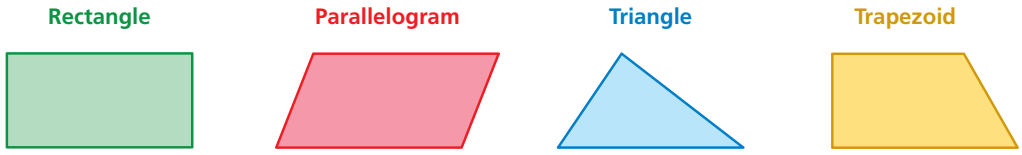
Essential Question How can you derive a formula for the area of a parallelogram?

A **polygon** is a closed figure in a plane that is made up of three or more line segments that intersect only at their endpoints. Several examples of polygons are parallelograms, triangles, and trapezoids.

The formulas for the areas of polygons can be derived from one area formula, the area of a rectangle. Recall that the area of a rectangle is the product of its length ℓ and its width w. The process you use to derive these other formulas is called *deductive reasoning*.



Area $= \ell w$



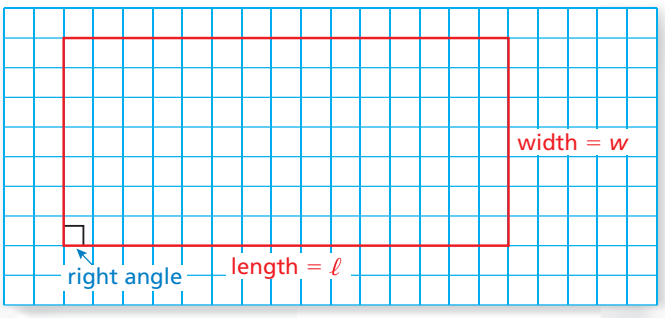
Defined → **Derive formula.** → **Derive formula.** → **Derive formula.**

Grades 4 and 5 | *Lesson 4.1* | *Lesson 4.2* | *Lesson 4.3*

1 ACTIVITY: Deriving the Area Formula of a Parallelogram

Work with a partner.

a. Draw *any* rectangle on a piece of grid paper. An example is shown below. Label the length and width. Then find the area of your rectangle.



b. Cut your rectangle into two pieces to form a parallelogram. Compare the area of the rectangle with the area of the parallelogram. What do you notice? Use your results to write a formula for the area A of a parallelogram.

$A =$ _____ Formula

Geometry

In this lesson, you will

- find areas of parallelograms.
- solve real-life problems.

Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** In this chapter students will derive area formulas from a known formula using deductive reasoning. Students should listen carefully to the statements made by classmates and critique their reasoning.

Motivate

- **Story Time:** Tell students you worked as a short-order cook at a diner while in college. Combining your love of math with cooking, you cut a piece of toast on a diagonal. Then you arranged the pieces so they were no longer square.
- Distribute two right triangle pieces, or a square piece of paper that students can fold on a diagonal and tear apart.

? "How many different shapes can you make with the toast?" three

? "Did the amount of toast change?" no "What did change?" the shape

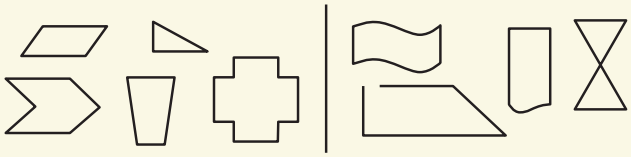
- Use student responses to assess knowledge of simple geometric shapes: rectangle, parallelogram, triangle, and trapezoid.
- Explain that in the next few lessons students will start with a shape, cut and rearrange it to make a shape for which they already know the formula.

Activity Notes

Discuss

- Discuss the definition of a polygon. Draw the following on the board and then ask students, "So what's a polygon?"

Polygons | Not Polygons



- Explain that in this chapter they will use deductive reasoning to derive area formulas for other polygons from the area formula for a rectangle.

Activity 1

- Suggest that students draw a rectangle large enough to manipulate. They may count squares to find the area or find the product of the dimensions.
- **Extension:** Students may want to explore several rectangles and several types of cuts to make a parallelogram.
- **MP3:** Ask a volunteer to share his or her rectangle and parallelogram. Did all students try essentially the same cut or are there multiple versions to present? Are there any cuts that did not allow a parallelogram to be made?
- Students should clearly state the formula. It is not width × length. If they are stuck, ask them where the width and length of the rectangle are found in the parallelogram.

Common Core State Standards

6.G.1 Find the area of . . . special quadrilaterals . . . by composing into rectangles . . .; apply these techniques in the context of solving real-world and mathematical problems.

Previous Learning

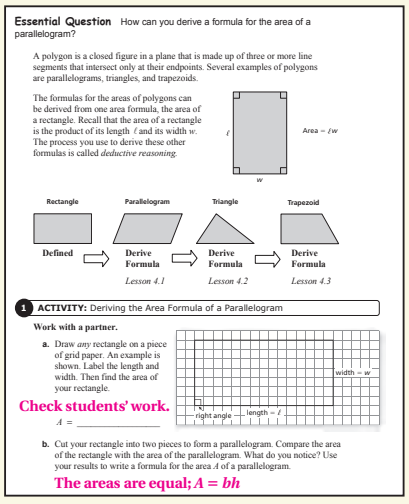
In earlier grades, students have used an area model to represent multiplication. Students have found the area of a rectangle.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

4.1 Record and Practice Journal



Essential Question How can you derive a formula for the area of a parallelogram?

A polygon is a closed figure in a plane that is made up of three or more line segments that intersect only at their endpoints. Several examples of polygons are parallelograms, triangles, and trapezoids.

The formulas for the areas of polygons can be derived from one area formula, the area of a rectangle. Recall that the area of a rectangle is the product of its length ℓ and its width w. The process you use to derive these other formulas is called *deductive reasoning*.

ℓ, w, Area = ℓw

Rectangle — Defined ⇨ Parallelogram — Derive Formula (Lesson 4.1) ⇨ Triangle — Derive Formula (Lesson 4.2) ⇨ Trapezoid — Derive Formula (Lesson 4.3)

1 ACTIVITY: Deriving the Area Formula of a Parallelogram

Work with a partner.

a. Draw *any* rectangle on a piece of grid paper. An example is shown. Label the length and width. Then find the area of your rectangle.

Check students' work.

A = _____

width = w, length = ℓ, right angle

b. Cut your rectangle into two pieces to form a parallelogram. Compare the area of the rectangle with the area of the parallelogram. What do you notice? Use your results to write a formula for the area A of a parallelogram.

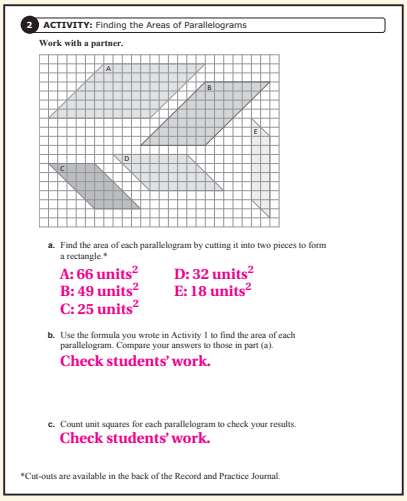
The areas are equal; $A = bh$

Differentiated Instruction

Auditory

Students may have difficulty remembering which measurement (units or *square* units) should be used for perimeter. Remind students that a *meter* is a unit of length, so the perimeter of a figure has units of length (*not* square units of area).

4.1 Record and Practice Journal



2 ACTIVITY: Finding the Areas of Parallelograms

Work with a partner.

a. Find the area of each parallelogram by cutting it into two pieces to form a rectangle.*

A: 66 units^2 D: 32 units^2
B: 49 units^2 E: 18 units^2
C: 25 units^2

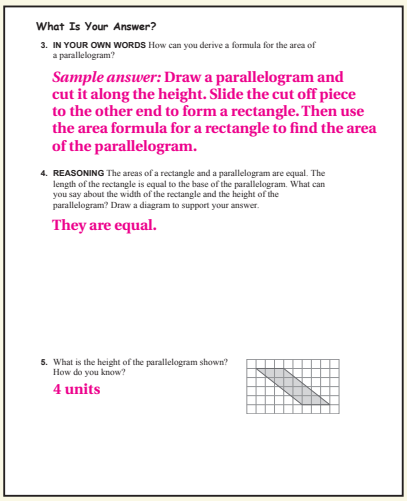
b. Use the formula you wrote in Activity 1 to find the area of each parallelogram. Compare your answers to those in part (a).

Check students' work.

c. Count unit squares for each parallelogram to check your results.

Check students' work.

*Cut-outs are available in the back of the Record and Practice Journal.



What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you derive a formula for the area of a parallelogram?

Sample answer: Draw a parallelogram and cut it along the height. Slide the cut off piece to the other end to form a rectangle. Then use the area formula for a rectangle to find the area of the parallelogram.

4. **REASONING** The areas of a rectangle and a parallelogram are equal. The length of the rectangle is equal to the base of the parallelogram. What can you say about the width of the rectangle and the height of the parallelogram? Draw a diagram to support your answer.

They are equal.

5. What is the height of the parallelogram shown? How do you know?

4 units

Laurie's Notes

Activity 2

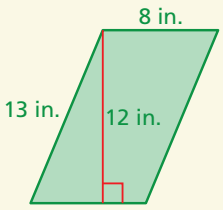
- This activity is the converse of the previous activity. Knowing how they cut the rectangle to make the parallelogram should give students a sense of how to start with the parallelogram and cut to make a rectangle.
- Students should try to visualize the cut and how the pieces would be rearranged to form the rectangle before they actually make the cut.
- Display the parallelograms as you ask for a volunteer to count the unit squares and half-unit squares. The volunteer should count aloud for the class.
- Check to see that students understand how to find and label the height of the parallelogram.

What Is Your Answer?

- **Neighbor Check:** Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.

Closure

- **Exit Ticket:** Find the area and perimeter of the parallelogram shown.



96 in.^{2}; 42 in.

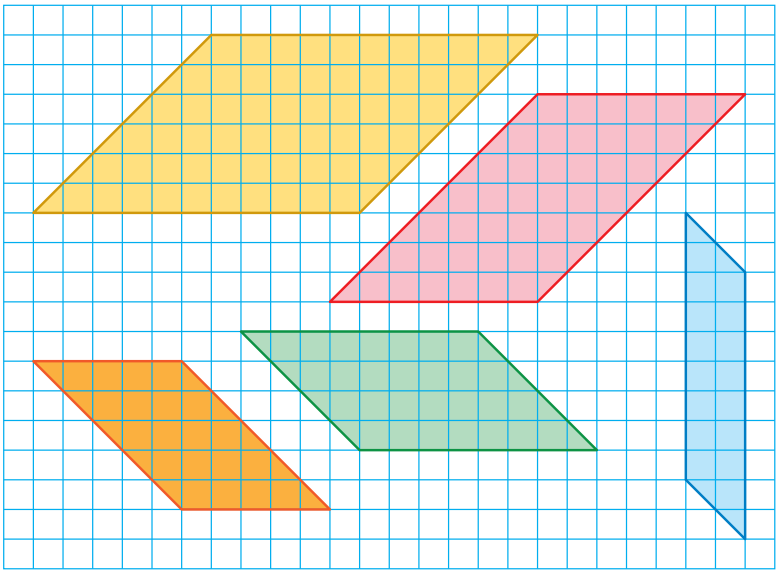
2 ACTIVITY: Finding Areas of Parallelograms

Math Practice

Use Assumptions

How are rectangles and parallelograms similar? How can you use this information to solve the problem?

Work with a partner.



a. Find the area of each parallelogram by cutting it into two pieces to form a rectangle.

b. Use the formula you wrote in Activity 1 to find the area of each parallelogram. Compare your answers to those in part (a).

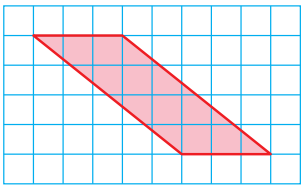
c. Count unit squares for each parallelogram to check your results.

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you derive a formula for the area of a parallelogram?

4. **REASONING** The areas of a rectangle and a parallelogram are equal. The length of a rectangle is equal to the base of the parallelogram. What can you say about the width of the rectangle and the height of the parallelogram? Draw a diagram to support your answer.

5. What is the height of the parallelogram shown? How do you know?



Practice Use what you learned about the areas of parallelograms to complete Exercises 3–5 on page 156.

4.1 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
polygon, *p. 152*

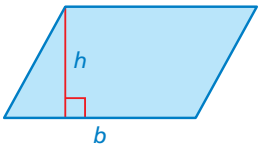
The *area* of a polygon is the amount of surface it covers. You can find the area of a parallelogram in much the same way as you can find the area of a rectangle.

Key Idea

Area of a Parallelogram

Words The area A of a parallelogram is the product of its base b and its height h.

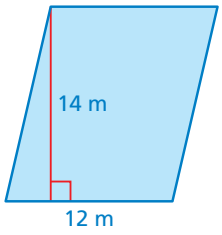
Algebra $A = bh$



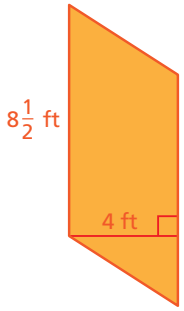
EXAMPLE 1 Finding Areas of Parallelograms

Find the area of each parallelogram.

a.



b.



Remember
Area is measured in square units.

$A = bh$	Write formula.	$A = bh$
$= 12(14)$	Substitute values.	$= 8\frac{1}{2}(4)$
$= 168$	Multiply.	$= 34$

∴ The area of the parallelogram is 168 square meters.

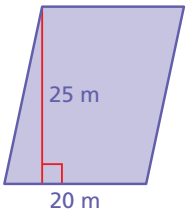
∴ The area of the parallelogram is 34 square feet.

On Your Own

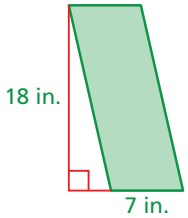
Now You're Ready
Exercises 3–8

Find the area of the parallelogram.

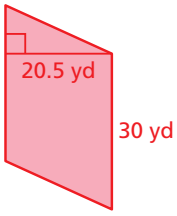
1.



2.



3.



Multi-Language Glossary at BigIdeasMath.com

Laurie's Notes

Introduction

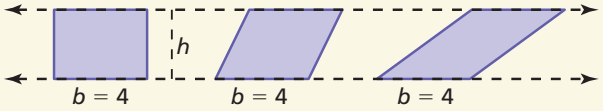
Connect

- **Yesterday:** Students derived the formula for the area of a parallelogram. (MP3)
- **Today:** Students will use the formula to find areas of parallelograms.

Motivate

- Draw a sketch similar to the one below. Ask each student to answer:

? "Which of the shaded figures has the greatest area?" The areas are all equal. How students answer this will give insight into how well they have internalized yesterday's activity.

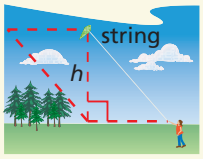


- Ask a student to explain this result. Hopefully the student will talk about cutting off a piece of the parallelogram and moving the piece to form a rectangle that is congruent (the same size and same shape) to the first quadrilateral.

Lesson Notes

Key Idea

- The *height* of a parallelogram is the perpendicular distance from a base to the opposite side.
- It is very important that students recognize that the height of a parallelogram is NOT a side of the parallelogram, unless the figure is a rectangle or square.
- Try to connect the concept of the height of a parallelogram to the height (altitude) of a kite. The length of the string is *not* the height of the kite.



- Ask students to think about b and h in the formula as variables, asking them what value(s) they may stand for, getting them to realize that they are ranges of values (> 0).

Example 1

? "What type of units do you use to measure area?" square units

? "Why is area measured using square units?" There are two dimensions with multiplication in the formula. cm $\times$ cm = cm^2

- Review how square units are written: cm^2, sq cm, or square centimeters.
- **MP6 Attend to Precision:** There is a difference between 168 meters and 168 square meters. Be sure that students are labeling results correctly.
- Begin good habits now. Be explicit in writing the formula, substituting the known values, and computing the answer.

On Your Own

- Students should note that in Question 2 the height is shown outside the parallelogram. The same could have been done for the other two questions.

Goal Today's lesson is finding the areas of parallelograms.

Technology for the Teacher

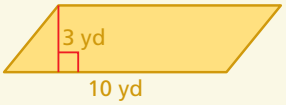
Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

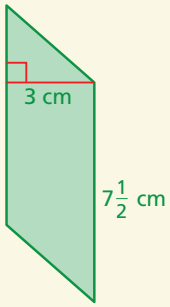
Find the area of each parallelogram.

a.



30 yd^2

b.



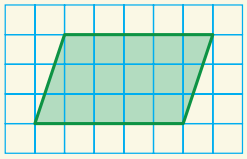
$22\frac{1}{2}$ cm^2

On Your Own

1. 500 m^2
2. 126 $in.^2$
3. 615 yd^2

Extra Example 2

Find the area of the parallelogram.



15 square units

On Your Own

4. 12 square units

Extra Example 3

In Example 3, you cut an 8-inch square out of the piece of wood. What is the area of the photo prop? 4544 in.^{2}

On Your Own

5. 32 ft^2

6. 4464 in.^{2}

English Language Learners

Vocabulary

English language learners may be confused by the abbreviations used for customary and metric measures. Have students write the measure and its abbreviation in their notebooks. List the customary measures separate from the metric measures.

Customary

inches	in.
feet	ft
yard	yd
miles	mi

Metric

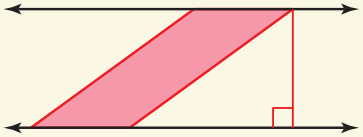
millimeter	mm
centimeter	cm
meter	m
kilometer	km

Laurie's Notes

Example 2

- This example follows from yesterday's activity.
- Students may need help finding the height. This is an example of the height being outside of the polygon. This example is a preview of sorts to working in the coordinate plane in Lesson 4.4 and Lesson 6.5.

? To assist students in seeing the height, you could extend the lines through the bases and ask, "What is the vertical distance between the two bases?"



Example 3

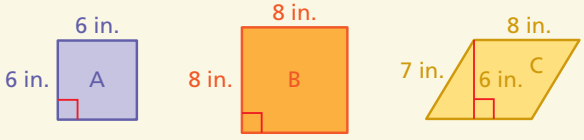
- Discuss each shape and the formulas used.
- The piece of wood is measured in feet and the square is measured in inches. The question asks what is left after the square is removed. So, the first step is to convert feet to inches.
- Students may ask why they can't just multiply by 12. The dimensional analysis is a strategy that students will use throughout mathematics and science.
- Take the time to write the verbal model. Again, this develops good habits in problem solving.

On Your Own

- **Think-Pair-Share:** Students should read each question independently and then work in pairs to answer the questions. When they have answered the questions, the pair should compare their answers with another group and discuss any discrepancies.

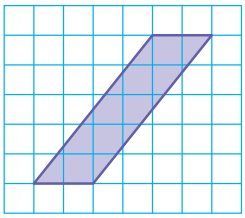
Closure

? Draw all three quadrilaterals and ask, "Are all of these polygons parallelograms?" yes



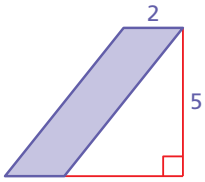
- Determine the area and perimeter of each figure. Then order the quadrilaterals from least to greatest area and least to greatest perimeter.
  A: 36 in.^{2}, 24 in.; B: 64 in.^{2}, 32 in.; C: 48 in.^{2}, 30 in.; A, C, B; A, C, B

EXAMPLE 2 Finding the Area of a Parallelogram on a Grid



Find the area of the parallelogram.

Count grid lines to find the dimensions.
The base b is 2 units, and the height h is 5 units.



$A = bh$ Write formula.

$= 2(5)$ Substitute values.

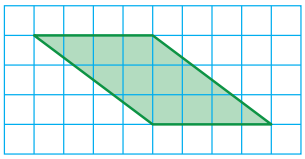
$= 10$ Multiply.

∴ The area of the parallelogram is 10 square units.

On Your Own

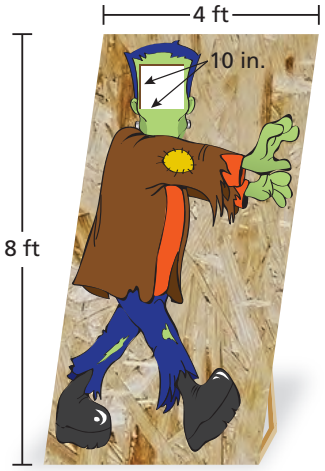
Now You're Ready
Exercises 11–13

4. Find the area of the parallelogram.



EXAMPLE 3 Real-Life Application

You make a photo prop for a school fair. You cut a 10-inch square out of a parallelogram-shaped piece of wood. What is the area of the photo prop?



Convert the dimensions of the piece of wood to inches.

There are 12 inches in 1 foot, so the base is $4 \cdot 12 = 48$ inches and the height is $8 \cdot 12 = 96$ inches.

Use a verbal model to solve the problem.

area of photo prop = area of wood − area of square

$= 96(48) - 10^2$ Substitute.

$= 96(48) - 100$ Evaluate 10^2.

$= 4608 - 100$ Multiply 96 and 48.

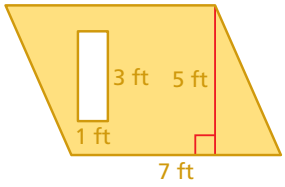
$= 4508$ Subtract 100 from 4608.

∴ The area of the photo prop is 4508 square inches.

On Your Own

Now You're Ready
Exercises 14–16

5. Find the area of the shaded region.



6. **WHAT IF?** In Example 3, you cut a 12-inch square out of the piece of wood. What is the area of the photo prop?

4.1 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **WRITING** What is the area of a polygon? Explain how the perimeter and the area of the polygon are different.
2. **CHOOSE TOOLS** Construct a parallelogram that has an area of 24 square inches. Explain your method.

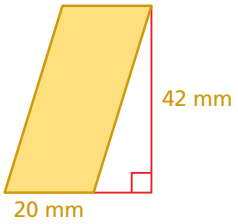
Practice and Problem Solving

Find the area of the parallelogram.

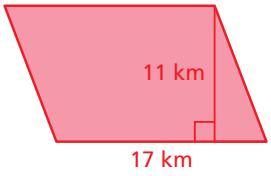
1

3.

4.



5.



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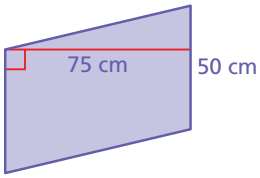

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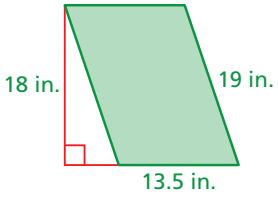
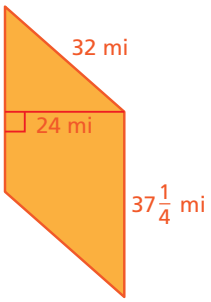
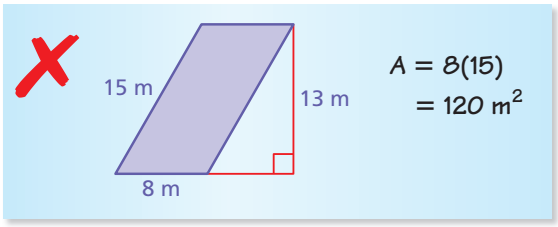


8.



9. **ERROR ANALYSIS** Describe and correct the error in finding the area of the parallelogram.

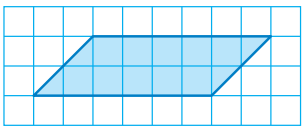


10. **CERAMIC TILE** A ceramic tile in the shape of a parallelogram has a base of 4 inches and a height of 1.5 inches. What is the area of the tile?

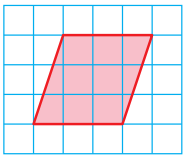
Find the area of the parallelogram.

2

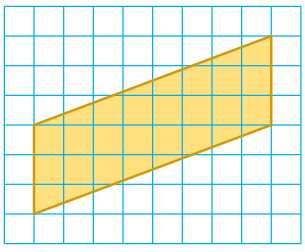
11.



12.



13.



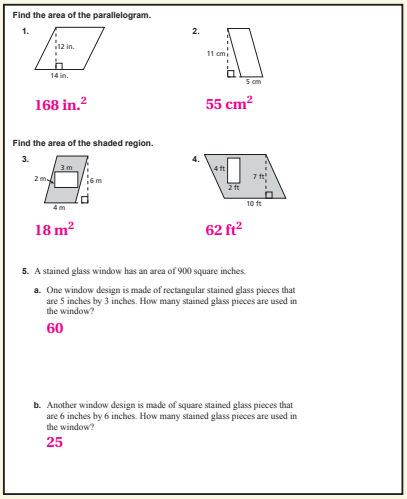
Assignment Guide and Homework Check

Level	Day 1 Activity Assignment	Day 2 Lesson Assignment	Homework Check
Basic	3–5, 22–26	1, 2, 7–11 odd, 12, 13–21 odd	7, 12, 15, 17, 21
Average	3–5, 22–26	1, 2, 6–10 even, 9, 11–14, 16, 17, 19–21	10, 12, 14, 17, 21
Advanced	3–5, 22–26	1, 2, 6–16 even, 9, 17–21	10, 12, 14, 18, 21

Common Errors

- **Exercises 3–13** Students may use the formula for perimeter instead of the formula for area. Tell students to write the formula and then identify the value of each variable before substituting.
- **Exercises 14–16** Students may forget to subtract the area of the white rectangle or parallelogram when finding the area of the shaded region. Draw the parallelogram on a piece of paper and cut out the part that the students must subtract to find the area.
- **Exercises 14–16** Some students may try to break the shaded region into smaller regions and then find the area of each smaller region. This is much more difficult.

4.1 Record and Practice Journal



Find the area of the parallelogram.

1. 12 in., 14 in. — **168 in.^{2}**

2. 11 cm, 5 cm — **55 cm^2**

Find the area of the shaded region.

3. 3 m, 2 m, 6 m, 4 m — **18 m^2**

4. 4 ft, 2 ft, 7 ft, 10 ft — **62 ft^2**

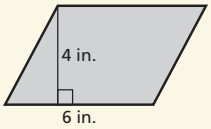
5. A stained glass window has an area of 900 square inches.

a. One window design is made of rectangular stained glass pieces that are 5 inches by 3 inches. How many stained glass pieces are used in the window?
60

b. Another window design is made of square stained glass pieces that are 6 inches by 6 inches. How many stained glass pieces are used in the window?
25

Vocabulary and Concept Check

1. The area of a polygon is the amount of surface it covers. The perimeter of a polygon is the distance around the polygon.
2. *Sample answer:* Any parallelogram where the product of the base and height is 24 in.^{2}.



Practice and Problem Solving

3. 18 ft^2
4. 840 mm^2
5. 187 km^2
6. 3750 cm^2
7. 243 in.^{2}
8. 894 mi^2
9. 15 meters was used for the height instead of 13 meters. $A = 8(13) = 104$ m^2
10. 6 in.^{2}
11. 12 units^2
12. 9 units^2
13. 24 units^2

Practice and Problem Solving

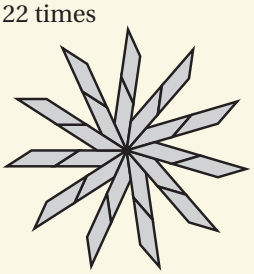
14. 64 cm^2

15 72 m^2

16. 96 ft^2

17. See *Taking Math Deeper*.

18. 22 times



19. 287 $in.^2$

20–21. See Additional Answers.

Fair Game Review

22. 13 **23.** 1640

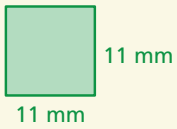
24. 480 **25.** 118

26. B

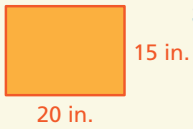
Mini-Assessment

Find the area of the square, rectangle, or parallelogram.

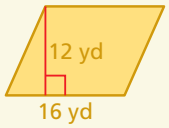
1. 121 mm^2



2. 300 $in.^2$

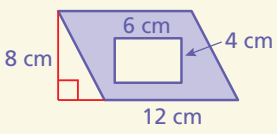


3. 192 yd^2



4. Find the area of the shaded region.

72 cm^2



Taking Math Deeper

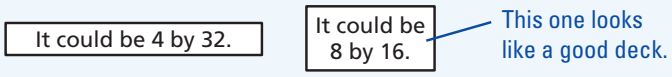
Exercise 17

Remind students that they don't have to see how to start the solution before they put their pencil to paper.

- Doodle some notes.
- Draw a diagram.
- Jot down some facts.
- Rewrite the problem.
- Underline the important words.

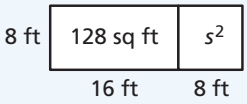
These things kick students' brains into gear. Soon they see a way to the solution.

1 How can an area be represented by $s^2 + 128$?
128 could be a rectangle. Draw a couple.

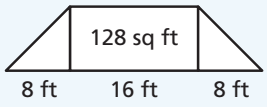


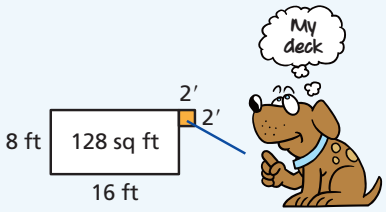
2 Here is a straightforward solution.

- You have the 128. How about the s^2?
- It could be the area of a square. Add this to your diagram.



3 Here are more creative solutions.





Project

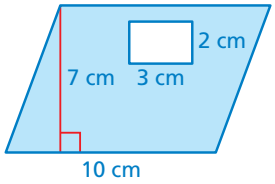
A builder needs to present four different decks to a customer using the information in Exercise 17. Design four different decks for him to use in his sales presentation. Explain how they meet the requirements of Exercise 17.

Reteaching and Enrichment Strategies

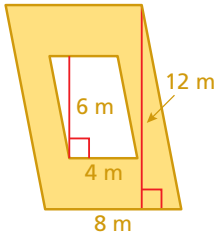
If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Start the next section

Find the area of the shaded region.

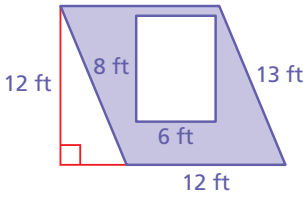
14.



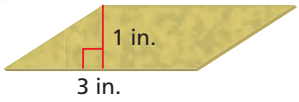
15.



16.

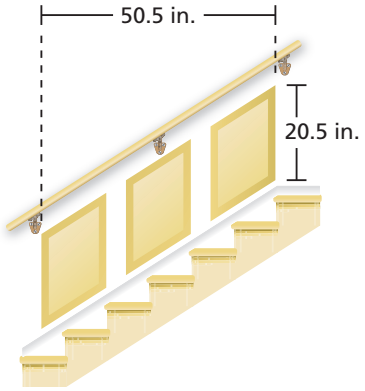


17. DECK Your deck has an area of 128 square feet. After adding a section, the area will be $s^2 + 128$ square feet. Draw a diagram of how this can happen.



18. T-SHIRT DESIGN You use the parallelogram-shaped sponge to create the T-shirt design. The area of the design is 66 square inches. How many times do you use the sponge to create the design? Draw a diagram to support your answer.

19. STAIRCASE The staircase has three parallelogram-shaped panels that are the same size. The horizontal distance between each panel is 4.25 inches. What is the area of one panel?



20. REASONING Find the missing dimensions in the table.

Parallelogram	Base	Height	Area
A	$x + 4$		$5x + 20$
B		8	$8x - 24$
C	6		$12x + 6y$

21. Logic Each dimension of a parallelogram is multiplied by a positive number n. Write an expression for the area of the new parallelogram.

Fair Game Review *What you learned in previous grades & lessons*

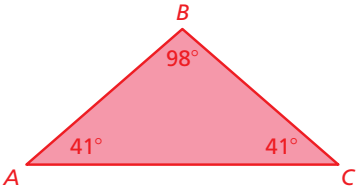
Use mental math to multiply. *(Skills Review Handbook)*

22. $\frac{1}{2} \times 26$ **23.** 82×20 **24.** 16×30 **25.** $\frac{1}{2} \times 236$

26. MULTIPLE CHOICE Which of the following describes angle B? *(Skills Review Handbook)*

Ⓐ acute Ⓑ obtuse

Ⓒ right Ⓓ isosceles



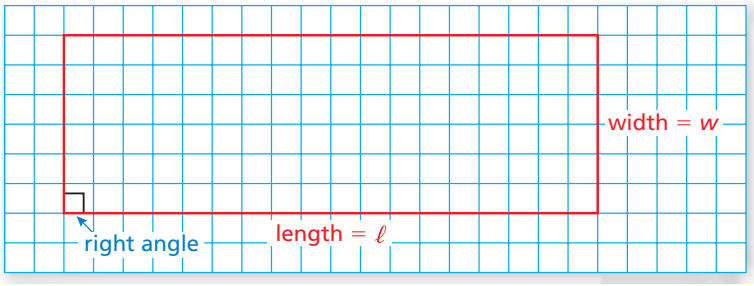
4.2 Areas of Triangles

Essential Question How can you derive a formula for the area of a triangle?

1 ACTIVITY: Deriving the Area Formula of a Triangle

Work with a partner.

a. Draw *any* rectangle on a piece of grid paper. An example is shown below. Label the length and width. Then find the area of your rectangle.



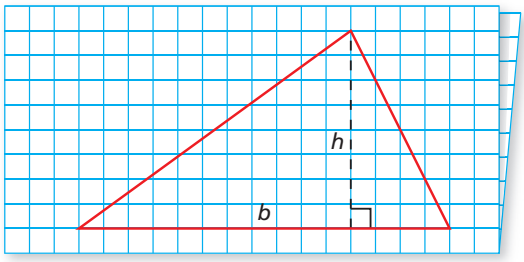
b. Draw a diagonal from one corner of your rectangle to the opposite corner. Cut along the diagonal. Compare the area of the rectangle with the area of the two pieces you cut. What do you notice? Use your results to write a formula for the area A of a triangle.

$A =$ _____ Formula

2 ACTIVITY: Deriving the Area Formula of a Triangle

Work with a partner.

a. Fold a piece of grid paper in half. Draw a triangle so that its base lies on one of the horizontal lines of the paper. Do not use a right triangle. Label the height and the base *inside* the triangle.



fold

b. Estimate the area of your triangle by counting unit squares.

Area ≈ _____ Estimate

c. Cut out the triangle so that you end up with two identical triangles. Form a quadrilateral whose area you know. What type of quadrilateral is it? Explain how you *know* it is this type.

d. Use your results to write a formula for the area of a triangle. Then use your formula to find the exact area of your triangle. Compare this area with your estimate in part (b).

Geometry

In this lesson, you will

- find areas of triangles.
- solve real-life problems.

Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Deductive reasoning is again used to make a conjecture about a new formula, the area of a triangle. Students should be able to analyze the two approaches used to derive the formula for the area of a triangle.

Motivate

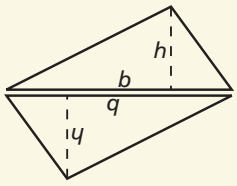
- The Flatiron (or Fuller) Building in New York City was built in 1902 on a triangular block formed by Fifth Avenue, Broadway and East 22nd Street.
- "Why do you think it is called the Flatiron Building?" Its shape is similar to a flatiron (for pressing clothes).
- "If you had an office in that building, where would you want it to be and why?" I personally would want the top floor at the most acute vertex!
- Triangles are a fairly uncommon shape for a building.

Activity Notes

Activity 1

- **MP3:** Listen for a logical argument: because there are two triangles of equal area, each must be half of the area of the rectangle. Students will likely write $A = \ell w \div 2$
- "Is there another way to write the formula?" yes; $A = \frac{1}{2}\ell w$
- Remind students that when two variables, or a variable and number, are written adjacent to one another it implies multiplication.
- **Extension:** Ask, "Can you can start with a non-right parallelogram and cut along the diagonal?" yes; Take time to explore this question.

Activity 2



- **Common Error:** In part (c), students may arrange their triangles with the bases together. The labels for the base and height have no meaning for this quadrilateral. Students need to arrange the triangles so that the base of the triangle is also the base of the quadrilateral.
- When students have finished, ask how they estimated the area of their triangle. You want to hear students describe how they dealt with nonsquare pieces of the grid paper.
- "Did anyone end up with a rectangle when they arranged their triangles? How would this be possible?" Their triangle was a right triangle.
- "Did anyone end up with a square when they arranged their triangles? How would this be possible?" Their triangle was a right isosceles triangle.
- "How are the first two activities the same?" Both activities are deriving the formula for area of a triangle, but using different methods.

Common Core State Standards

6.G.1 Find the area of right triangles, other triangles, . . . by composing into rectangles . . . ; apply these techniques in the context of solving real-world and mathematical problems.

Previous Learning

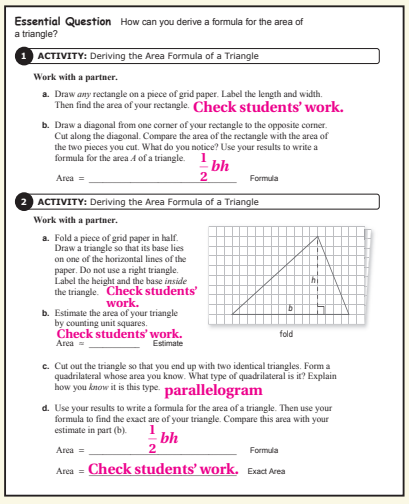
Students have found the area of a square, rectangle, and parallelogram.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

4.2 Record and Practice Journal



Essential Question How can you derive a formula for the area of a triangle?

1 ACTIVITY: Deriving the Area Formula of a Triangle

Work with a partner.

a. Draw *any* rectangle on a piece of grid paper. Label the length and width. Then find the area of your rectangle. **Check students' work.**

b. Draw a diagonal from one corner of your rectangle to the opposite corner. Cut along the diagonal. Compare the area of the rectangle with the area of the two pieces you cut. What do you notice? Use your results to write a formula for the area A of a triangle. $\frac{1}{2}bh$

Area = _____ Formula

2 ACTIVITY: Deriving the Area Formula of a Triangle

Work with a partner.

a. Fold a piece of grid paper in half. Draw a triangle so that its base lies on one of the horizontal lines of the paper. Do not use a right triangle. Label the height and the base *inside* the triangle. **Check students' work.**

b. Estimate the area of your triangle by counting unit squares. **Check students' work.**

Area = _____ Estimate

fold

c. Cut out the triangle so that you end up with two identical triangles. Form a quadrilateral whose area you know. What type of quadrilateral is it? Explain how you *know* it is this type. **parallelogram**

d. Use your results to write a formula for the area of a triangle. Then use your formula to find the exact are of your triangle. Compare this area with your estimate in part (b). $\frac{1}{2}bh$

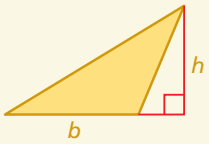
Area = _____ Formula

Area = **Check students' work.** Exact Area

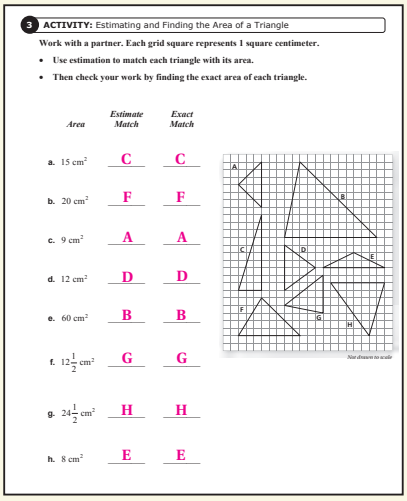
English Language Learners

Vocabulary

Review the definition of perpendicular. Two lines are perpendicular if the lines form a right angle. The corner of a sheet of paper is a right angle. For obtuse triangles, the height (the line perpendicular to the base) may lie outside of the triangle.



4.2 Record and Practice Journal



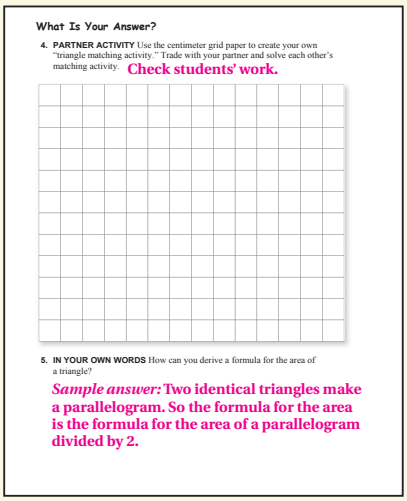
3 **ACTIVITY:** Estimating and Finding the Area of a Triangle

Work with a partner. Each grid square represents 1 square centimeter.

- **Use estimation to match each triangle with its area.**
- **Then check your work by finding the exact area of each triangle.**

Area	Estimate Match	Exact Match
a. 15 cm²	C	C
b. 20 cm²	F	F
c. 9 cm²	A	A
d. 12 cm²	D	D
e. 60 cm²	B	B
f. $12\frac{1}{2}$ cm²	G	G
g. $24\frac{1}{2}$ cm²	H	H
h. 8 cm²	E	E

Not drawn to scale



What Is Your Answer?

4. **PARTNER ACTIVITY** Use the centimeter grid paper to create your own "triangle matching activity." Trade with your partner and solve each other's matching activity. **Check students' work.**

5. **IN YOUR OWN WORDS** How can you derive a formula for the area of a triangle?

Sample answer: **Two identical triangles make a parallelogram. So the formula for the area is the formula for the area of a parallelogram divided by 2.**

Laurie's Notes

Activity 3

- Students will first estimate the area of each triangle. Again, they will have to take into account nonsquare pieces of the grid paper.
- Given the different orientations of the triangles, students will need to decide which side of the triangle should be the base. If they need assistance, suggest that they use the horizontal or vertical side of the triangle as the base.

? "How well did you do with your estimate?" Answers will vary.

- For each triangle, ask a student to explain how they found the exact area.

What Is Your Answer?

- **FYI:** If one of the sides of the triangle is not horizontal or vertical, it will be difficult to find the exact area.

Closure

- Another famous triangle is in the Atlantic Ocean. It is the Bermuda Triangle. Draw a sketch of the triangle and ask students to compute the area of this triangular region. about 450,000 mi^2

Math Practice

Calculate Accurately

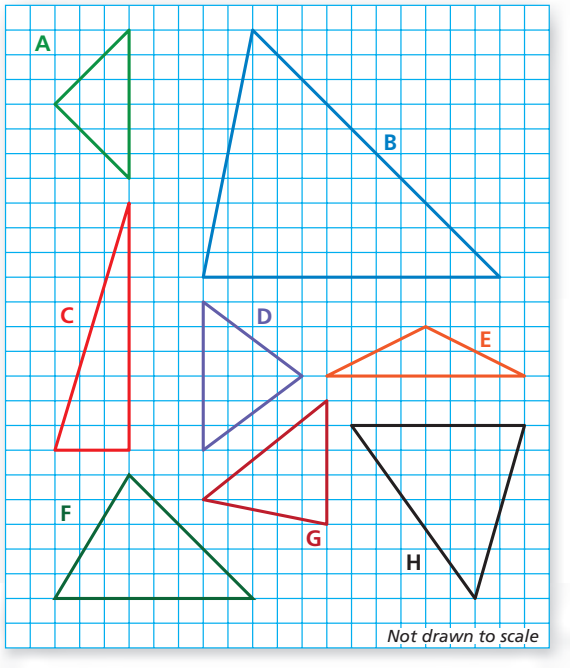
How can you estimate the area of each triangle so that the answer is close to the exact area?

3 ACTIVITY: Estimating and Finding the Area of a Triangle

Work with a partner. Each grid square represents 1 square centimeter.

- **Use estimation to match each triangle with its area.**
- **Then check your work by finding the exact area of each triangle.**

	Area	*Estimate Match*	*Exact Match*
a.	$15\ \text{cm}^2$		
b.	$20\ \text{cm}^2$		
c.	$9\ \text{cm}^2$		
d.	$12\ \text{cm}^2$		
e.	$60\ \text{cm}^2$		
f.	$12\frac{1}{2}\ \text{cm}^2$		
g.	$24\frac{1}{2}\ \text{cm}^2$		
h.	$8\ \text{cm}^2$		



What Is Your Answer?

4. **PARTNER ACTIVITY** Use a piece of centimeter grid paper to create your own "triangle matching activity." Trade with your partner and solve each other's matching activity.

5. **IN YOUR OWN WORDS** How can you derive a formula for the area of a triangle?

Practice Use what you learned about the areas of triangles to complete Exercises 3–5 on page 162.

4.2 Lesson

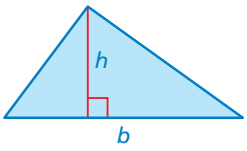
Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Idea

Area of a Triangle

Words The area A of a triangle is one-half the product of its base b and its height h.

Algebra $A = \frac{1}{2}bh$



EXAMPLE 1 Finding the Area of a Triangle

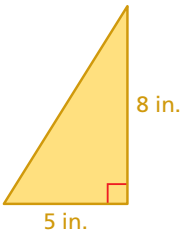
Find the area of the triangle.

$A = \frac{1}{2}bh$ Write formula.

$= \frac{1}{2}(5)(8)$ Substitute 5 for b and 8 for h.

$= \frac{1}{2}(40)$ Multiply 5 and 8.

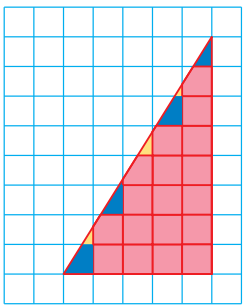
$= 20$ Multiply $\frac{1}{2}$ and 40.



∴ The area of the triangle is 20 square inches.

Remember
In Example 1, use the Associative Property of Multiplication to multiply 5 and 8 first.

Reasonable? Draw the triangle on grid paper and count unit squares. Each square in the grid represents 1 square inch.



Squares full or nearly full: 18

Squares about half full: 4

The area is $18(1) + 4\left(\frac{1}{2}\right) = 20$ square inches.

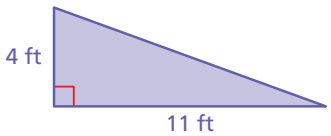
So, the answer is reasonable. ✓

On Your Own

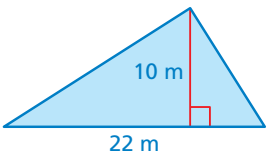
Now You're Ready
Exercises 3–8

Find the area of the triangle.

1.



2.



Laurie's Notes

Introduction

Connect

- **Yesterday:** Students derived the formula for the area of a triangle. (MP3)
- **Today:** Students will use the formula to find the areas of several triangles.

Motivate

- **Preparation:** Before class, make a triangular hat by folding a newspaper. Alternatively, you could search the Internet for a video of the hat (newspaper sailor hat) or a napkin (pyramid napkin folding) being folded. You want students to see a common folded triangular shape.
- The goal is to have students think about how much area of the original newspaper (or napkin) is showing.

Lesson Notes

Key Idea

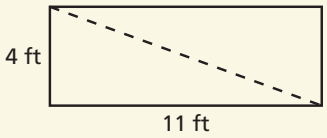
- The *base* of a triangle can be any of its sides.
- The *height* of a triangle is the perpendicular distance from a base to the opposite vertex.
- **FYI:** The formula $A = \frac{1}{2}bh$ is sometimes written $A = \frac{bh}{2}$. Students may make computational mistakes due to the fraction in this formula.

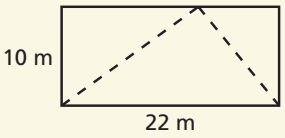
Example 1

- The idea of dividing by 2 as another way of taking $\frac{1}{2}$ of a number makes sense to students. For example: $\frac{1}{2}$ of 10 is the same as $10 \div 2$.
- **Common Error:** Students may want to take $\frac{1}{2}$ of 5 *and* $\frac{1}{2}$ of 8 because they think it is the Distributive Property.
- Note that the product of 5 and 8 was found (Associative Property), and then $\frac{1}{2}$ of that product was computed. Because multiplication is commutative, it would also be correct to find the product of $\frac{1}{2}$ and 8 (which is 4), and then multiply 4 by 5.
- The visual check for reasonableness is the approach used.

On Your Own

- **Extension:** Draw the following on the board and ask students how the area of the rectangle is related to the answers they found for Questions 1 and 2.





Goal Today's lesson is finding the area of a triangle.

Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

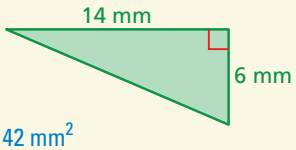
Differentiated Instruction

Inclusion

When using the formula for the area of a triangle, $A = \frac{1}{2}bh$, tell students to simplify the calculation by dividing b or h by 2, whichever is even.

Extra Example 1

Find the area of the triangle.



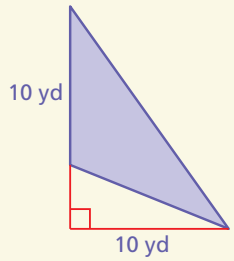
42 mm^2

On Your Own

1. 22 ft^2
2. 110 m^2

Extra Example 2

Find the area of the triangle.



50 yd^2

Extra Example 3

A triangle has a base of 2 inches and a height of 3 inches. The base and height of a second triangle are four times greater than the base and height of the first triangle. How many times greater is the area of the second triangle than the area of the first triangle? 16 times greater

On Your Own

3. 60 cm^2

4. 9 times greater

Laurie's Notes

Example 2

- **MP1 Make Sense of Problems and Persevere in Solving Them:** Mathematically proficient students will check their answers to problems using different methods. Finding $\frac{1}{2}$ of 12 and then multiplying by 9 is different from finding the product of 12 and 9 and then taking $\frac{1}{2}$. Ask students if they can think of examples where one of the two approaches is easier than the other. Example: $\frac{1}{2}(2.5)(4)$

? "How do you find $\frac{1}{2} \times 12 \times 9$?" Listen for the strategies of taking $\frac{1}{2}$ of 12 first, and then multiplying by 9; or finding the product of 12 and 9, and then taking $\frac{1}{2}$ of the product.

- In some cases, the base of a triangle needs to be extended in order to find the height.

Example 3

- Take time to write the formula for the area of a triangle. It is good practice for students to go through the substitution process.
- **Common Error:** The question asks how many times greater, not how much greater. Students may give an answer of 3 square centimeters because they subtracted the two areas.

On Your Own

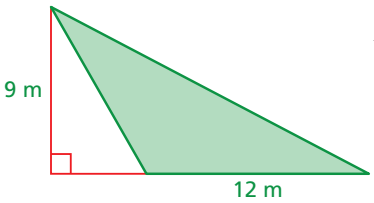
- A diagram of this type often confuses students. The red segments are not part of the lengths of the sides of the triangle.

Closure

- Have students find the area of the newspaper hat (or triangular napkin) by measuring the necessary dimensions.
- If you did not make the hat or napkin, you could ask students to finish the following writing prompt.
  The area of a triangle is similar to the area of a parallelogram because. . .

EXAMPLE 2 Finding the Area of a Triangle

Find the area of the triangle.



$A = \frac{1}{2}bh$ Write formula.

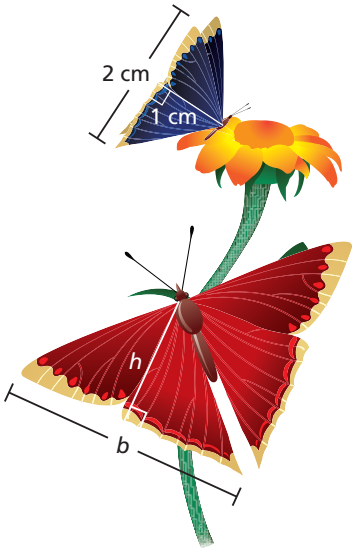
$= \frac{1}{2}(12)(9)$ Substitute 12 for b and 9 for h.

$= 54$ Multiply.

∴ The area of the triangle is 54 square meters.

EXAMPLE 3 Real-Life Application

The base and height of the red butterfly wing are two times greater than the base and height of the blue butterfly wing. How many times greater is the area of the red wing than the area of the blue wing?



Find the area of the blue wing.

$A = \frac{1}{2}bh$ Write formula.

$= \frac{1}{2}(2)(1)$ Substitute 2 for b and 1 for h.

$= 1 \text{ cm}^2$ Multiply.

The red wing dimensions are 2 times greater, so the base is $2 \times 2 = 4$ cm and the height is $2 \times 1 = 2$ cm. Find the area of the red wing.

$A = \frac{1}{2}bh$ Write formula.

$= \frac{1}{2}(4)(2)$ Substitute 4 for b and 2 for h.

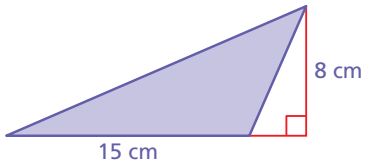
$= 4 \text{ cm}^2$ Multiply.

∴ Because $\frac{4 \text{ cm}^2}{1 \text{ cm}^2} = 4$, the area of the red wing is 4 times greater.

On Your Own

Now You're Ready
Exercises 12–14

3. Find the area of the triangle.



4. **WHAT IF?** In Example 3, the base and the height of the red butterfly wing are three times greater than those of the blue wing. How many times greater is the area of the red wing?

4.2 Exercises

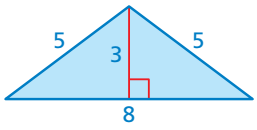
Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **CRITICAL THINKING** Can *any* side of a triangle be labeled as its base? Explain.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

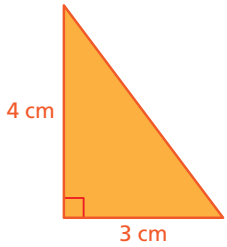
What is the area of the triangle?	What is the distance around the triangle?
How many unit squares fit in the triangle?	What is one-half the product of the base and the height?



Practice and Problem Solving

Find the area of the triangle.

1

3.


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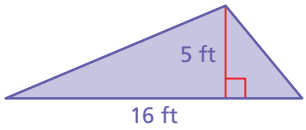

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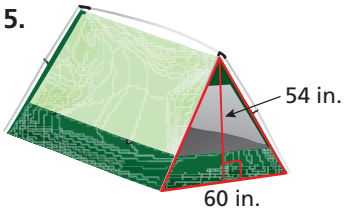

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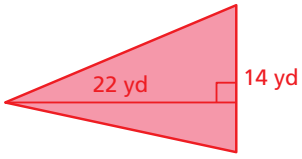

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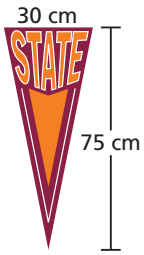

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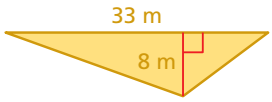
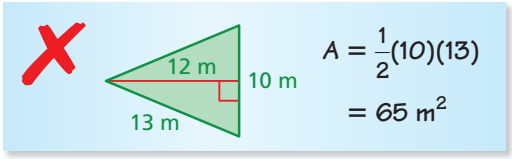
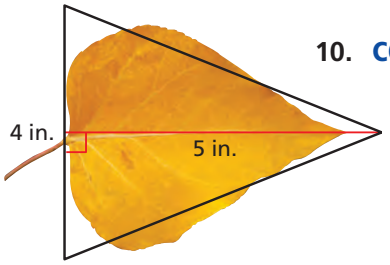


9. **ERROR ANALYSIS** Describe and correct the error in finding the area of the triangle.





10. **COTTONWOOD LEAF** Estimate the area of the cottonwood leaf.

11. **CORNER SHELF** A shelf has the shape of a triangle. The base of the shelf is 36 centimeters, and the height is 18 centimeters. Find the area of the shelf.

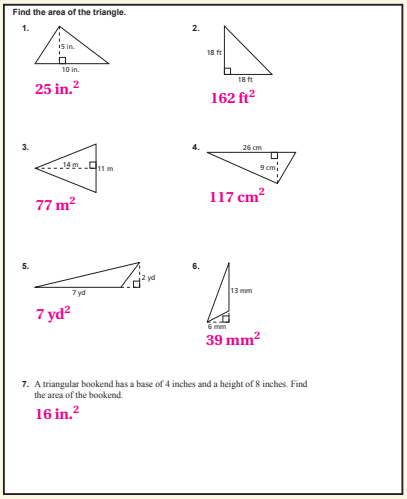
Assignment Guide and Homework Check

Level	Day 1 Activity Assignment	Day 2 Lesson Assignment	Homework Check
Basic	3–5, 21–24	1, 2, 6–13, 15	8, 11, 13, 15
Average	3–5, 21–24	1, 2, 6–14, 16, 18, 19	8, 11, 14, 16
Advanced	3–5, 21–24	1, 2, 6–9, 12–14, 16, 17, 19, 20	8, 12, 14, 19

Common Errors

- **Exercises 3–8** Students may multiply the base and the height and forget to multiply by $\frac{1}{2}$. Tell students to write the formula, and then identify the value of each variable before substituting.
- **Exercises 3–8** Students may mentally find the area by multiplying one-half the base and one-half the height. The answer will be one-half of the value of the correct answer. Tell students to write the formula, and then identify the value of each variable before substituting.
- **Exercises 12–14** Students may be confused by the drawing and labeling of obtuse triangles. Remind students to identify the base of the triangle. Then the height of the triangle is the perpendicular distance from the base to the opposite vertex. For obtuse triangles, the height may be labeled outside of the triangle.

4.2 Record and Practice Journal


Find the area of the triangle.

1. 5 in., 10 in. — 25 in.^{2}

2. 18 ft, 18 ft — 162 ft^2

3. 14 m, 11 m — 77 m^2

4. 26 cm, 9 cm — 117 cm^2

5. 7 yd, 2 yd — 7 yd^2

6. 13 mm, 6 mm — 39 mm^2

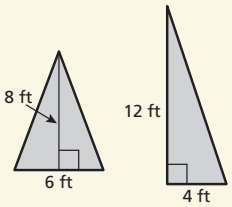
7. A triangular bookend has a base of 4 inches and a height of 8 inches. Find the area of the bookend.
16 in.^{2}

Vocabulary and Concept Check

1. yes; To find the area of the triangle, you must also know the height of the triangle. That is, the perpendicular distance from the base to the opposite vertex.
2. What is the distance around the triangle?; 18 units; 12 units^2

Practice and Problem Solving

3. 6 cm^2
4. 40 ft^2
5. 1620 in.^{2}
6. 154 yd^2
7. 1125 cm^2
8. 132 m^2
9. The side length of 13 meters was used instead of the height.
   $A = \frac{1}{2}(10)(12) = 60$ m^2
10. about 10 in.^{2}
11. 324 cm^2
12. 68 m^2
13. 90 mi^2
14. 189 mm^2
15. *Sample answer:*

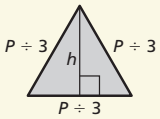


Practice and Problem Solving

16. See *Taking Math Deeper*.

17. x^2 times greater

18. Find the length of the base by dividing the perimeter by 3. Then multiply one-half of the base by the height.



19. 4 times greater

20. 6 ft

Fair Game Review

21. Mult. Prop. of One

22. Comm. Prop. of Mult.

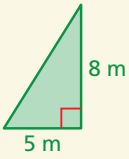
23. Assoc. Prop. of Add.

24. C

Mini-Assessment

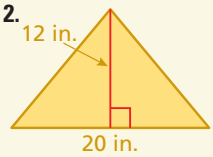
Find the area of the triangle.

1.



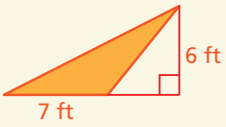
20 m^2

2.



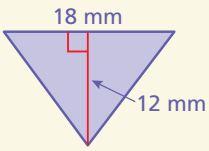
120 $in.^2$

3.



21 ft^2

4.



108 mm^2

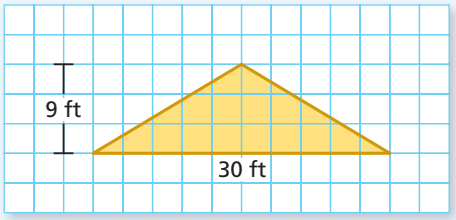
5. A right triangle has a base of 4 inches and a height of 2 inches. The base and height of a second right triangle are two times greater than the base and the height of the first right triangle. How many times greater is the area of the second triangle than the area of the first triangle? 4 times greater

Taking Math Deeper

Exercise 16

Some questions involving mathematics have a straightforward "math" answer. But when you think about the real-life situation, you realize that the answer is more complicated. This is an example of one of those types of questions.

1 Finding the area of the sail of the hang glider is straightforward.



a. $A = \frac{1}{2}bh$

$= \frac{1}{2} \cdot 30 \cdot 9$

$= 135 \text{ ft}^2$

2 We have found the area of the fabric for the completed hang glider. You will most likely choose to give full credit for this answer. However, you might also choose to take the question more literally. Anyone who sews knows that you need extra fabric, to account for seams and losses in cutting.

From the photograph, you can see the seams in the hang glider. Suppose the fabric comes in a width of 3 feet. How could you layout the fabric to produce an isosceles triangle with a base of 30 feet and a height of 9 feet? This is a challenging question.

3 **b.** In general, the greater the weight of the pilot, the greater the area of the hang glider's sail.

For example:

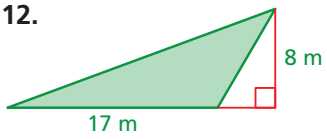
Sail Area	*Pilot Weight*
126 ft^2	110–176 lbs
148 ft^2	150–215 lbs
160 ft^2	185–275 lbs

Reteaching and Enrichment Strategies

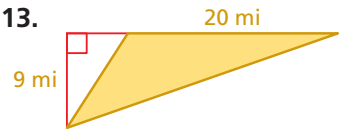
If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Start the next section

Find the area of the triangle.

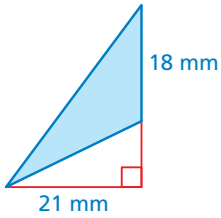
12.



13.



14.

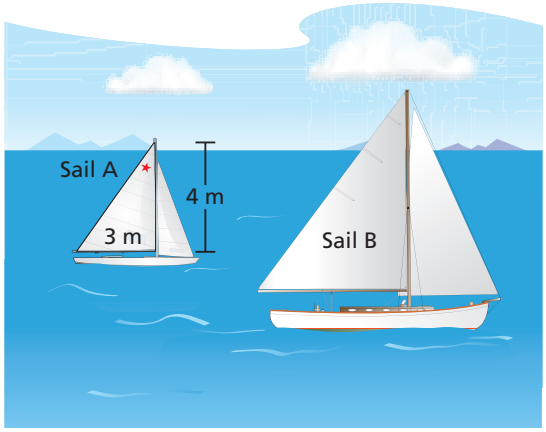


15. OPEN-ENDED Draw and label two triangles that each have an area of 24 square feet.

16. HANG GLIDING The wingspan of the triangular hang glider is 30 feet.

a. How much fabric is needed to make the sail?

b. RESEARCH Use the Internet or some other source to find how the area of the sail is related to the weight limit of the pilot.



17. SAILBOATS The base and the height of Sail B are x times greater than the base and the height of Sail A. How many times greater is the area of Sail B? Write your answer as a power.

18. WRITING You know the height and the perimeter of an equilateral triangle. Explain how to find the area of the triangle. Draw a diagram to support your reasoning.

19. REASONING The base and the height of Triangle A are half the base and the height of Triangle B. How many times greater is the area of Triangle B?

20. Critical Thinking The total area of the polygon is 176 square feet. Find the value of x.

Fair Game Review What you learned in previous grades & lessons

Tell which property is illustrated by the statement. *(Section 3.3)*

21. $n \cdot 1 = n$ **22.** $4 \cdot m = m \cdot 4$ **23.** $(x + 2) + 5 = x + (2 + 5)$

24. MULTIPLE CHOICE What is the first step when using order of operations? *(Section 1.3)*

(A) Multiply or divide from left to right. (B) Add or subtract from left to right.

(C) Perform operations in parentheses. (D) Evaluate numbers with exponents.

4 Study Help

Check It Out
Graphic Organizer
BigIdeasMath.com

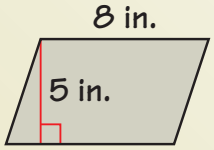
You can use a **four square** to organize information about a topic. Each of the four squares can be a category, such as *definition, vocabulary, example, non-example, words, algebra, table, numbers, visual, graph,* or *equation*. Here is an example of a four square for the area of a parallelogram.

Area of a parallelogram

Words
The area A of a parallelogram is the product of its base b and its height h.

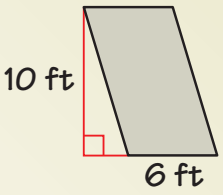
Algebra
$A = bh$

Example
$A = bh$
$= 8(5)$
$= 40$



The area of the parallelogram is 40 square inches.

Example
$A = bh$
$= 6(10)$
$= 60$



The area of the parallelogram is 60 square feet.

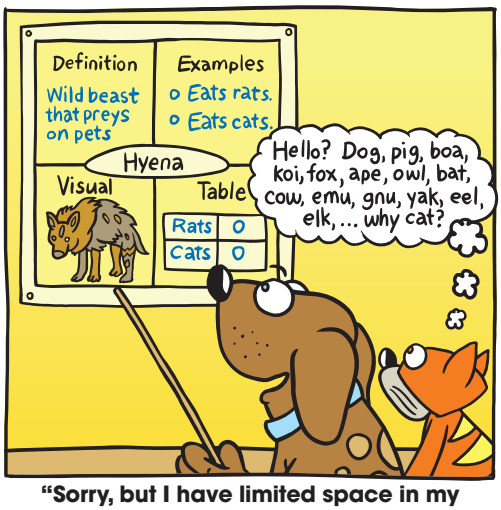
On Your Own

Make a four square to help you study the topic.

1. area of a triangle

After you complete this chapter, make four squares for the following topics.

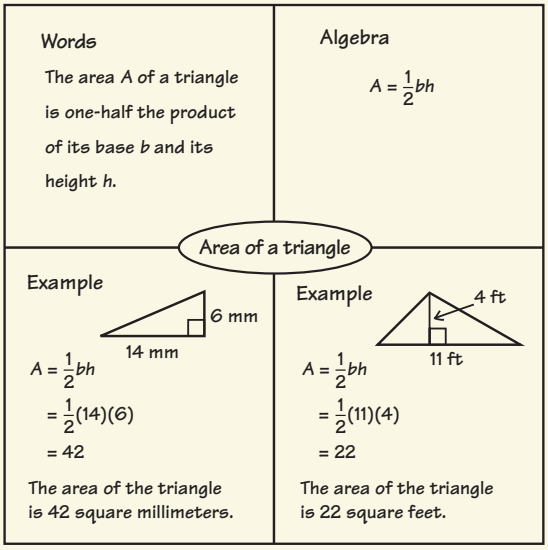
2. area of a trapezoid
3. area of a composite figure
4. drawing a polygon in a coordinate plane
5. finding distances in the first quadrant



"Sorry, but I have limited space in my four square. I needed pet names with only three letters."

Sample Answer

1.



List of Organizers

Available at *BigIdeasMath.com*

Comparison Chart
Concept Circle
Definition (Idea) and Example Chart
Example and Non-Example Chart
Formula Triangle
Four Square
Information Frame
Information Wheel
Notetaking Organizer
Process Diagram
Summary Triangle
Word Magnet
Y Chart

About this Organizer

A **Four Square** can be used to organize information about a topic. Students write the topic in the "bubble" in the middle of the four square. Then students write concepts related to the topic in the four squares surrounding the bubble. Any concept related to the topic can be used. Encourage students to include concepts that will help them learn the topic. Students can place their four squares on note cards to use as a quick study reference.

Technology for the Teacher

Editable Graphic Organizer

Answers

1. 14 cm^2
2. 5950 in.^2
3. 384 yd^2
4. 882 mi^2
5. 28 cm^2
6. 78 m^2
7. 9 mi^2
8. 3 sheets

Alternative Quiz Ideas

100% Quiz	Math Log
Error Notebook	Notebook Quiz
Group Quiz	Partner Quiz
Homework Quiz	Pass the Paper

Group Quiz

Students work in groups. Give each group a large index card. Each group writes five questions that they feel evaluate the material they have been studying. On a separate piece of paper, students solve the problems. When they are finished, they exchange cards with another group. The new groups work through the questions on the card.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Lesson Tutorials *BigIdeasMath.com*	Resources by Chapter • Enrichment and Extension • Technology Connection Game Closet at *BigIdeasMath.com* Start the next section

Technology for the Teacher

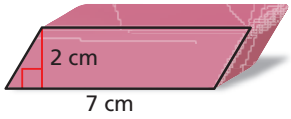
Online Assessment
Assessment Book
ExamView® Assessment Suite

4.1–4.2 Quiz

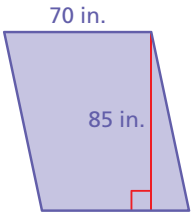
Check It Out
Progress Check
BigIdeasMath.com

Find the area of the parallelogram. *(Section 4.1)*

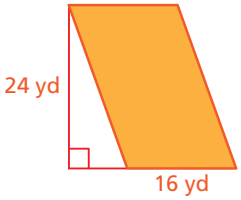
1.



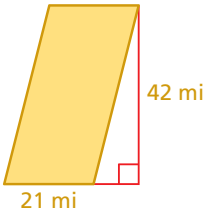
2.



3.

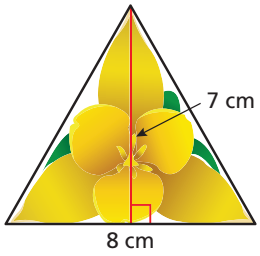


4.

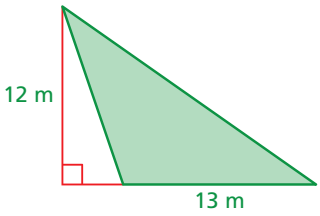


Find the area of the triangle. *(Section 4.2)*

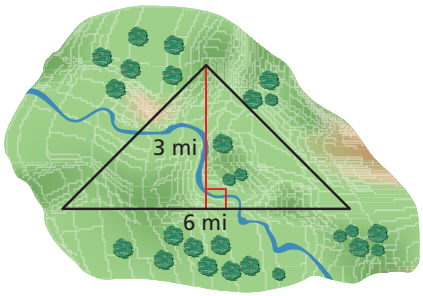
5.



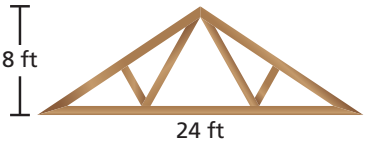
6.



7. LAND A wildlife conservation group buys a plot of land. How much land does it buy? *(Section 4.2)*



8. FRAMING A sheet of plywood is 4 feet wide by 8 feet long. What is the minimum number of sheets of plywood needed to cover the frame? Justify your answer. *(Section 4.2)*



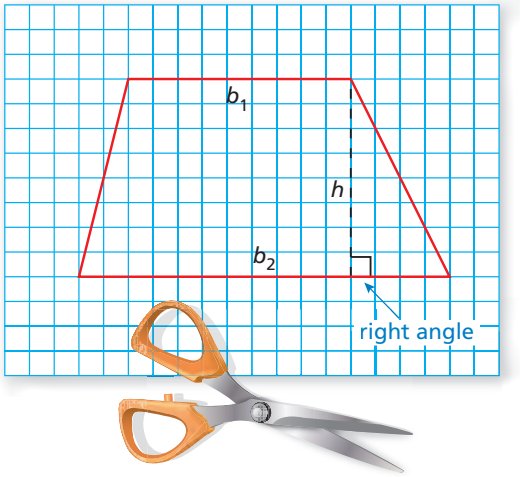
4.3 Areas of Trapezoids

Essential Question How can you derive a formula for the area of a trapezoid?

1 ACTIVITY: Deriving the Area Formula of a Trapezoid

Work with a partner. Use a piece of centimeter grid paper.

a. Draw *any* trapezoid so that its base lies on one of the horizontal lines of the paper.

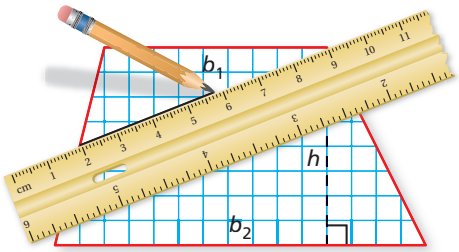


b. Estimate the area of your trapezoid (in square centimeters) by counting unit squares.

Area ≈ _____ Estimate

c. Label the height and the bases *inside* the trapezoid.

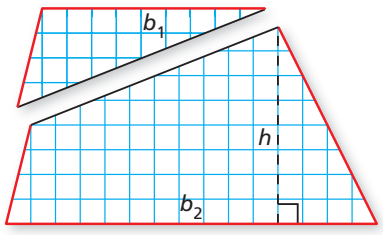
d. Cut out the trapezoid. Mark the midpoint of the side opposite the height. Draw a line from the midpoint to the opposite upper vertex.



e. Cut along the line. You will end up with a triangle and a quadrilateral. Arrange these two figures to form a figure whose area you know.

f. Use your result to write a *formula* for the area of a trapezoid.

Area = _____ Formula



g. Use your formula to find the area of your trapezoid (in square centimeters).

Area = _____ Exact Area

h. Compare this area with your estimate in part (b).

Geometry

In this lesson, you will

- find areas of trapezoids.
- solve real-life problems.

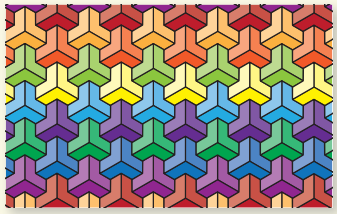
Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Deductive reasoning is again used to make a conjecture about a new formula, the area of a trapezoid. Students should be able to construct an argument for deriving this formula.

Motivate



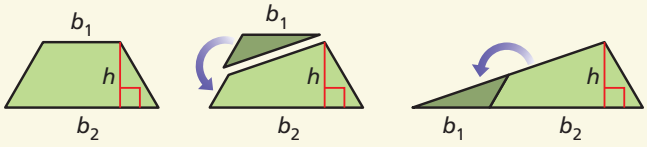
- Display this common optical design. Identify the concave hexagons that fit together to make this tessellation.
- "Can you draw a segment to change each hexagon into two congruent quadrilaterals? yes What type of quadrilateral will you have?" trapezoid

- Explain that today's activity involves exploring the area of a trapezoid.

Activity Notes

Activity 1

- "How are trapezoids and parallelograms alike? different?" Both are 4-sided. A parallelogram has two pairs of opposite sides that are parallel and congruent. A trapezoid has only one pair of opposite sides that are parallel.
- Review the dimensions of a trapezoid: the two parallel bases are often called b_1 and b_2. The dimensions are read "*b* sub 1" and "*b* sub 2." The two bases are not the same length, so to distinguish between them the same variable is used with different subscripts.
- Students may need assistance with part (d). If the height is an even number, students can locate the midpoint by moving along the vertical grid line.



- Students will need to experiment to see how to form a new shape from the triangle and quadrilateral. The new triangle will have the same height as the original trapezoid. The base of the new triangle is the sum of the two bases of the trapezoid. Students may need help in writing the formula for the area of a trapezoid because parentheses are needed.
- Discuss the formula students wrote for the area of a trapezoid. Ask them to explain how they derived the formula.
- **Alternate Approach:** Have students cut out 2 congruent trapezoids and label the bases and height. Arrange the trapezoids to form a parallelogram.
  - **MP3:** Students should be able to construct an argument to find the area of one trapezoid. The constructed parallelogram has an area of $(b_1 + b_2)h$. Half of the area of this parallelogram is the area of the trapezoid.

Common Core State Standards

6.G.1 Find the area of . . . special quadrilaterals, . . . by . . . decomposing into triangles . . . ; apply these techniques in the context of solving real-world and mathematical problems.

Previous Learning

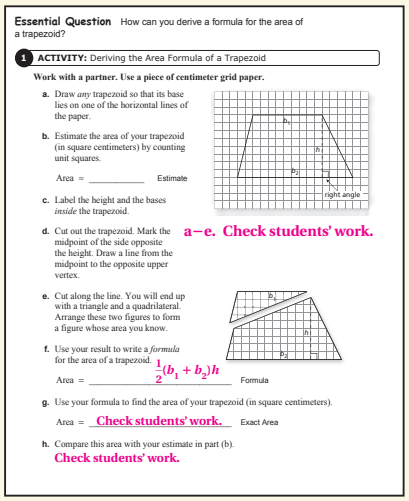
Students have found the area of a square, rectangle, parallelogram, and triangle.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

4.3 Record and Practice Journal



Essential Question How can you derive a formula for the area of a trapezoid?

1 ACTIVITY: Deriving the Area Formula of a Trapezoid

Work with a partner. Use a piece of centimeter grid paper.

a. Draw *any* trapezoid so that its base lies on one of the horizontal lines of the paper.

b. Estimate the area of your trapezoid (in square centimeters) by counting unit squares.

Area = _____ Estimate

c. Label the height and the bases *inside* the trapezoid.

d. Cut out the trapezoid. Mark the midpoint of the side opposite the height. Draw a line from the midpoint to the opposite upper vertex.

a–e. **Check students' work.**

e. Cut along the line. You will end up with a triangle and a quadrilateral. Arrange these two figures to form a figure whose area you know.

f. Use your result to write a *formula* for the area of a trapezoid.

Area = $\frac{1}{2}(b_1 + b_2)h$ Formula

g. Use your formula to find the area of your trapezoid (in square centimeters).

Area = **Check students' work.** Exact Area

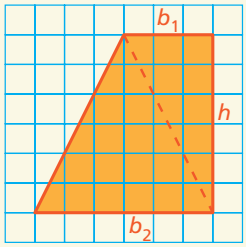
h. Compare this area with your estimate in part (b).

Check students' work.

Differentiated Instruction

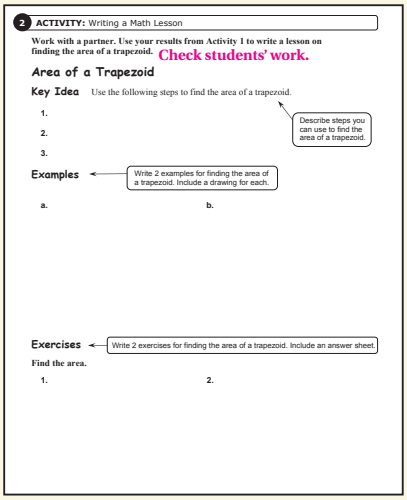
Kinesthetic

Have students draw a trapezoid on grid paper. Then draw a diagonal to form two triangles that are the same height as the trapezoid.



The bases of the trapezoid become the bases of the triangles. Find the area of the trapezoid by summing the areas of the two triangles.

4.3 Record and Practice Journal



2 **ACTIVITY:** Writing a Math Lesson

Work with a partner. Use your results from Activity 1 to write a lesson on finding the area of a trapezoid. Check students' work.

Area of a Trapezoid

Key Idea Use the following steps to find the area of a trapezoid.

1.
2.
3.

Describe steps you can use to find the area of a trapezoid.

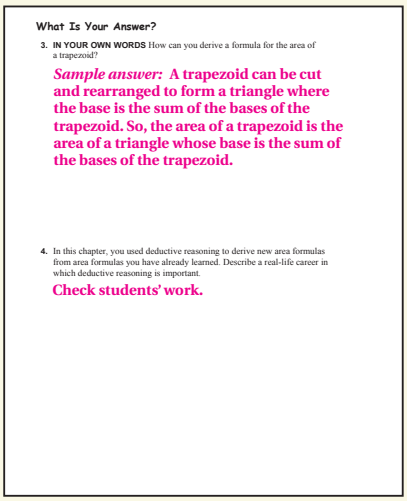
Examples Write 2 examples for finding the area of a trapezoid. Include a drawing for each.

a. b.

Exercises Write 2 exercises for finding the area of a trapezoid. Include an answer sheet.

Find the area.

1. 2.



What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you derive a formula for the area of a trapezoid?

Sample answer: A trapezoid can be cut and rearranged to form a triangle where the base is the sum of the bases of the trapezoid. So, the area of a trapezoid is the area of a triangle whose base is the sum of the bases of the trapezoid.

4. In this chapter, you used deductive reasoning to derive new area formulas from area formulas you have already learned. Describe a real-life career in which deductive reasoning is important.

Check students' work.

Laurie's Notes

Activity 2

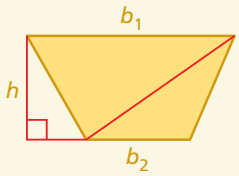
- This activity will give you insight into how students think about finding the area of a trapezoid.
- When partners have finished, have them exchange their lessons with another pair of students. Everyone should read the lesson they receive and write a critique to the authors of the lesson. They should provide feedback on what is clear in the lesson and what can be made clearer.
- Have students complete the exercises of the lesson they received.

What Is Your Answer?

- For Question 4, there are several popular television shows which feature deductive reasoning. Hopefully students will be able to list several careers, such as crime investigation and medical research.

Closure

- Draw the following figure for students. Explain that a diagonal of the trapezoid has been drawn.



- Have students explain to their partners how they might derive the area of a trapezoid from this figure.

2 ACTIVITY: Writing a Math Lesson

Work with a partner. Use your results from Activity 1 to write a lesson on finding the area of a trapezoid.

Math Practice

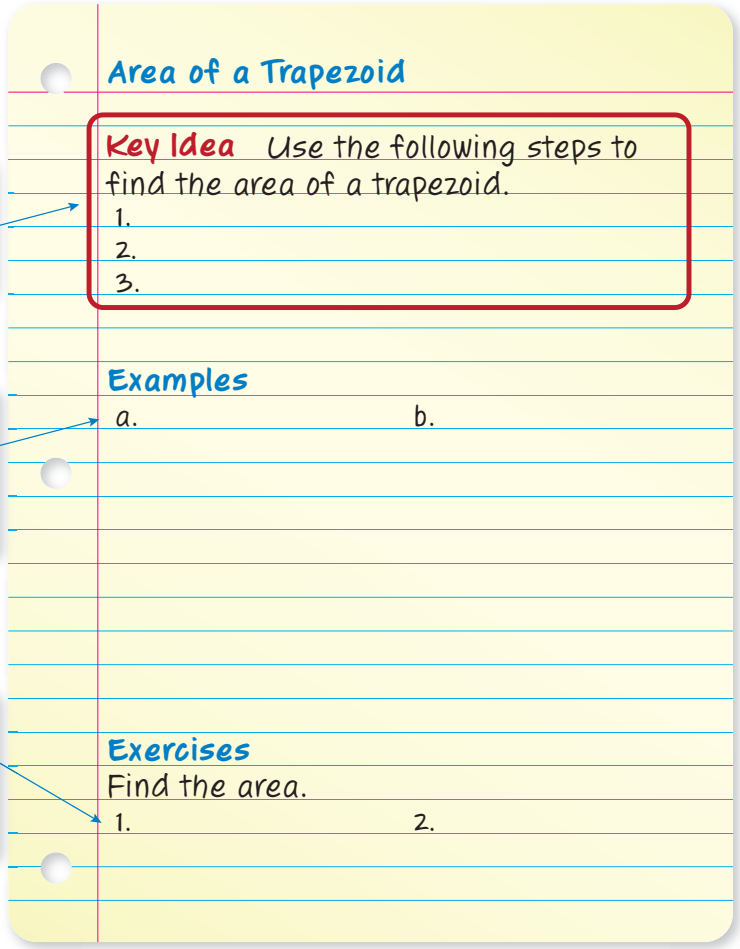
Use Clear Definitions

Do your steps for the *Key Idea* help another person understand how to solve the problem? Do the examples follow your steps?

Describe steps you can use to find the area of a trapezoid.

Write two examples for finding the area of a trapezoid. Include a drawing for each.

Write two exercises for finding the area of a trapezoid. Include an answer sheet.



What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you derive a formula for the area of a trapezoid?

4. In this chapter, you used deductive reasoning to derive new area formulas from area formulas you have already learned. Describe a real-life career in which deductive reasoning is important.

Practice Use what you learned about the areas of trapezoids to complete Exercises 4–6 on page 170.

4.3 Lesson

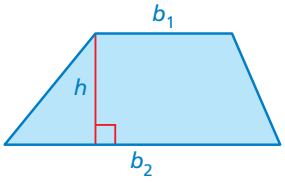
Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Idea

Area of a Trapezoid

Words The area A of a trapezoid is one-half the product of its height h and the sum of its bases b_1 and b_2.

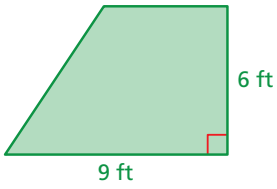
Algebra $A = \frac{1}{2}h(b_1 + b_2)$



EXAMPLE 1 Finding Areas of Trapezoids

Find the area of each trapezoid.

a.



$A = \frac{1}{2}h(b_1 + b_2)$ Write formula.

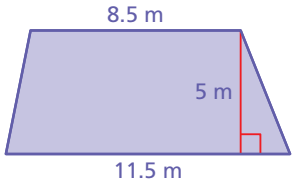
$= \frac{1}{2}(6)(5 + 9)$ Substitute.

$= \frac{1}{2}(6)(14)$ Add.

$= 42$ Multiply.

∴ The area of the trapezoid is 42 square feet.

b.



$A = \frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{2}(5)(8.5 + 11.5)$

$= \frac{1}{2}(5)(20)$

$= 50$

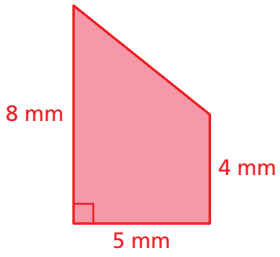
∴ The area of the trapezoid is 50 square meters.

On Your Own

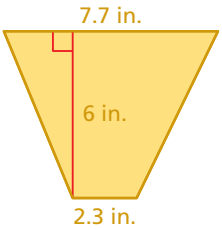
Now You're Ready
Exercises 7–9

Find the area of the trapezoid.

1.



2.



Laurie's Notes

Introduction

Connect

- **Yesterday:** Students derived the formula for the area of a trapezoid. (MP3)
- **Today:** Students will use the formula to find the areas of several trapezoids.

Motivate

- Historically, the word *trapezoid* refers to a quadrilateral with no parallel sides and a *trapezium* refers to a quadrilateral with only one pair of opposite sides that are parallel. In the 1700s a mathematical dictionary published in the United States reversed the meanings of the words and the definition of a trapezoid as a quadrilateral with only one pair of opposite sides that are parallel became accepted. Some countries today, such as England, still use the historical definitions.

Lesson Notes

Key Idea

- The *height* of a trapezoid is the perpendicular distance between the parallel bases.
- Write and discuss the area of a trapezoid formula.
- Again, remind students that to find $\frac{1}{2}$ of a number you can divide the number by 2.

Example 1

- Work through both examples.
- Write the formula first. Show how the values are substituted for the variables. In performing the operations, find the sum first. Then find the product of the sum and the height of the trapezoid. Finally, divide the product by 2.
- Discuss the factors and terms involved in the formula: $\frac{1}{2}$, h, and $(b_1 + b_2)$ and the order of operations.

? "How else might the computations be done in the first example?" Listen for: First multiply $\frac{1}{2}$ and 6. Then use the Distributive Property, or add $5 + 9$ and then multiply by 3.

- **MP7 Look for and Make Use of Structure:** You want students to be able to recognize that while there are three variables, $(b_1 + b_2)$ is one factor in the formula. There are three factors, $\frac{1}{2}$, h, and $(b_1 + b_2)$, and there are different ways in which the factors can be multiplied. If the height is an odd number, you might not want to take $\frac{1}{2}$ of it as the first step.

On Your Own

- In Question 1, note that the height is a side of the trapezoid, and that it is not a vertical segment.

Goal Today's lesson is finding the area of a trapezoid.

Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

English Language Learners

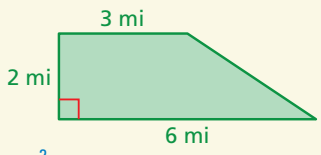
Illustrate

Have students look around the classroom, school building, and school grounds for shapes they have studied in this chapter. Students can find the perimeter or the area of a shape and can identify what types of workers would need to know this information for their jobs.

Extra Example 1

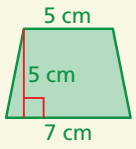
Find the area of each trapezoid.

a.



9 mi^2

b.



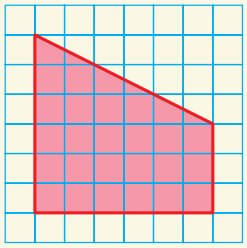
30 cm^2

On Your Own

1. 30 mm^2
2. 30 in.^2

Extra Example 2

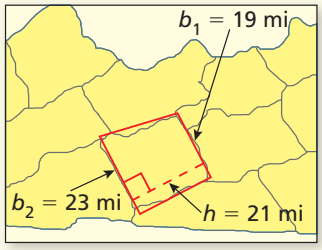
Find the area of the trapezoid.



27 units^2

Extra Example 3

A trapezoid can be used to approximate the shape of Smyth County, Virginia. The population is about 32,000. About how many people are there per square mile?



about 73 people per square mile

On Your Own

3. $10\frac{1}{2}$ units^2

4. about 1 person per square mile;
$\frac{22{,}650 \text{ people}}{530 \text{ mi}^2} \approx 43$
people per square mile

Laurie's Notes

Example 2

"How can you find the dimensions of the trapezoid?" Count the grid lines.

- **MP1 Make Sense of Problems and Persevere in Solving Them:** Mathematically proficient students will check their answers to problems using different methods. Finding one-half of 6 and then multiplying by 3 is different from finding the product of 6 and 3 and then taking one-half. Ask students if they can think of examples where one of the two approaches is easier than the other. Example: $\frac{1}{2}(2.5)(4)$
- **Common Error:** Be sure that students count grid lines correctly. Often they will count points versus the segments between the points. Students who do this will have bases of 2 and 3, and a height of 7.

Example 3

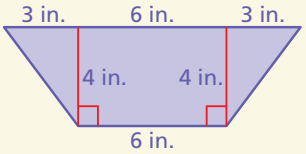
- Students will be dividing whole numbers where the remainder is not zero. From the context of the problem, it is clear that the nearest whole number is an appropriate answer. In Grade 7, students will study decimal remainders (7.NS.2d).
- This is an example of finding population density. Help students think about the size of 1 square mile. Relate it to a physical location in your area that students would be familiar with. In Scott County, there are approximately 44 people living in every square mile.
- **Extension:** Although the county you live and/or teach in may not be similar to a common geometric figure, you can find the land areas and population estimates at *quickfacts.census.gov/qfd/index.html*.

On Your Own

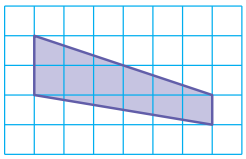
- It is good for students to practice *reading* the dimensions of a figure drawn on grid paper.
- Encourage students to estimate their answers first by counting.
- This is an example where both the height and the sum of the bases are odd. Ask a student to explain how he or she found the answer. A common approach will be $3 \times (3 + 4)$ is 21 and one-half of 21 is 10.5, so the area is 10.5 units^2.
- For Question 4, guide students to make an approximation without calculating. The decrease is close to the number of square miles in the country, so there should be about 1 person per square mile decrease.

Closure

- Find the area of the trapezoid in two different ways. Which method do you prefer? Explain. 36 in.^{2}; Method 1: Area of a Trapezoid Formula, Method 2: sum the areas of the two triangles and the rectangle



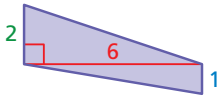
EXAMPLE 2 Finding the Area of a Trapezoid on a Grid



What is the area of the trapezoid?

Ⓐ 6 units^2 Ⓑ 7 units^2 Ⓒ 9 units^2 Ⓓ 12 units^2

Count grid lines to find the dimensions. The height h is 6 units, base b_1 is 1 unit, and base b_2 is 2 units.



$$A = \frac{1}{2}h(b_1 + b_2)$$ Write formula.

$$= \frac{1}{2}(6)(1 + 2)$$ Substitute values.

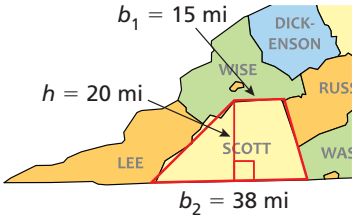
$$= \frac{1}{2}(6)(3)$$ Add.

$$= 9$$ Multiply.

∴ The area of the trapezoid is 9 square units.
The correct answer is Ⓒ.

EXAMPLE 3 Real-Life Application

You can use a trapezoid to approximate the shape of Scott County, Virginia. The population is about 23,200. About how many people are there per square mile?



Find the area of Scott County.

$$A = \frac{1}{2}h(b_1 + b_2)$$ Write formula for area of a trapezoid.

$$= \frac{1}{2}(20)(15 + 38)$$ Substitute 20 for h, 15 for b_1, and 38 for b_2.

$$= \frac{1}{2}(20)(53) = 530$$ Simplify.

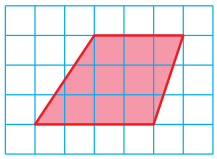
The area of Scott County is about 530 square miles. Divide the population by the area to find the number of people per square mile.

∴ So, there are about $\frac{23{,}200 \text{ people}}{530 \text{ mi}^2} \approx 44$ people per square mile.

On Your Own

Now You're Ready
Exercises 11–13

3. Find the area of the trapezoid.



4. **WHAT IF?** In Example 3, the population of Scott County decreases by 550. By how much does the number of people per square mile change? Explain.

4.3 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** Identify the bases and the height of the trapezoid.

2. **REASONING** What measures do you need to find the area of a trapezoid?

3. **WHICH ONE DOESN'T BELONG?** Which one does *not* belong with the other three? Explain your reasoning.

$\frac{1}{2}bh$ $\quad \ell w \quad$ $2\ell + 2w$ $\quad \frac{1}{2}h(b_1 + b_2)$

Practice and Problem Solving

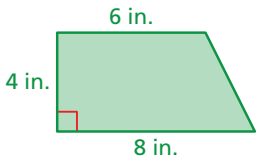
Find the area of the trapezoid.

4. $b_1 = 4, b_2 = 8, h = 2$

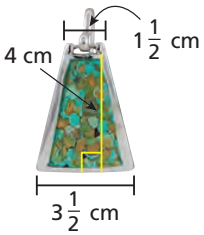
5. $b_1 = 5, b_2 = 7, h = 4$

6. $b_1 = 12, b_2 = 6, h = 3$

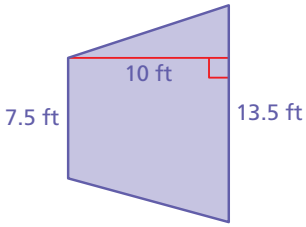
1 7.



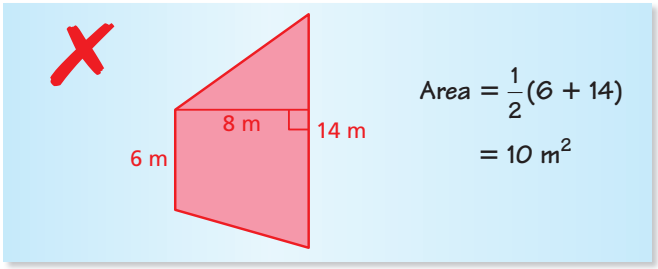
8.



9.

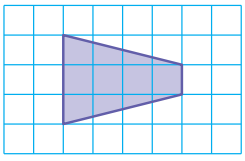


10. **ERROR ANALYSIS** Describe and correct the error in finding the area of the trapezoid.

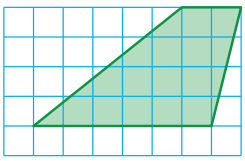


Find the area of the trapezoid.

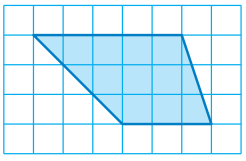
2 11.



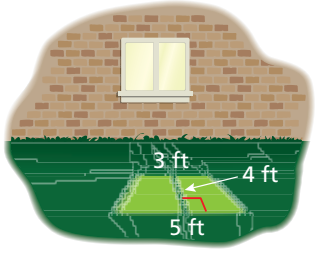
12.



13.



14. **LIGHT** Light shines through a window. What is the area of the trapezoid-shaped region created by the light?



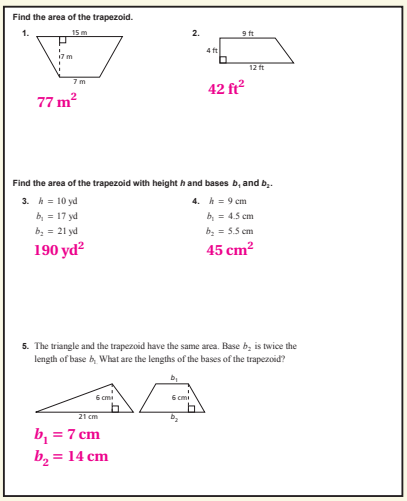
Assignment Guide and Homework Check

Level	Day 1 Activity Assignment	Day 2 Lesson Assignment	Homework Check
Basic	4–6, 23–27	1–3, 7–15, 17, 19	9, 13, 14, 15
Average	4–6, 23–27	1–3, 7–16, 19–21 odd	9, 13, 16, 21
Advanced	4–6, 23–27	1–3, 8–16, 18–22	8, 12, 18, 21

Common Errors

- **Exercises 7–9** Students may forget to multiply by $\frac{1}{2}$. Tell students to write out the formula for the area of a trapezoid, and then identify the value of each variable before substituting.
- **Exercise 9** Students may forget to align the decimal points when adding decimals.
- **Exercises 11–13** Students may not count grid lines correctly. Often they will count points versus the segments between the points. When this happens the bases and the heights will be one greater than they should be.
- **Exercises 15–18** Students may substitute the height for one of the bases. Tell students to write out the formula for the area of a trapezoid, and then identify the value of each variable before substituting.

4.3 Record and Practice Journal



Find the area of the trapezoid.

1. 15 m, 7 m, 7 m
77 m²

2. 9 ft, 4 ft, 12 ft
42 ft²

Find the area of the trapezoid with height h and bases b_1 and b_2.

3. h = 10 yd, b_1 = 17 yd, b_2 = 21 yd
190 yd²

4. h = 9 cm, b_1 = 4.5 cm, b_2 = 5.5 cm
45 cm²

5. The triangle and the trapezoid have the same area. Base b_2 is twice the length of base b_1. What are the lengths of the bases of the trapezoid?

6 cm, 21 cm; b_1, 6 cm, b_2

b_1 = 7 cm
b_2 = 14 cm

Vocabulary and Concept Check

1. bases: 4 ft and 7 ft; height: 15 ft
2. height h and bases b_1 and b_2
3. $2\ell + 2w$; This is an expression for the perimeter of a rectangle. The other three are expressions for area (triangle, rectangle, and trapezoid).

Practice and Problem Solving

4. 12 units^2
5. 24 units^2
6. 27 units^2
7. 28 in.^{2}
8. 10 cm^2
9. 105 ft^2
10. The height was not included in the formula.
$A = \frac{1}{2}(8)(6 + 14) = 80$ m^2
11. 8 units^2
12. 16 units^2
13. 12 units^2
14. 16 ft^2
15. 60 in.^{2}
16. 253 cm^2
17. 78 mi^2
18. 301 m^2
19. 18 ft

Practice and Problem Solving

20. *Sample answers:*
$b_1 = 2$ ft, $b_2 = 3$ ft;
$b_1 = 1.5$ ft, $b_2 = 3.5$ ft

21. See *Taking Math Deeper*.

22. **a.** $x > 0$ and $x < 15$ inches; For $x = 15$ inches the area of the trapezoid is twice the area of the triangle, so x must be less than 15 inches.

b. no; When $x = 15$ inches the quadrilateral is a rectangle, not a trapezoid.

Fair Game Review

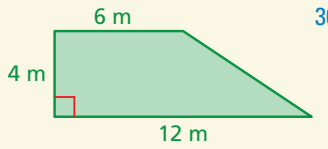
23–26. See Additional Answers.

27. C

Mini-Assessment

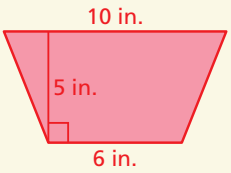
Find the area of the trapezoid.

1.



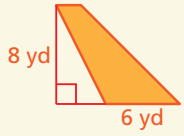
36 m^2

2.



40 in.^{2}

3.



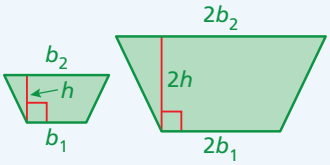
32 yd^2

4. $h = 10$ cm, $b_1 = 3$ cm, $b_2 = 5$ cm
40 cm^2

Taking Math Deeper

Exercise 21

This problem gives students several options for problem-solving strategies. One is to assign values for h, b_1, and b_2 and then *Solve a Simpler Problem.*



1 *Solve a Simpler Problem:* Let $h = 2$, $b_1 = 4$, and $b_2 = 6$.

Area of Small Trapezoid	Area of Large Trapezoid
$= \frac{1}{2}h(b_1 + b_2)$	$= \frac{1}{2}(2h)(2b_1 + 2b_2)$
$= \frac{1}{2}(2)(4 + 6)$	$= \frac{1}{2}(4)(8 + 12)$
$= 10$ units^2	$= 40$ units^2

So, the area of the floor covered by the larger speaker is 4 times greater than the area of the floor covered by the smaller speaker.

2 At this stage in their study of algebra, most students do not know enough about simplifying algebraic expressions to solve the problem abstractly.

$$\frac{\text{Area of Large Trapezoid}}{\text{Area of Small Trapezoid}} = \frac{\frac{1}{2}(2h)(2b_1 + 2b_2)}{\frac{1}{2}h(b_1 + b_2)} = \frac{4h(b_1 + b_2)}{h(b_1 + b_2)} = 4$$

So, the best approach is to try a few other values for h, b_1, and b_2, and then use inductive reasoning to conclude that the area of the large trapezoid is always 4 times greater than the area of the small trapezoid.

3 **Extension:** Have students review Example 3 on page 161. In that example, they can see that doubling the dimensions has the effect of making the area 4 times greater.

Ask students to experiment with all the shapes they have studied in this chapter. In each case, they can conclude that doubling the dimensions of the figure increases the area by a factor of 4.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Start the next section

Find the area of a trapezoid with height h and bases b_1 and b_2.

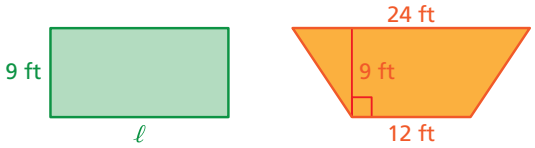
15. $h = 6$ in.
$b_1 = 9$ in.
$b_2 = 11$ in.

16. $h = 22$ cm
$b_1 = 10.5$ cm
$b_2 = 12.5$ cm

17. $h = 12$ mi
$b_1 = 5.6$ mi
$b_2 = 7.4$ mi

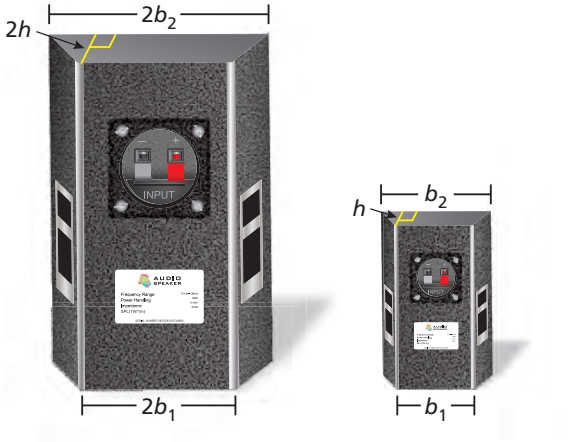
18. $h = 14$ m
$b_1 = 21$ m
$b_2 = 22$ m

19. REASONING The rectangle and the trapezoid have the same area. What is the length ℓ of the rectangle?

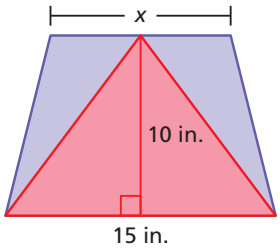


20. OPEN-ENDED The area of the trapezoidal student election sign is 5 square feet. Find two possible values for each base length.

21. AUDIO How many times greater is the area of the floor covered by the larger speaker than by the smaller speaker?



22. Critical Thinking The triangle and the trapezoid share a 15-inch base and a height of 10 inches.



a. The area of the trapezoid is less than twice the area of the triangle. Find the values of x. Explain your reasoning.

b. Can the area of the *trapezoid* be exactly twice the area of the triangle? Explain your reasoning.

Fair Game Review What you learned in previous grades & lessons

Plot the ordered pair in a coordinate plane. *(Skills Review Handbook)*

23. (5, 0)

24. (2, 4)

25. (0, 3)

26. (6, 1)

27. MULTIPLE CHOICE Which expression represents "6 more than x"? *(Section 3.2)*

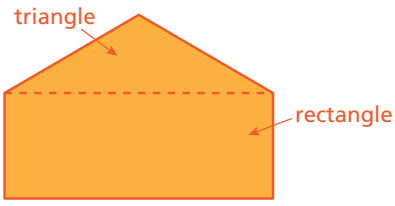
Ⓐ $6 - x$ Ⓑ $6x$ Ⓒ $x + 6$ Ⓓ $\frac{6}{x}$

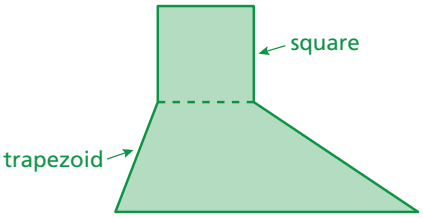
Extension 4.3 Areas of Composite Figures

Check It Out Lesson Tutorials BigIdeasMath.com

Key Vocabulary
composite figure, *p. 172*

A **composite figure** is made up of triangles, squares, rectangles, and other two-dimensional figures. Here are two examples.

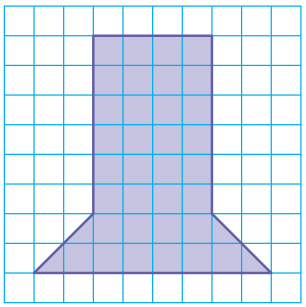




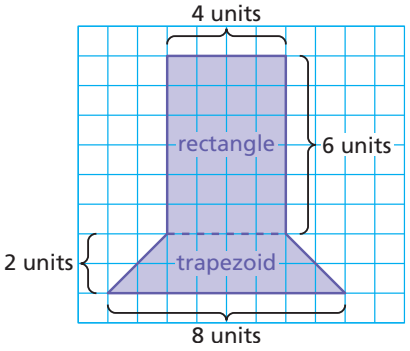
To find the area of a composite figure, separate it into figures with areas you know how to find. Then find the sum of the areas of those figures.

EXAMPLE 1 Finding the Area of a Composite Figure

Find the area of the purple figure.



You can separate the figure into a rectangle and a trapezoid. Count grid lines to find the dimensions of each figure. Then find the area of each figure.



Area of Rectangle

$A = \ell w$

$= 6(4)$

$= 24$

Area of Trapezoid

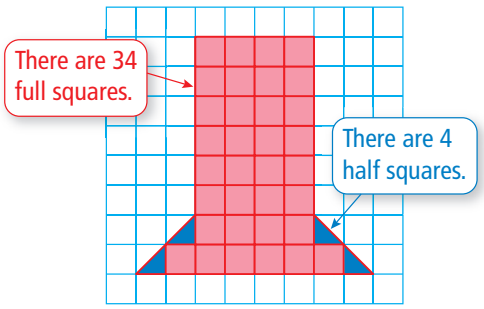
$A = \frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{2}(2)(4 + 8)$

$= 12$

∴ So, the area of the purple figure is $24 + 12 = 36$ square units.

Reasonable? You can check your result by counting unit squares.



Full squares: 34

Half squares: 4

The area is

$34(1) + 4\left(\frac{1}{2}\right) = 36$ square units.

So, the answer is reasonable. ✓

Study Tip
There is often more than one way to separate composite figures. In Example 1, you can separate the figure into one rectangle and two triangles.

Geometry
In this extension, you will
- find areas of composite figures.
- solve real-life problems.

Multi-Language Glossary at BigIdeasMath.com

Laurie's Notes

Introduction

Connect

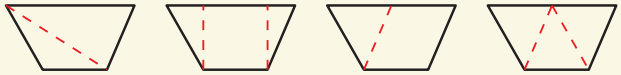
- **Yesterday:** Students have learned to find the areas of several polygons. (MP1)
- **Today:** Students will use different area formulas to find the area of a composite figure.

FYI

- A trapezoid can be decomposed into different polygons. Students have already worked with composite figures. Thinking of a trapezoid as a composite figure was actually how its area formula was derived.

Motivate

- Give students a piece of paper with four copies of a trapezoid drawn on it. Ask them to "decompose" the trapezoid into polygons that they would be able to find the area of. Sample answers are shown.

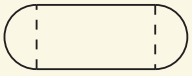


- Give time for students to share their thinking with the class. In each case, they should identify the names of the shapes they formed.
- Discuss that the area of the trapezoid is the sum of the areas of the figures of which it is composed.

Discuss

? Draw a couple of samples of composite figures and ask, "These are examples of a composite figure. What do you think a composite figure is?" Listen for a shape that is made up of other polygons like triangles and rectangles.

- **FYI:** Composite figures do not have to be composed of only polygons. The oval track shown is two semi-circles and a rectangle.



Lesson Notes

Example 1

? "What are the dimensions of this shape?" The rectangle is 6 by 4 units. The trapezoid has bases of 4 and 8 and a height of 2.

- Write the formula for the area of each polygon, substitute the known values, and perform the computations.
- Check the reasonableness of the answer as shown.

? "Could the figure have been decomposed a different way? Explain." Yes; have students show different methods.

- If time allows, work through different methods so that students understand that the total area will not change.

Common Core State Standards

6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Goal Today's lesson is using different area formulas to find the area of a **composite figure**.

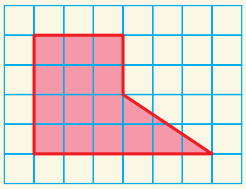
Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

Find the area of the red figure.



15 square units

Record and Practice Journal Extension 4.3 Practice

1. 32 units^2
2. 22 units^2
3. 26 units^2
4. 12 units^2
5. 24 cm^2
6. 28 m^2
7. 64 ft^2
8. 36 in.^{2}
9. 64 ft^2

Extra Example 2

Find the area of the swimming pool.

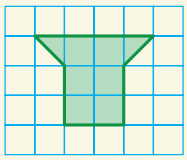
136.5 square feet

Practice

1. 36 units^2
2. 32 units^2
3. 20 units^2
4. 120 ft^2
5. $126\frac{1}{2} \text{ cm}^2$
6. 132 in.^2
7. See Additional Answers.

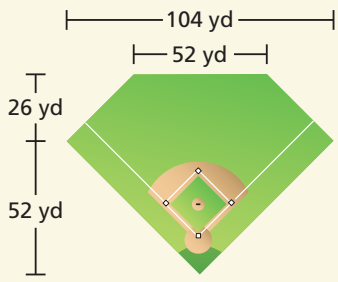
Mini-Assessment

1. Find the area of the figure.



7 square units

2. Find the area of the softball field.



4732 square feet

Laurie's Notes

Example 2

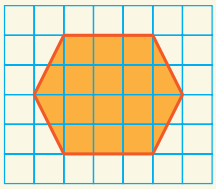
- Draw the composite figure shown and label all of the given dimensions.
- Ask students to talk with their partners about different ways in which the figure could be decomposed. Suggest to them that their methods need to result in figures where the dimensions are known or could be found.
- Give students a few minutes and then share as a class.
- **MP1 Make Sense of Problems and Persevere in Solving Them:** Instead of modeling the problem for them, have students work the problem with their partners. Students should have a pathway into the problem so that they can be successful.
- Ask different students to present their solutions, looking for a selection of methods.

Practice

- **Neighbor Check:** Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.

Closure

- Describe how to find the area of the following.

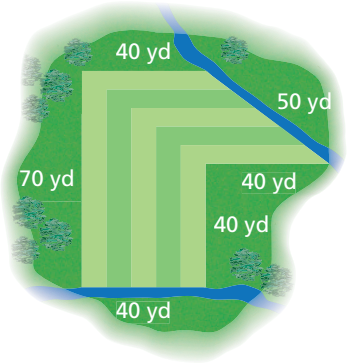


Sample answer:

Separate the hexagon with a horizontal line to create two trapezoids.

$$A = \frac{1}{2}(2)(3 + 5) + \frac{1}{2}(2)(5 + 3)$$
$$= 8 + 8$$
$$= 16 \text{ square units}$$

EXAMPLE 2 Real-Life Application

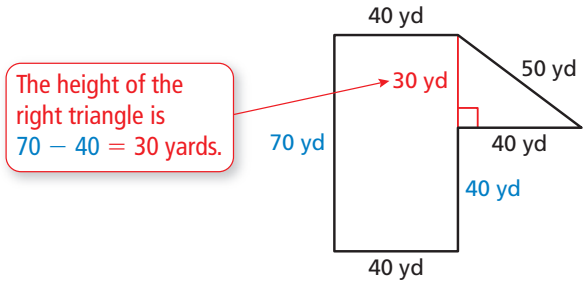


STEM VIDEO

Find the area of the fairway between two streams on a golf course.

There are several ways to separate the fairway into figures whose areas you can find using formulas. It appears that one way is to separate it into a right triangle and a rectangle.

Identify each shape and find any missing dimensions.

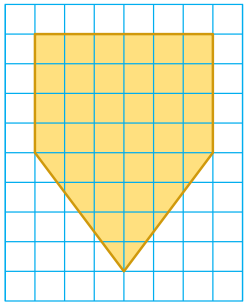


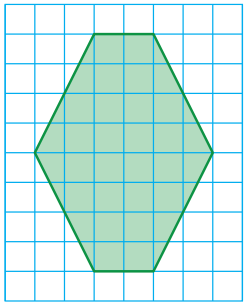
Area of Rectangle	*Area of Right Triangle*
$A = \ell w$	$A = \frac{1}{2}bh$
$= 70(40)$	$= \frac{1}{2}(40)(30)$
$= 2800$	$= 600$

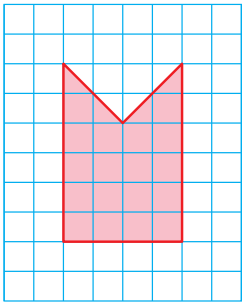
∴ So, the area of the fairway is $2800 + 600 = 3400$ square yards.

Practice

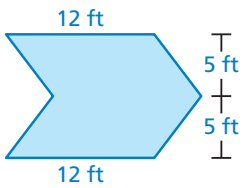
Find the area of the shaded figure.

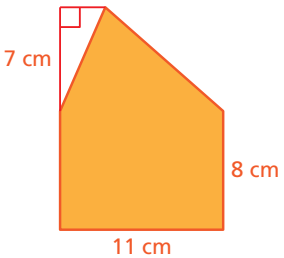
1. 

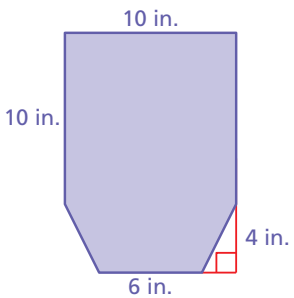
2. 

3. 

Find the area of the figure.

4. 

5. 

6. 

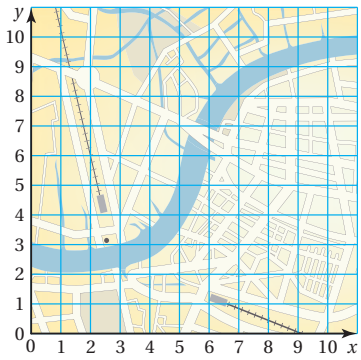
7. **ANOTHER METHOD** Find the area in Example 2 using a different method.

4.4 Polygons in the Coordinate Plane

Essential Question How can you find the lengths of line segments in a coordinate plane?

1 ACTIVITY: Finding Distances on a Map

Work with a partner. The coordinate grid shows a portion of a city. Each square on the grid represents one square mile.



a. A public library is located at (4, 5). City Hall is located at (7, 5). Plot and label these points.

b. How far is the public library from City Hall?

c. A stadium is located 4 miles from the public library. Give the coordinates of several possible locations of the stadium. Justify your answers by graphing.

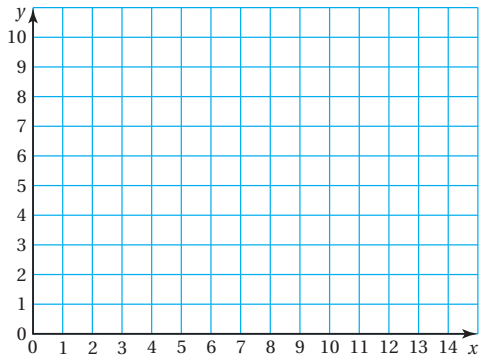
d. Connect the three locations of the public library, City Hall, and the stadium using your answers in part (c). What shapes are formed?

2 ACTIVITY: Graphing Polygons

Work with a partner. Plot and label each set of points in the coordinate plane. Then connect each set of points to form a polygon.

Rectangle: $A(2, 3)$, $B(2, 10)$, $C(6, 10)$, $D(6, 3)$

Triangle: $E(8, 3)$, $F(14, 8)$, $G(14, 3)$



Geometry

In this lesson, you will
- draw polygons in the coordinate plane.
- find distances in the coordinate plane.
- solve real-life problems.

Laurie's Notes

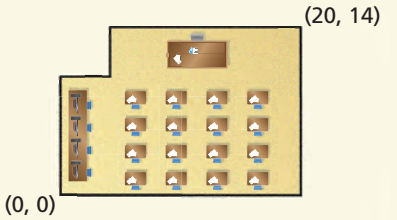
Introduction

Standards for Mathematical Practice

- **MP8 Look for and Express Regularity in Repeated Reasoning:** Mathematically proficient students recognize patterns and make connections among mathematical ideas.

Motivate

- "Who has played the game of *Battleship* before?" Chances are most students have or will have a familiarity with it.
- Tell them that you have hidden something in the classroom. It's their job to guess "coordinates" of the location.
- Identify the corner of the room that will be the origin (0, 0). Give some sense of size by labeling the opposite corner of your room. A sketch of my classroom is shown.



- Have students guess coordinates of the location of your "hide." This will allow you to review concepts necessary for this lesson, such as how to start at the origin, (0, 0) and locate an ordered pair.
- **FYI:** Graphing in the first quadrant is a Grade 5 standard. (5.G.2)

Activity Notes

Activity 1

- Students should be able to plot the ordered pairs without difficulty if you have done the motivation described above.
- **Connection:** Students have found the dimensions of a polygon on a grid. Finding distances of segments is an equivalent skill.
- Ask students to share their answers to parts (c) and (d).
- **Extension:** Ask if there are other points that would also be 4 miles from the library. The solution is a circle with radius of 4 centered at the library.

Activity 2

- When students have finished, have students share their results at the overhead, document camera, or interactive board. This will ensure that students have successfully plotted the points prior to beginning Activity 3.

Common Core State Standards

6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Previous Learning

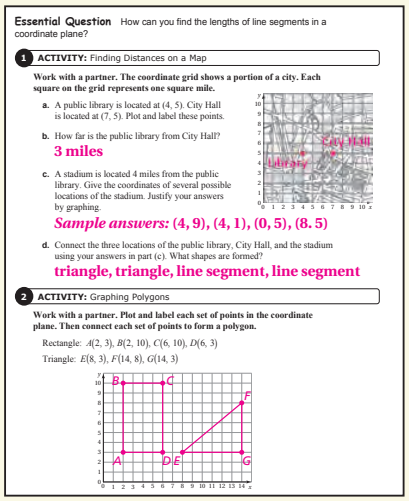
Students need to be familiar with plotting points in the first quadrant of the coordinate plane.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

4.4 Record and Practice Journal



Essential Question How can you find the lengths of line segments in a coordinate plane?

1 ACTIVITY: Finding Distances on a Map

Work with a partner. The coordinate grid shows a portion of a city. Each square on the grid represents one square mile.

a. A public library is located at (4, 5). City Hall is located at (7, 5). Plot and label these points.

b. How far is the public library from City Hall?

3 miles

c. A stadium is located 4 miles from the public library. Give the coordinates of several possible locations of the stadium. Justify your answers by graphing.

Sample answers: (4, 9), (4, 1), (0, 5), (8, 5)

d. Connect the three locations of the public library, City Hall, and the stadium using your answers in part (c). What shapes are formed?

triangle, triangle, line segment, line segment

2 ACTIVITY: Graphing Polygons

Work with a partner. Plot and label each set of points in the coordinate plane. Then connect each set of points to form a polygon.

Rectangle: $A(2, 3)$, $B(2, 10)$, $C(6, 10)$, $D(6, 3)$

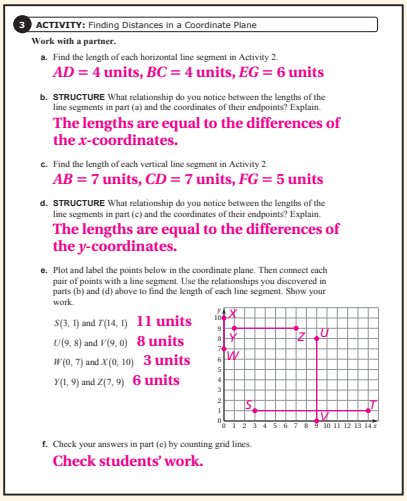
Triangle: $E(8, 3)$, $F(14, 8)$, $G(14, 3)$

Differentiated Instruction

Organization

Point out to students that the coordinate plane is made up of two number lines. The horizontal line is similar to the number line with which they are familiar. This is the *x*-axis and the *x*-coordinate corresponds to the first number of a coordinate pair. The second number line is perpendicular to the first. This is the *y*-axis and the *y*-coordinate corresponds to the second number of a coordinate pair.

4.4 Record and Practice Journal



3 ACTIVITY: Finding Distances in a Coordinate Plane

Work with a partner.

a. Find the length of each horizontal line segment in Activity 2.

$AD = 4$ units, $BC = 4$ units, $EG = 6$ units

b. STRUCTURE What relationship do you notice between the lengths of the line segments in part (a) and the coordinates of their endpoints? Explain.

The lengths are equal to the differences of the *x*-coordinates.

c. Find the length of each vertical line segment in Activity 2.

$AB = 7$ units, $CD = 7$ units, $FG = 5$ units

d. STRUCTURE What relationship do you notice between the lengths of the line segments in part (c) and the coordinates of their endpoints? Explain.

The lengths are equal to the differences of the *y*-coordinates.

e. Plot and label the points below in the coordinate plane. Then connect each pair of points with a line segment. Use the relationships you discovered in parts (b) and (d) above to find the length of each line segment. Show your work.

$S(3, 1)$ and $T(14, 1)$ 11 units

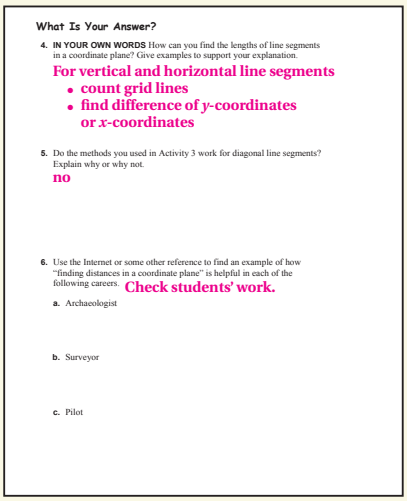
$U(9, 8)$ and $V(9, 0)$ 8 units

$W(0, 7)$ and $X(0, 10)$ 3 units

$Y(1, 9)$ and $Z(7, 9)$ 6 units

f. Check your answers in part (e) by counting grid lines.

Check students' work.



What Is Your Answer?

4. IN YOUR OWN WORDS How can you find the lengths of line segments in a coordinate plane? Give examples to support your explanation.

For vertical and horizontal line segments

- count grid lines
- find difference of *y*-coordinates or *x*-coordinates

5. Do the methods you used in Activity 3 work for diagonal line segments? Explain why or why not.

no

6. Use the Internet or some other reference to find an example of how "finding distances in a coordinate plane" is helpful in each of the following careers. Check students' work.

a. Archaeologist

b. Surveyor

c. Pilot

Laurie's Notes

Activity 3

- Give students sufficient time to work through the activity. As you circulate, make sure that students are paying attention to the *x*- and *y*-coordinates of the ordered pairs.
- **MP7 Look for and Make Use of Structure:** Mathematically proficient students will observe the same *x*-coordinate of vertical segments and the same *y*-coordinate of horizontal segments.
- Sometimes students will simply note that the points are "up and down" or "straight across" from one another. Focus their attention on the actual ordered pairs versus the location of the points in the coordinate grid.
- Students have no formal conception of absolute value so they will describe the length of the segments as the difference between the *x*- or *y*-coordinate. At this point, it is fine that they notice you simply subtract the lesser coordinate from the greater coordinate to find the length of the line segment.

What Is Your Answer?

- In Question 4, students will only reference horizontal and vertical segments.
- Depending upon your students, you may want to plant the seed that in later mathematics classes they will not be restricted to horizontal and vertical segments. Segments drawn on a diagonal in the coordinate plane also have a length, and there will be a method to determine that length. This is the question posed in Question 5.
- Ask if there are parents whose jobs involve finding distances. Ask students to share information.

Closure

- Plot a rectangle with a perimeter of 20. What are the coordinates?
  Sample answer: (1, 2), (7, 2), (7, 6), (1, 6)

3 ACTIVITY: Finding Distances in a Coordinate Plane

Work with a partner.

a. Find the length of each horizontal line segment in Activity 2.

b. **STRUCTURE** What relationship do you notice between the lengths of the line segments in part (a) and the coordinates of their endpoints? Explain.

c. Find the length of each vertical line segment in Activity 2.

d. **STRUCTURE** What relationship do you notice between the lengths of the line segments in part (c) and the coordinates of their endpoints? Explain.

e. Plot and label the points below in the coordinate plane. Then connect each pair of points with a line segment. Use the relationships you discovered in parts (b) and (d) above to find the length of each line segment. Show your work.

$S(3, 1)$ and $T(14, 1)$ $U(9, 8)$ and $V(9, 0)$

$W(0, 7)$ and $X(0, 10)$ $Y(1, 9)$ and $Z(7, 9)$

f. Check your answers in part (e) by counting grid lines.

Math Practice

Repeat Calculations

What calculations are repeated? How can you use this information to write a rule about the length of a line segment?

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you find the lengths of line segments in a coordinate plane? Give examples to support your explanation.

5. Do the methods you used in Activity 3 work for diagonal line segments? Explain why or why not.

6. Use the Internet or some other reference to find an example of how "finding distances in a coordinate plane" is helpful in each of the following careers.

a.

Archaeologist

b.

Surveyor

c.

Pilot

Practice Use what you learned about finding the lengths of line segments to complete Exercises 3–5 on page 178.

4.4 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

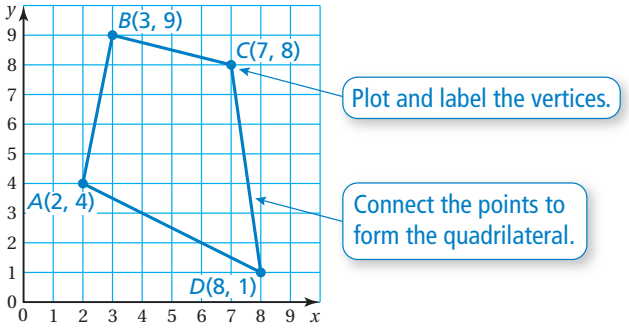
You can use ordered pairs to represent vertices of polygons. To draw a polygon in a coordinate plane, plot and connect the ordered pairs.

EXAMPLE 1 Drawing a Polygon in a Coordinate Plane

The vertices of a quadrilateral are $A(2, 4)$, $B(3, 9)$, $C(7, 8)$, and $D(8, 1)$. Draw the quadrilateral in a coordinate plane.

Study Tip

After you plot the vertices, connect them *in order* to draw the polygon.



On Your Own

Now You're Ready
Exercises 6–11

Draw the polygon with the given vertices in a coordinate plane.

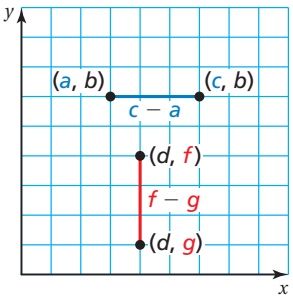
1. $A(0, 0)$, $B(5, 7)$, $C(7, 4)$
2. $W(4, 4)$, $X(7, 4)$, $Y(7, 1)$, $Z(4, 1)$
3. $F(1, 3)$, $G(3, 6)$, $H(5, 6)$, $J(3, 3)$
4. $P(1, 4)$, $Q(3, 5)$, $R(7, 3)$, $S\left(6, \frac{1}{2}\right)$, $T\left(2, \frac{1}{2}\right)$

Key Idea

Finding Distances in the First Quadrant

You can find the length of a horizontal or vertical line segment in a coordinate plane by using the coordinates of the endpoints.

- When the x-coordinates are the same, the vertical distance between the points is the difference of the y-coordinates.
- When the y-coordinates are the same, the horizontal distance between the points is the difference of the x-coordinates.



Be sure to subtract the lesser coordinate from the greater coordinate.

Laurie's Notes

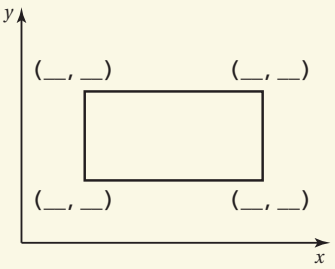
Introduction

Connect

- **Yesterday:** Students plotted ordered pairs and explored lengths of line segments in a coordinate plane. (MP7, MP8)
- **Today:** Students will draw polygons in the coordinate plane and find the length of their sides.

Motivate

- Draw two axes without any scale, and a rectangle, as shown.



- Tell students to use the numbers 2, 5, 6, and 8 to fill in all 8 blanks so that the ordered pairs represent the vertices of a rectangle.
- Yes, each number is used twice, but that is not something that needs to be stated at the beginning.
- *Sample answer:* (2, 5), (8, 5), (8, 6), and (2, 6)
- Ask students to discuss their reasoning and process. The goal is for students to think about the *x*-coordinates of vertical segments and the *y*-coordinates of horizontal segments.

Lesson Notes

Example 1

- Write the ordered pairs and ask students to plot them. Note that the labels: *A*, *B*, *C*, and D are also written next to the ordered pair in the coordinate plane. In this book, the vertices will always be in alphabetical order.

? "When the ordered pairs are connected in order, which polygon is formed?" quadrilateral

? "Is it a special quadrilateral with a more specific name?" No

On Your Own

- Students should plot each problem on a different coordinate plane.
- Have volunteers share their polygons with the class. Have students identify the type of polygon they have plotted.

Key Idea

- Write the Key Idea. Sketch a horizontal segment and a vertical segment that can be referenced as you discuss how to find the length of the segment. Be sure to identify what the term *endpoint* means.
- There are many words used to describe how to find distances in the first quadrant. Having sample segments to reference will help students see that it is a simple subtraction problem.
- All of the ordered pairs will be in the first quadrant. For clarity, the Key Idea tells students to subtract the lesser number from the greater number. In Section 6.5, students will plot ordered pairs in all four quadrants.

Goal Today's lesson is drawing polygons in the coordinate plane and finding the lengths of the sides of the polygons.

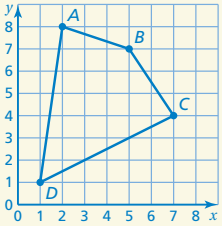
Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

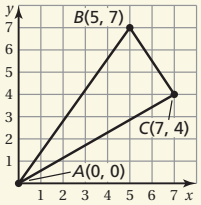
Extra Example 1

The vertices of a quadrilateral are $A(2, 8)$, $B(5, 7)$, $C(7, 4)$, and $D(1, 1)$. Draw the quadrilateral in a coordinate plane.

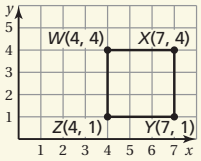


On Your Own

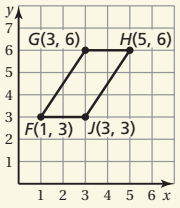
1.



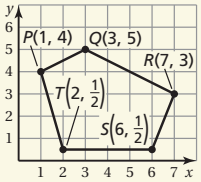
2.



3.



4.



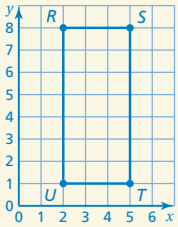
English Language Learners

Vocabulary

Encourage English language learners to answer in complete sentences. For instance, in Example 3, a student could respond, "The area of the giraffe exhibit is 4800 square feet." This provides language support and reinforces the units of measure in the answer.

Extra Example 2

The vertices of a rectangle are $R(2, 8)$, $S(5, 8)$, $T(5, 1)$, and $U(2, 1)$. Draw the rectangle in a coordinate plane, and find its perimeter.



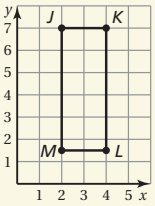
20 units

Extra Example 3

In a grid of a housing development, the vertices of a children's play area are $J(20, 60)$, $K(80, 70)$, $L(80, 0)$, and $M(20, 0)$. The coordinates are measured in feet. What is the area of the children's play area? 3900 square feet

On Your Own

5.



15 units; 11 square units

6. Increases it by 600 square feet for a total of 5400 square feet.

Laurie's Notes

Example 2

- Plot the ordered pairs as shown.
- "What information is needed to find the perimeter of the rectangle?" the length of each side
- "What is the length of the base? Explain." $6; 7 - 1$
- "What is the height of the rectangle? Explain." $4; 6 - 2$
- "What is the perimeter? Explain." $20; 6 \times 2 + 4 \times 2$
- Note that some students may find the perimeter by adding, $6 + 6 + 4 + 4$.
- Because no units have been specified, write the answer as 20 units.

Example 3

- **MP6 Attend to Precision:** The last example asked students to find a perimeter. This example asks students to find area. Students should pay attention to units in labeling their answers.
- Ask a student to read the problem.
- "What information is needed to find the area of the trapezoid?" the lengths of both bases and the height
- "How do you find the length of each base?" The longer base has a length of $100 - 0 = 100$. The shorter base has a length of $90 - 0 = 90$.
- "What ordered pairs can you use to find the height of the trapezoid?" H(0, 30) and E(0, 90)
- "What is the height?" $90 - 30 = 60$
- Write the formula for the area of a trapezoid and substitute the known values.
- Finish the problem as shown. Be sure that the answer is labeled with the correct units.

On Your Own

- **Think-Pair-Share:** Students should read each question independently and then work in pairs to answer the questions. When they have answered the questions, the pair should compare their answers with another group and discuss any discrepancies.

Closure

- Give the endpoints of a horizontal segment with a length of 6. *Sample answer:* (1, 3) and (12, 3)
- Give the endpoints of a vertical segment with a length of 11. *Sample answer:* (3, 2) and (3, 8)

EXAMPLE 2 Finding a Perimeter

The vertices of a rectangle are $F(1, 6)$, $G(7, 6)$, $H(7, 2)$, and $J(1, 2)$. Draw the rectangle in a coordinate plane and find its perimeter.

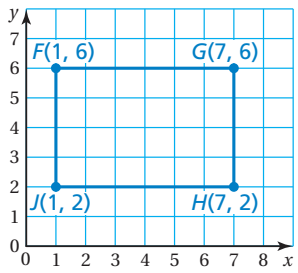
Draw the rectangle and use the vertices to find its dimensions.

The length is the horizontal distance between $F(1, 6)$ and $G(7, 6)$, which is the difference of the x-coordinates.

$$\text{length} = 7 - 1 = 6 \text{ units}$$

The width is the vertical distance between $G(7, 6)$ and $H(7, 2)$, which is the difference of the y-coordinates.

$$\text{width} = 6 - 2 = 4 \text{ units}$$



∴ So, the perimeter of the rectangle is $2(6) + 2(4) = 20$ units.

Study Tip

You can also find the length using vertices H and J. You can find the width using vertices F and J.

EXAMPLE 3 Real-Life Application

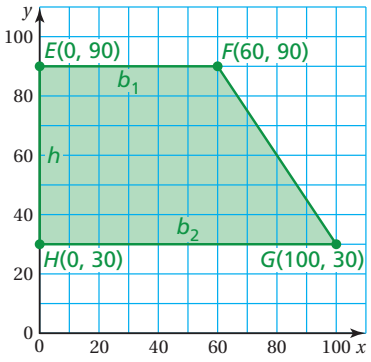
In a grid of the exhibits at a zoo, the vertices of the giraffe exhibit are $E(0, 90)$, $F(60, 90)$, $G(100, 30)$, and $H(0, 30)$. The coordinates are measured in feet. What is the area of the giraffe exhibit?

Plot and connect the vertices using a coordinate grid to form a trapezoid. Use the coordinates to find the lengths of the bases and the height.

$$b_1 = 60 - 0 = 60$$

$$b_2 = 100 - 0 = 100$$

$$h = 90 - 30 = 60$$



Use the formula for the area of a trapezoid.

$$A = \frac{1}{2}(60)(60 + 100)$$

$$= \frac{1}{2}(60)(160) = 4800$$

∴ The area of the giraffe exhibit is 4800 square feet.

Common Error

You can count grid lines to find the dimensions, but make sure you consider the scale of the axes.

On Your Own

Now You're Ready
Exercises 12–15

5. The vertices of a rectangle are $J(2, 7)$, $K(4, 7)$, $L(4, 1.5)$, and $M(2, 1.5)$. Find the perimeter and the area of the rectangle.
6. **WHAT IF?** In Example 3, the giraffe exhibit is enlarged by moving vertex F to $(80, 90)$. How does this affect the area? Explain.

4.4 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **WRITING** How can you use a coordinate plane to draw a polygon?
2. **WRITING** How can you find the perimeter of a rectangle in a coordinate plane?

Practice and Problem Solving

Plot and label each pair of points in a coordinate plane. Find the length of the line segment connecting the points.

3. $C(0, 1), D(8, 1)$
4. $K(5, 2), L(5, 6)$
5. $Q(3, 4), R(3, 9)$

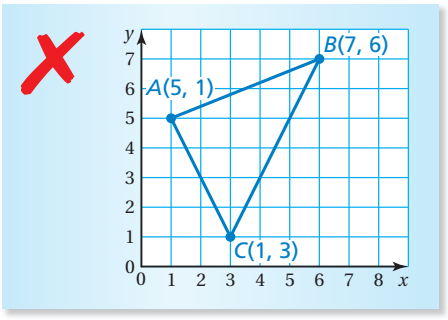
Draw the polygon with the given vertices in a coordinate plane.

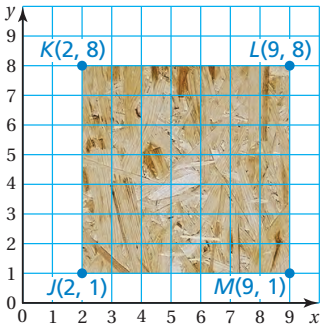
1 6. $A(4, 7), B(6, 2), C(0, 0)$
7. $D\left(\frac{1}{2}, 2\right), E(5, 5), F(4, 1)$
8. $G\left(1\frac{1}{2}, 4\right), H\left(1\frac{1}{2}, 8\right), J(5, 8), K(5, 4)$
9. $L(3, 2), M(3, 5), N(9, 5), P(9, 2)$
10. $Q(0, 4), R(10, 8), S(7, 4), T(10, 2), U(5, 0)$
11. $V(2, 2), W\left(3, 7\frac{1}{2}\right), X\left(8, 7\frac{1}{2}\right), Y(10, 4), Z(7, 0)$

Find the perimeter and the area of the polygon with the given vertices.

2 12. $C(1, 1), D(1, 4), E(4, 4), F(4, 1)$
13. $J(1, 2), K(7, 2), L(7, 8), M(1, 8)$
14. $N(0, 2), P(5, 2), Q(5, 5), R(0, 5)$
15. $S(3, 0), T(3, 9), U(8, 9), V(8, 0)$

16. **ERROR ANALYSIS** Describe and correct the error in drawing a triangle with vertices $A(5, 1)$, $B(7, 6)$, and $C(1, 3)$.





17. **TREE HOUSE** You design a tree house using a coordinate plane. You plot the vertices of the floor at $J(2, 1)$, $K(2, 8)$, $L(9, 8)$, and $M(9, 1)$. The coordinates are measured in feet.
   a. What is the shape of the floor?
   b. What are the perimeter and the area of the floor?

Assignment Guide and Homework Check

Level	Day 1 Activity Assignment	Day 2 Lesson Assignment	Homework Check
Basic	3–5, 27–31	1, 2, 7–15 odd, 16, 17, 19	7, 9, 13, 17
Average	3–5, 27–31	1, 2, 6–16 even, 17, 19, 22–24	6, 10, 14, 24
Advanced	3–5, 27–31	1, 2, 8–24 even, 25, 26	10, 12, 14, 24

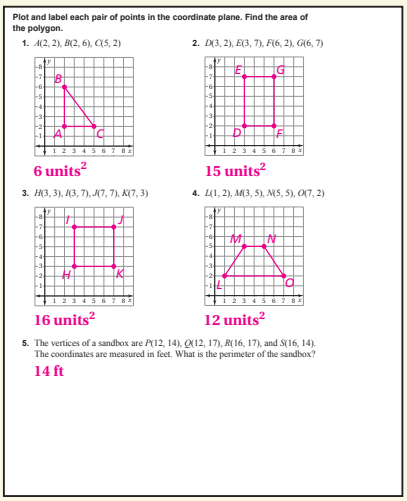
For Your Information

- **Exercise 23** Students may be confused by the statement, "Each grid square represents 9 square miles." Students need to realize that if 1 grid square is 9 square miles, then 1 unit length is equal to 3 miles.

Common Errors

- **Exercises 7, 8, and 11** The fractions in the coordinates may confuse students. Refresh the concept of graphing a coordinate involving a fraction.
- **Exercises 12–15** Students may confuse perimeter and area. Have them write the formulas and then substitute the values. Students should also include "units" or "square units" as part of the answer.

4.4 Record and Practice Journal


Plot and label each pair of points in the coordinate plane. Find the area of the polygon.

1. $A(2, 2)$, $B(2, 6)$, $C(5, 2)$

6 units^2

2. $D(3, 2)$, $E(3, 7)$, $F(6, 2)$, $G(6, 7)$

15 units^2

3. $H(3, 3)$, $I(3, 7)$, $J(7, 7)$, $K(7, 3)$

16 units^2

4. $L(1, 2)$, $M(3, 5)$, $N(5, 5)$, $O(7, 2)$

12 units^2

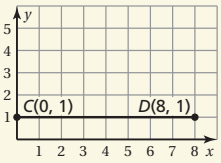
5. The vertices of a sandbox are $P(12, 14)$, $Q(12, 17)$, $R(16, 17)$, and $S(16, 14)$. The coordinates are measured in feet. What is the perimeter of the sandbox?

14 ft

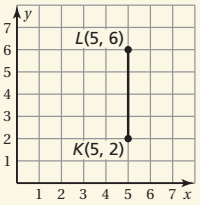
Vocabulary and Concept Check

1. Plot the points that represent the vertices of the polygon and connect the points in order.
2. Assume that all of the sides are vertical or horizontal. Find the points with the same y-coordinates. Subtract the x-coordinates of those points to find the length of the horizontal sides. Find the points with the same x-coordinates. Subtract the y-coordinates of those points to find the length of the vertical sides. Last, add the lengths of all four sides to find the perimeter.

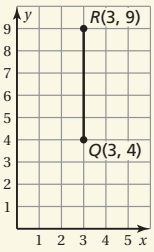
Practice and Problem Solving

3. 

Length of CD is 8 units.

4. 

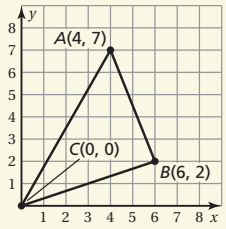
Length of KL is 4 units.

5. 

Length of QR is 5 units.

Practice and Problem Solving

6.



7–25. See Additional Answers.

26. See *Taking Math Deeper*.

Fair Game Review

27. 2

28. $8\frac{4}{5}$

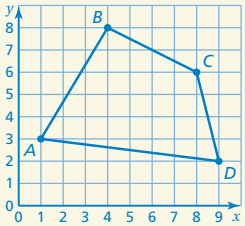
29. $\frac{5}{16}$

30. $3\frac{19}{27}$

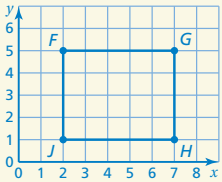
31. D

Mini-Assessment

1. The vertices of a quadrilateral are $A(1, 3)$, $B(4, 8)$, $C(8, 6)$, and $D(9, 2)$. Draw the quadrilateral in a coordinate plane.



2. The vertices of a rectangle are $F(2, 5)$, $G(7, 5)$, $H(7, 1)$, and $J(2, 1)$. Draw the rectangle in a coordinate plane and find its perimeter.



18 units

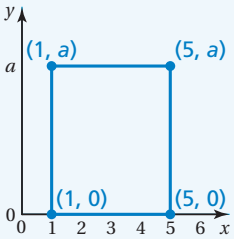
3. In a grid of the swimming area at a beach, the vertices are $E(1, 0)$, $F(1, 5)$, $G(4, 7)$, and $H(4, 0)$. The coordinates are measured in meters. What is the area of the swimming area? 18 square meters

Taking Math Deeper

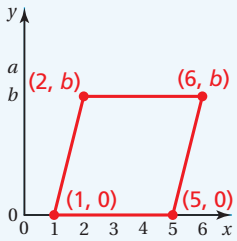
Exercise 26

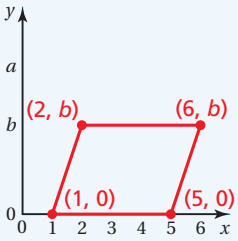
Note that in this exercise, we are assuming a and b are positive because students have not yet plotted ordered pairs outside of Quadrant I.

1 Draw the figures. Place a tick mark at some value a on the vertical axis. Then use the given vertices to draw a rectangle.



Place a tick mark at some value b on the vertical axis. Because the value of a is greater than the value of b, b must be between 0 and a on the vertical axis. Use the given vertices to draw a parallelogram. Here are two possible examples.





2 Look at the dimensions. The base of each figure can be represented by the side length with endpoints (1, 0) and (5, 0). So, the bases have the same length. The heights of the quadrilaterals are different. The height a of the rectangle is greater than the height b of the parallelogram.

3 Answer the question. The area of each figure can be found by multiplying the base and height. The height of the rectangle is always greater, so you are multiplying the base by a greater number compared to the height of the parallelogram. So, the rectangle has the greater area.

Project

Research the discovery of the coordinate plane. Who invented it? When was it invented? Explain why you think the coordinate plane is helpful in mathematics.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Start the next section

OPEN-ENDED **Draw a polygon with the given conditions in a coordinate plane.**

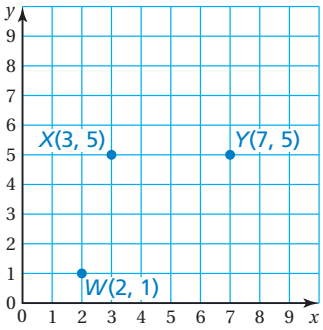
18. a square with a perimeter of 20 units

19. a rectangle with a perimeter of 18 units

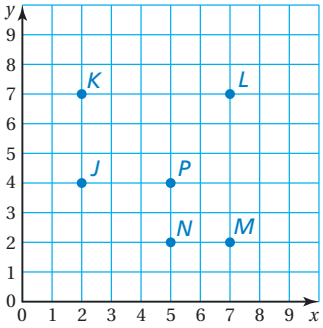
20. a rectangle with an area of 24 units^2

21. a triangle with an area of 15 units^2

22. **STRUCTURE** The coordinate plane shows three vertices of a parallelogram. Find two possible points that could represent the fourth vertex.



23. **BUS ROUTE** Polygon *JKLMNP* represents a bus route. Each grid square represents 9 square miles. What is the shortest distance, in miles, from station *P* to station *L* using the bus route? Explain your reasoning.



24. **CITY LIMITS** In a topographical map of a city, the vertices of the city limits are $A(10, 9)$, $B(18, 9)$, $C(18, 2)$, $D(14, 4.5)$, and $E(10, 4.5)$. The coordinates are measured in miles. What is the area of the city?

25. **BACKYARD** The vertices of a backyard are $W(10, 30)$, $X(10, 100)$, $Y(110, 100)$, and $Z(50, 30)$. The coordinates are measured in feet. The line segment XZ separates the backyard into a lawn and a garden. The area of the lawn is greater than the area of the garden. How many times larger is the lawn than the garden?

26. **Precision** The vertices of a rectangle are $(1, 0)$, $(1, a)$, $(5, a)$, and $(5, 0)$. The vertices of a parallelogram are $(1, 0)$, $(2, b)$, $(6, b)$, and $(5, 0)$. The value of a is greater than the value of b. Which polygon has a greater area? Explain your reasoning.

Fair Game Review *What you learned in previous grades & lessons*

Divide. Write the answer in simplest form. *(Section 2.3)*

27. $1\frac{1}{3} \div \frac{2}{3}$ **28.** $6\frac{3}{5} \div \frac{3}{4}$ **29.** $2\frac{1}{2} \div 8$ **30.** $4\frac{1}{6} \div 1\frac{1}{8}$

31. **MULTIPLE CHOICE** You are filling bottles from 5 gallons of lemonade. How many bottles can you fill when each bottle is $\frac{3}{8}$ of a gallon? *(Section 2.2)*

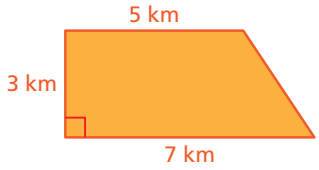
Ⓐ $1\frac{7}{8}$ Ⓑ 3 Ⓒ 8 Ⓓ $13\frac{1}{3}$

4.3–4.4 Quiz

Check It Out
Progress Check
BigIdeasMath.com

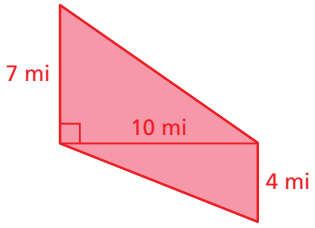
Find the area of the trapezoid. *(Section 4.3)*

1.



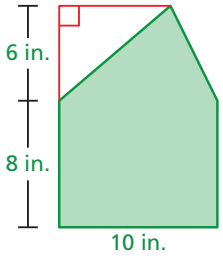
2.

3.

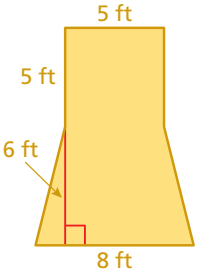


Find the area of the figure. *(Section 4.3)*

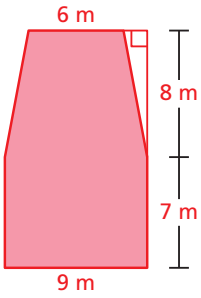
4.



5.



6.



Draw the polygon with the given vertices in a coordinate plane. *(Section 4.4)*

7. $A(1, 2)$, $B(3, 5)$, $C(6, 1)$

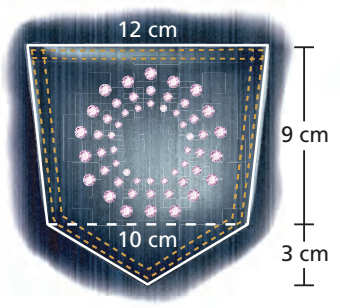
8. $E(1, 2)$, $F(3, 6)$, $G(8, 6)$, $H(6, 2)$

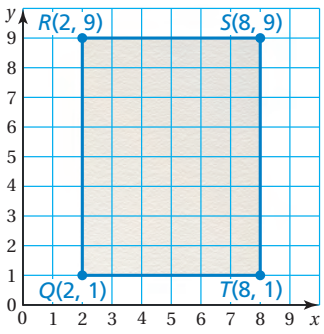
Find the perimeter and the area of the polygon with the given vertices. *(Section 4.4)*

9. $J(1, 3)$, $K(1, 8)$, $L(5, 8)$, $M(5, 3)$

10. $P(1, 2)$, $Q(1, 7)$, $R(7, 7)$, $S(7, 2)$

11. **BACK POCKET** How much material do you need to make two back pockets? *(Section 4.3)*





12. **PATIO** Plans for a patio are shown in the coordinate plane at the left. The coordinates are measured in feet. Find the perimeter and the area of the patio. *(Section 4.4)*

Alternative Assessment Options

Math Chat
Structured Interview
Student Reflective Focus Question
Writing Prompt

Student Reflective Focus Question

Ask students to use their own words to summarize the similarities and differences among the area formulas for a rectangle, parallelogram, triangle, and trapezoid. Be sure that they include examples. Select students at random to present their summaries to the class.

Study Help Sample Answers

Remind students to complete Graphic Organizers for the rest of the chapter.

2.

Area of a trapezoid

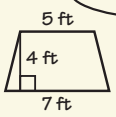
Words

The area A of a trapezoid is one-half the product of its height h and the sum of its bases b_1 and b_2.

Algebra

$A = \frac{1}{2}h(b_1 + b_2)$

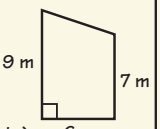
Example



$A = \frac{1}{2}h(b_1 + b_2)$
$= \frac{1}{2}(4)(5 + 7)$
$= 24$

The area of the trapezoid is 24 square feet.

Example



$A = \frac{1}{2}h(b_1 + b_2)$
$= \frac{1}{2}(6)(7 + 9)$
$= 48$

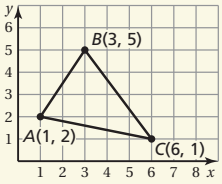
The area of the trapezoid is 48 square meters.

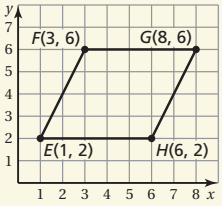
3–5. Available at *BigIdeasMath.com.*

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Lesson Tutorials *BigIdeasMath.com*	Resources by Chapter • Enrichment and Extension • Technology Connection Game Closet at *BigIdeasMath.com* Start the Chapter Review

Answers

1. 18 km^2
2. 100 in.^2
3. 55 mi^2
4. 110 in.^2
5. 64 ft^2
6. 123 m^2
7. 

8. 

9. 18 units; 20 units^2
10. 22 units; 30 units^2
11. 228 cm^2
12. 28 ft; 48 ft^2

Technology for the Teacher

Online Assessment
Assessment Book
ExamView® Assessment Suite

For the Teacher

Additional Review Options

- *BigIdeasMath.com*
- Online Assessment
- Game Closet at *BigIdeasMath.com*
- Vocabulary Help
- Resources by Chapter

Answers

1. 500 yd^2
2. 242 mm^2
3. 80 km^2
4. 175 cm^2
5. 105 m^2
6. 6 in.^2
7. 49 mi^2
8. 76 ft^2
9. 120 cm^2
10. 90 in.^2

Review of Common Errors

Exercises 1 and 2

- Students may use the formula for perimeter instead of the formula for area. Tell students to write the formula, and then identify the value of each variable before substituting.

Exercises 3 and 4

- Students may multiply the base and the height and forget to multiply by $\frac{1}{2}$. Tell students to write the formula, and then identify the value of each variable before substituting.

Exercises 3 and 4

- Students may mentally find the area by multiplying one-half the base and one-half the height. The answer will be one-half of the value of the correct answer. Tell students to write the formula, and then identify the value of each variable before substituting.

Exercise 4

- Students may be confused by the drawing and labeling of obtuse triangles. Remind students to identify the base of the triangle. Then the height of the triangle is the perpendicular distance from the base to the opposite vertex. For obtuse triangles, the height may be labeled outside of the triangle.

Exercises 5–7

- Students may forget to multiply by $\frac{1}{2}$. Tell students to write the formula for the area of a trapezoid, and then identify the value of each variable before substituting.

Exercises 5–7

- Students may substitute the height for one of the bases. Tell students to write the formula for the area of a trapezoid, and then identify the value of each variable before substituting.

Exercise 14

- The fractions in the coordinates may confuse students. Refresh the concept of graphing a coordinate involving a fraction.

Exercises 15–18

- Students may confuse perimeter and area. Have them write the formulas and then substitute the values. Students should also include "units" or "square units" as part of the answer.

4 Chapter Review

Check It Out
Vocabulary Help
BigIdeasMath.com

Review Key Vocabulary

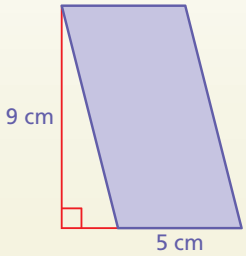
polygon, *p. 152*

composite figure, *p. 172*

Review Examples and Exercises

4.1 Areas of Parallelograms *(pp. 152–157)*

Find the area of the parallelogram.



$A = bh$ Write formula.

$= 5(9)$ Substitute 5 for b and 9 for h.

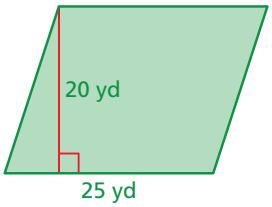
$= 45$ Multiply.

∴ The area of the parallelogram is 45 square centimeters.

Exercises

Find the area of the parallelogram.

1.



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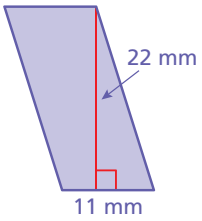


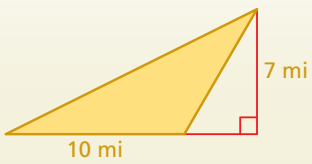
4.2 Areas of Triangles *(pp. 158–163)*

Find the area of the triangle.

$A = \frac{1}{2}bh$ Write formula.

$= \frac{1}{2}(10)(7)$ Substitute.

$= 35$ Multiply.

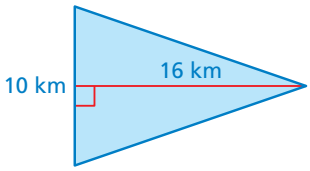


∴ The area of the triangle is 35 square miles.

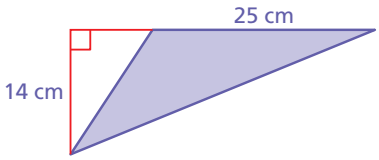
Exercises

Find the area of the triangle.

3.



4.



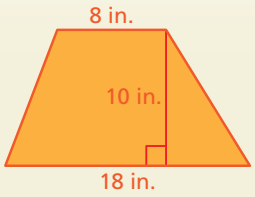
4.3 Areas of Trapezoids *(pp. 166–173)*

Find the area of the trapezoid.

$$A = \frac{1}{2}h(b_1 + b_2) \quad \text{Write formula.}$$

$$= \frac{1}{2}(10)(8 + 18) \quad \text{Substitute.}$$

$$= \frac{1}{2}(10)(26) = 130 \quad \text{Multiply.}$$

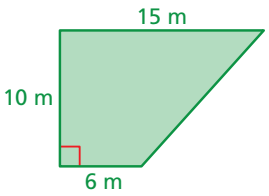


∴ The area of the trapezoid is 130 square inches.

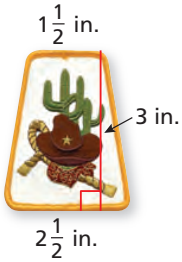
Exercises

Find the area of the trapezoid.

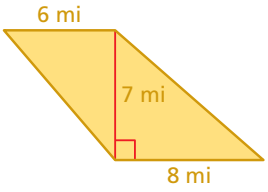
5.



6.

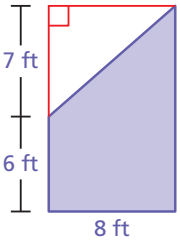


7.

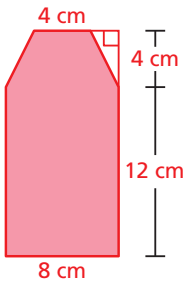


Find the area of the figure.

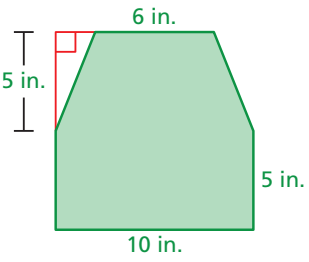
8.



9.



10.



Review Game

Area and Perimeter Tic-Tac-Toe

Materials

- Tic-Tac-Toe Game Boards (Each board will have three different parallelograms, three different triangles, and three different trapezoids with the lengths, widths, and heights labeled.)
- Coin
- 5 red and 5 blue chips

Players: 2

Directions

Each student has 5 chips of the same color.

The first player selects a shape and then tosses a coin. If the coin is heads, the student finds the area of the shape. If the coin is tails, the student finds the perimeter of the shape.

If the answer is correct, the student covers the shape with a chip. If the answer is incorrect, the student loses a turn and the second player has an opportunity to solve the problem and cover the shape with a chip.

Students continue taking turns until one player gets three chips in a row, vertically, horizontally, or diagonally.

Students can exchange boards with another group and continue to play another round.

Who Wins?

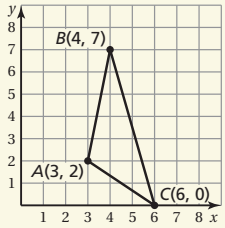
A player earns one point for getting Tic-Tac-Toe. Continue to play until one student gets three points or until the teacher calls time.

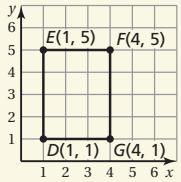
For the Student

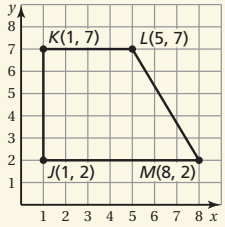
Additional Practice

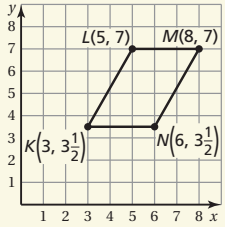
- Lesson Tutorials
- Multi-Language Glossary
- Self-Grading Progress Check
- *BigIdeasMath.com*
  Dynamic Student Edition
  Student Resources

Answers

11. 

12. 

13. 

14. 

15. 18 units; 20 units^2

16. 10 units; 6 units^2

17. 18 units; 18 units^2

18. 28 units; 33 units^2

My Thoughts on the Chapter

What worked. . .

Teacher Tip

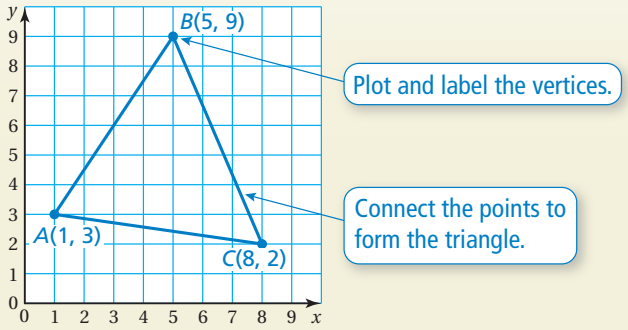
Not allowed to write in your teaching edition? Use sticky notes to record your thoughts.

What did not work. . .

What I would do differently. . .

4.4 Polygons in the Coordinate Plane *(pp. 174–179)*

a. The vertices of a triangle are $A(1, 3)$, $B(5, 9)$, and $C(8, 2)$. Draw the triangle in a coordinate plane.



b. The vertices of a rectangle are $F(2, 6)$, $G(8, 6)$, $H(8, 1)$, and $J(2, 1)$. Draw the rectangle in a coordinate plane and find its perimeter.

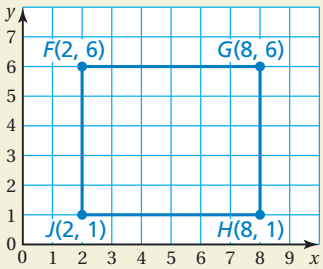
Draw the rectangle and use the vertices to find its dimensions.

The length is the horizontal distance between $F(2, 6)$ and $G(8, 6)$, which is the difference of the x-coordinates.

$$\text{length} = 8 - 2 = 6 \text{ units}$$

The width is the vertical distance between $G(8, 6)$ and $H(8, 1)$, which is the difference of the y-coordinates.

$$\text{width} = 6 - 1 = 5 \text{ units}$$



∴ So, the perimeter of the rectangle is $2(6) + 2(5) = 22$ units.

Exercises

Draw the polygon with the given vertices in a coordinate plane.

11. $A(3, 2)$, $B(4, 7)$, $C(6, 0)$

12. $D(1, 1)$, $E(1, 5)$, $F(4, 5)$, $G(4, 1)$

13. $J(1, 2)$, $K(1, 7)$, $L(5, 7)$, $M(8, 2)$

14. $K\left(3, 3\frac{1}{2}\right)$, $L(5, 7)$, $M(8, 7)$, $N\left(6, 3\frac{1}{2}\right)$

Find the perimeter and the area of the polygon with the given vertices.

15. $P(4, 3)$, $Q(4, 7)$, $R(9, 7)$, $S(9, 3)$

16. $T(2, 7)$, $U(2, 9)$, $V(5, 9)$, $W(5, 7)$

17. $W(11, 2)$, $X(11, 8)$, $Y(14, 8)$, $Z(14, 2)$

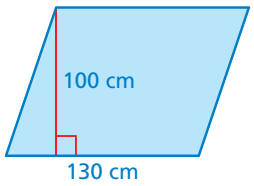
18. $A(12, 2)$, $B(12, 13)$, $C(15, 13)$, $D(15, 2)$

4 Chapter Test

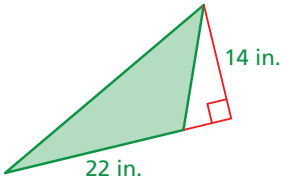
Check It Out
Test Practice
BigIdeasMath.com

Find the area of the parallelogram, triangle, or trapezoid.

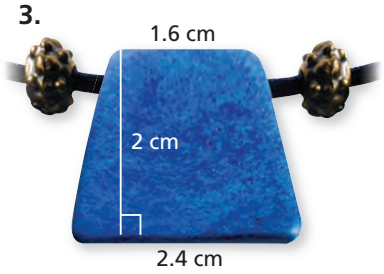
1.



2.

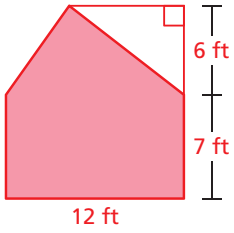


3.

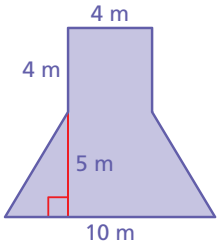


Find the area of the figure.

4.



5.



6.

Draw the polygon with the given vertices in a coordinate plane.

7. $A(4, 2)$, $B(5, 6)$, $C(7, 4)$

8. $D(3, 4)$, $E(5, 8)$, $F(8, 8)$, $G(6, 4)$

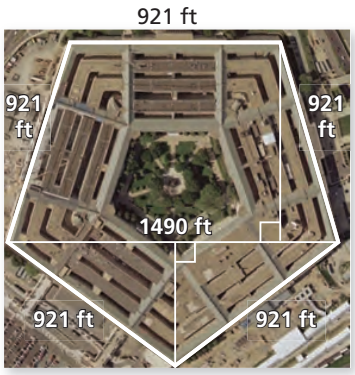
Find the perimeter and the area of the polygon with the given vertices.

9. $Q(5, 6)$, $R(5, 10)$, $S(9, 10)$, $T(9, 6)$

10. $W(2, 8)$, $X(2, 16)$, $Y(8, 16)$, $Z(8, 8)$

11. **TABLETOP** The base lengths of a trapezoidal tabletop are 6 feet and 8 feet. The height is 5 feet. What is the area of the tabletop?

12. **PENTAGON** The Pentagon in Arlington, Virginia, is the headquarters of the U.S. Department of Defense.



a. Find the perimeter of the Pentagon.

b. A pentagon is made of a triangle and a trapezoid. The height of the triangle shown is about 541 feet, and the height of the trapezoid shown is about 876 feet. Estimate the land area of the Pentagon.

13. **CAMPING** The vertices of a campsite are (25, 15), (25, 30), (55, 30), and (55, 15). The vertices of your tent are (30, 20), (30, 25), (40, 25), and (40, 20). The coordinates are measured in feet. What is the area of the campsite not covered by your tent?

Test Item References

Chapter Test Questions	Section to Review	Common Core State Standards
1	4.1	6.G.1
2	4.2	6.G.1
3–6, 11, 12	4.3	6.G.1
7–10, 13	4.4	6.G.3

Test-Taking Strategies

Remind students to quickly look over the entire test before they start so that they can budget their time. On this test, it is very important for students to **Stop** and **Think**. When students hurry on a test dealing with perimeter and area formulas, they may use the wrong formula. Encourage students to write the formula first, and then substitute the values for the variables.

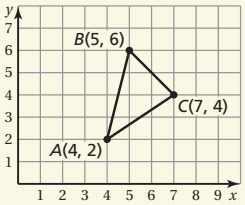
Common Errors

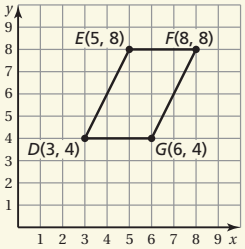
- **Exercises 1–3** Students may use the wrong formula. Tell students to write the formula, and then identify the value of each variable before substituting.
- **Exercise 2** Students may multiply the base and the height and forget to multiply by $\frac{1}{2}$. Tell students to write the formula, and then identify the value of each variable before substituting.
- **Exercise 2** Students may mentally find the area by multiplying one-half the base and one-half the height. The answer will be one-half of the value of the correct answer. Tell students to write the formula, and then identify the value of each variable before substituting.
- **Exercise 2** Students may be confused by the drawing and labeling of obtuse triangles. Remind students to identify the base of the triangle. Then the height of the triangle is the perpendicular distance from the base to the opposite vertex. For obtuse triangles, the height may be labeled outside of the triangle.
- **Exercises 9 and 10** Students may confuse perimeter and area. Have them write the formulas and then substitute the values. Students should also include "units" or "square units" as part of the answer.
- **Exercises 3, 11, and 12** Students may substitute the height for one of the bases. Tell students to write the formula for the area of a trapezoid, and then identify the value of each variable before substituting.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials *BigIdeasMath.com* Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Game Closet at *BigIdeasMath.com* Start Cumulative Assessment

Answers

1. 13,000 cm^2
2. 154 $in.^2$
3. 4 cm^2
4. 120 ft^2
5. 51 m^2
6. 128 $in.^2$
7. 

8. 

9. 16 units; 16 $units^2$
10. 28 units; 48 $units^2$
11. 35 ft^2
12. **a.** 4605 ft
    b. 1,459,063 ft^2
13. 400 ft^2

Technology for the Teacher

Online Assessment
Assessment Book
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Test-Taking Strategies

Available at *BigIdeasMath.com*

After Answering Easy Questions, Relax
Answer Easy Questions First
Estimate the Answer
Read All Choices before Answering
Read Question before Answering
Solve Directly or Eliminate Choices
Solve Problem before Looking at Choices
Use Intelligent Guessing
Work Backwards

About this Strategy

When taking a multiple choice test, be sure to read each question carefully and thoroughly. When taking a timed test, it is often best to skim the test and answer the easy questions first. Be careful that you record your answer in the correct position on the answer sheet.

Answers

1. B
2. 5832
3. F
4. C

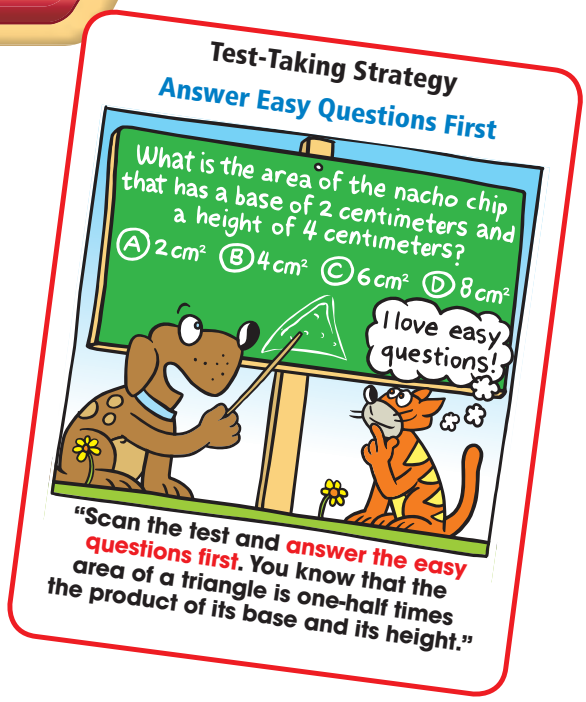
Technology for the Teacher

Performance Tasks
Online Assessment
Assessment Book
ExamView® Assessment Suite

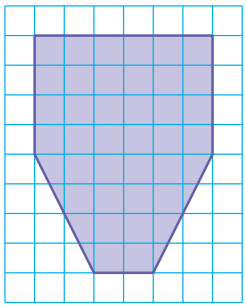
Item Analysis

1. **A.** The student finds the area of a trapezoid with bases of 2 and 6 and a height of 8.
   B. Correct answer
   C. The student counts all squares with any shading.
   D. The student uses $A = h(b_1 + b_2)$ to find the area of a trapezoid.
2. **Gridded Response:** Correct answer: 5832
   Common Error: The student adds 18 three times and gets 54.
3. **F.** Correct answer
   G. The student found a factor of 36 but not a factor of 42.
   H. The student found a factor of 36 but not a factor of 42.
   I. The student found a factor of 36 but not a factor of 42.
4. **A.** The student only finds the length.
   B. The student adds the length and width.
   C. Correct answer
   D. The student adds all of the *x*- and *y*-coordinates of the vertices.

4 Cumulative Assessment



1. What is the area of the shaded figure shown below?



A. 32 units^2

B. 40 units^2

C. 44 units^2

D. 56 units^2

2. What is the value of the expression below?

18^3

3. You have 36 red apples and 42 green apples. What is the greatest number of identical fruit baskets you can make with no apples left over?

F. 6

G. 9

H. 12

I. 18

4. What is the perimeter of the rectangle with the vertices shown below?

$A(4, 7)$, $B(4, 15)$, $C(9, 15)$, $D(9,7)$

A. 8 units

B. 13 units

C. 26 units

D. 70 units

5. What property was used to simplify the expression?

$$\begin{aligned} 5 \times 78 &= 5(70 + 8) \\ &= 5(70) + 5(8) \\ &= 350 + 40 \\ &= 390 \end{aligned}$$

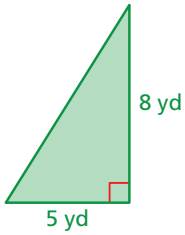
F. Associative Property of Multiplication

G. Commutative Property of Addition

H. Distributive Property

I. Multiplication Property of One

6. What is the area, in square yards, of the triangle below?



7. Which of the following is equivalent to $\frac{12}{35}$?

A. $\frac{5}{6} \div \frac{2}{7}$

B. $\frac{2}{7} \div \frac{6}{5}$

C. $\frac{2}{7} \div \frac{5}{6}$

D. $\frac{5}{6} \div \frac{7}{2}$

8. The description below represents the area of which polygon?

"one-half the product of its height and the sum of the lengths of its bases"

F. rectangle

G. square

H. trapezoid

I. triangle

Item Analysis (continued)

5. **F.** The student incorrectly identifies the property being used as the Associative Property of Multiplication.

   G. The student incorrectly identifies the property being used as the Commutative Property of Addition.

   H. Correct answer

   I. The student incorrectly identifies the property being used as the Multiplication Property of One.

6. **Gridded Response:** Correct answer: 20

   Common Error: The student multiplies 5 by 8 to get 40 and forgets to multiply by one-half.

7. **A.** The student incorrectly inverts the dividend and multiplies instead of inverting the divisor and multiplying.

   B. The student incorrectly multiplies the fractions instead of inverting the divisor and multiplying.

   C. Correct answer

   D. The student incorrectly inverts both the dividend and the divisor instead of inverting only the divisor before multiplying.

8. **F.** The student incorrectly identifies the description as the formula for the area of a rectangle.

   G. The student incorrectly identifies the description as the formula for the area of a square.

   H. Correct answer

   I. The student incorrectly identifies the description as the formula for the area of a triangle.

Answers

5. H
6. 20
7. C
8. H

Answers

9. B

10. F

11. *Part A* 435 ft^2

Part B 435 ft^2

Part C The trapezoid can be decomposed into two triangles and a rectangle. The formulas in Part A are simpler. Part B requires only one formula.

Item Analysis (continued)

9. **A.** The student incorrectly performs the order of operations.

B. Correct answer

C. The student incorrectly performs the order of operations.

D. The student incorrectly performs the order of operations.

10. **F.** Correct answer

G. The student writes an expression that represents addition instead of multiplication.

H. The student writes an expression that represents subtraction instead of multiplication.

I. The student writes an expression that represents division instead of multiplication.

11. **4 points** The student demonstrates a thorough understanding of finding areas of polygons. The student calculates the area of the yard correctly and gets an answer of 435 square feet. The student is able to explain why the two methods give the same result and provides an advantage for using each method. The student shows accurate, complete work for all parts and provides clear and complete explanations.

3 points The student demonstrates an understanding of finding areas of polygons, but the student's work and explanations demonstrate an essential but less than thorough understanding.

2 points The student demonstrates a partial understanding of finding areas of polygons. The student's work and explanations demonstrate a lack of essential understanding.

1 point The student demonstrates very limited understanding of finding areas of polygons. The student's response is incomplete and exhibits many flaws.

0 points The student provided no response, a completely incorrect or incomprehensible response, or a response that demonstrates insufficient understanding of finding areas of polygons.

9. Edward was evaluating the expression in the box.

$$\begin{aligned} 180 \div 9 + 3^4 - 1 &= 180 \div 9 + 81 - 1 \\ &= 180 \div 90 - 1 \\ &= 2 - 1 \\ &= 1 \end{aligned}$$

What should Edward do to correct the error that he made?

A. Add 9 and 81 then subtract 1 before dividing.

B. Divide 180 by 9 before adding or subtracting.

C. Divide 180 by 9 then subtract 1 before adding 3^4.

D. Subtract 1 from 90 before dividing.

10. You have 3 times as many guitar picks as your cousin. Let v be the number of guitar picks that your cousin has. Which expression represents the number of guitar picks you have?

F. $3v$

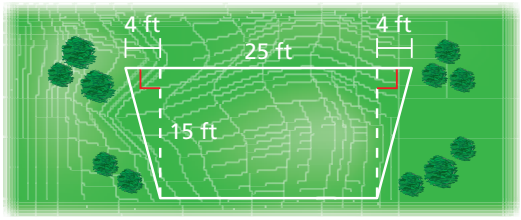
G. $v + 3$

H. $3 - v$

I. $\frac{v}{3}$

11. Your family hires a company to install invisible fencing around your yard.

Think
Solve
Explain



Part A Find the area of the yard using only the area formulas for rectangles and triangles. Show your work.

Part B Find the area of the yard using the area formula for trapezoids.

Part C Explain why the two methods of finding the area of the yard give the same result. Describe the advantages of each method.