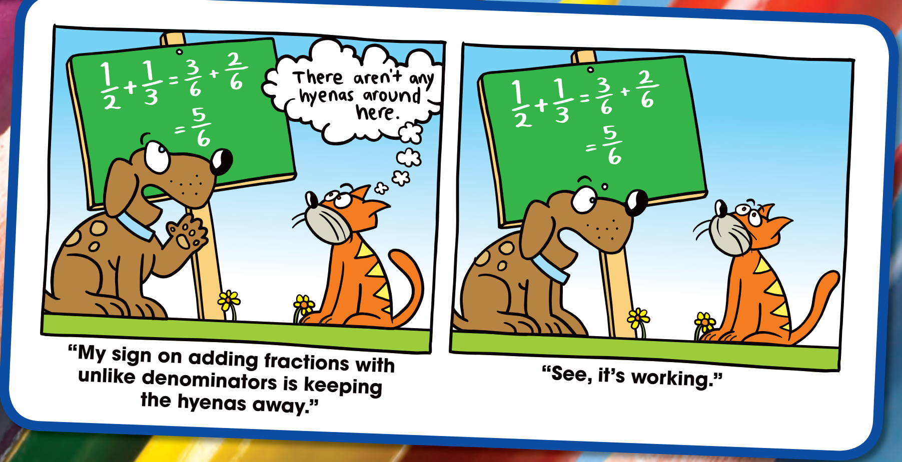
1 Numerical Expressions and Factors



Common Core Progression

4th Grade

- Fluently add and subtract.
- Find factors and multiples from 1–100.

5th Grade

- Fluently multiply.
- Multiply and divide by powers of 10.
- Evaluate expressions with whole number exponents with powers of 10.
- Use parenthesis, brackets, or braces in numerical expressions.

6th Grade

- Fluently divide.
- Write and evaluate with whole number exponents.
- Find the prime factorization of a number.
- Find the GCF of two whole numbers.
- Find the LCM of two whole numbers.

Chapter Summary

Section	Common Core State Standard	
1.1	Learning	6.NS.2 ★
1.2	Preparing for	6.EE.1
1.3	Learning	6.EE.1 ★
1.4	Preparing for	6.NS.4
1.5	Learning	6.NS.4, 6.EE.2b
1.6	Learning	6.NS.4

★ Teaching is complete. Standard can be assessed.

Pacing Guide for Chapter 1

Chapter Opener	1 Day
Section 1 Activity Lesson	 1 Day 1 Day
Section 2 Activity Lesson	 1 Day 1 Day
Section 3 Activity Lesson	 1 Day 1 Day
Study Help / Quiz	1 Day
Section 4 Activity Lesson	 1 Day 1 Day
Section 5 Activity Lesson	 1 Day 1 Day
Section 6 Activity Lesson Extension	 1 Day 1 Day 1 Day
Chapter Review/ Chapter Tests	2 Days
Total Chapter 1	17 Days
Year-to-Date	18 Days

Technology for the Teacher

BigIdeasMath.com
Chapter at a Glance
Complete Materials List
Parent Letters: English and Spanish

Common Core State Standards

4.OA.4 … Determine whether a given whole number in the range 1–100 is prime or composite.
4.NF.3c Add and subtract mixed numbers with like denominators.

Additional Topics for Review

- Dividing Whole Numbers
- Order of Operations (no exponents)
- Multiples of Whole Numbers

Try It Yourself

1. prime **2.** composite
3. prime **4.** prime
5. composite **6.** composite
7. prime **8.** composite
9. composite
10. $\frac{62}{9}$ or $6\frac{8}{9}$ **11.** $\frac{106}{11}$ or $9\frac{7}{11}$
12. $\frac{33}{4}$ or $8\frac{1}{4}$ **13.** $\frac{58}{13}$ or $4\frac{6}{13}$
14. $\frac{7}{2}$ or $3\frac{1}{2}$ **15.** $\frac{4}{3}$ or $1\frac{1}{3}$

Record and Practice Journal Fair Game Review

1. composite **2.** prime
3. prime **4.** composite
5. prime **6.** composite
7. composite **8.** prime
9. prime **10.** composite
11. yes **12.** no
13. $2\frac{4}{5}$ **14.** $5\frac{5}{7}$
15. $10\frac{7}{9}$ **16.** $8\frac{10}{11}$
17. $2\frac{1}{2}$ **18.** $1\frac{1}{2}$
19. $\frac{3}{5}$ **20.** $3\frac{1}{2}$
21. $4\frac{1}{2}$ c

Math Background Notes

Vocabulary Review

- Prime Number
- Composite Number
- Mixed Number
- Improper Fraction

Identifying Prime and Composite Numbers

- Students should be familiar with factor pairs, prime numbers, and composite numbers.
- Review the definition of a prime number. A *prime number* is a whole number greater than 1 whose only factors are 1 and itself.
- You may want to review factoring with students. To find the factors of a number, try to divide the number by prime numbers that are less than the given number.
- **Common error:** Students may remember the divisibility rules for 2, 3, and 5 but forget to check for divisibility for larger prime numbers. Remind students to keep checking all of the prime numbers less than the given number before deciding whether the number is prime or composite.

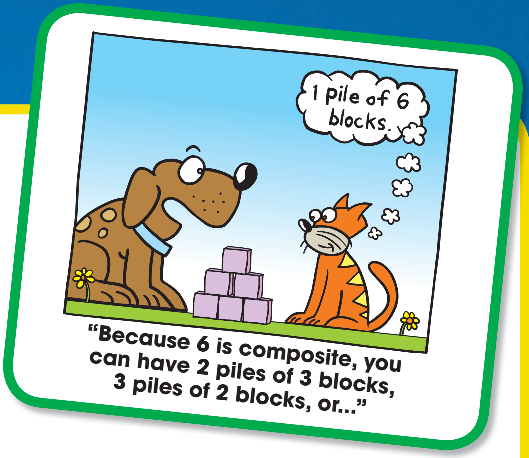
Adding and Subtracting Mixed Numbers with Like Denominators

- Remind students that adding and subtracting fractions always requires a common denominator.
- Students should be familiar with mixed numbers and improper fractions, although these vocabulary terms might be new to them.
- Remind students that mixed numbers and improper fractions represent the same number in two different ways. Both represent parts greater than or equal to one whole.
- When two fractions already share a common denominator, simply add or subtract their numerators and keep the common denominator.
- Students may want to add or subtract the whole numbers first, then the fractional parts. Although this method may have worked for students before, be sure to model the correct steps to ensure that students have a good foundation for adding and subtracting mixed numbers with unlike denominators.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Record and Practice Journal • Fair Game Review Skills Review Handbook Lesson Tutorials	Game Closet at *BigIdeasMath.com* Start the next section

What You Learned Before



"Because 6 is composite, you can have 2 piles of 3 blocks, 3 piles of 2 blocks, or..."

Identifying Prime and Composite Numbers

Example 1 **Determine whether 26 is prime or composite.**
Because the factors of 26 are 1, 2, 13, and 26, it is composite.

Example 2 **Determine whether 37 is prime or composite.**
Because the only factors of 37 are 1 and 37, it is prime.

Try It Yourself

Determine whether the number is prime or composite.

1. 5
2. 14
3. 17
4. 23
5. 28
6. 33
7. 43
8. 57
9. 64

Adding and Subtracting Mixed Numbers with Like Denominators

Example 3 **Find $2\frac{3}{5} + 4\frac{1}{5}$.**

$$2\frac{3}{5} + 4\frac{1}{5} = \frac{2 \cdot 5 + 3}{5} + \frac{4 \cdot 5 + 1}{5}$$ Rewrite the mixed numbers as improper fractions.

$$= \frac{13}{5} + \frac{21}{5}$$ Simplify.

$$= \frac{13 + 21}{5}$$ Add the numerators.

$$= \frac{34}{5}, \text{ or } 6\frac{4}{5}$$ Simplify.

Try It Yourself

Add or subtract.

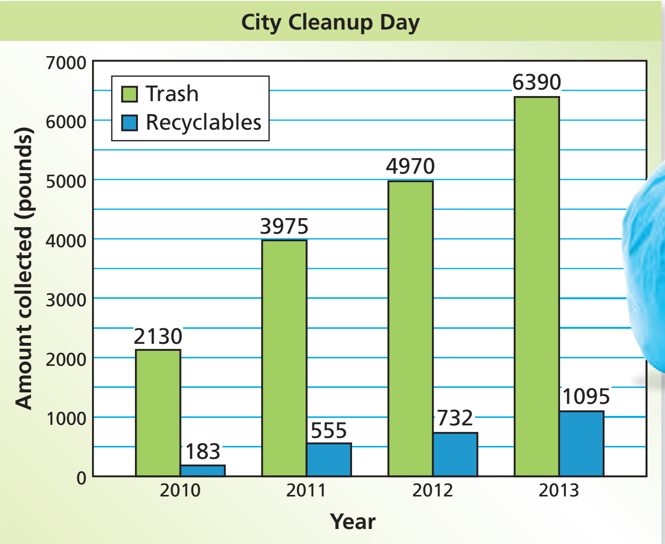
10. $4\frac{1}{9} + 2\frac{7}{9}$
11. $6\frac{1}{11} + 3\frac{6}{11}$
12. $3\frac{7}{8} + 4\frac{3}{8}$
13. $5\frac{8}{13} - 1\frac{2}{13}$
14. $7\frac{1}{4} - 3\frac{3}{4}$
15. $4\frac{1}{6} - 2\frac{5}{6}$

1.1 Whole Number Operations

Essential Question How do you know which operation to choose when solving a real-life problem?

1 ACTIVITY: Choosing an Operation

Work with a partner. The double bar graph shows the history of a citywide cleanup day.



- **Copy each question below.**
- **Underline a key word or phrase that helps you know which operation to use to answer the question. State the operation. Why do you think the key word or phrase indicates the operation you chose?**
- **Write an expression you can use to answer the question.**
- **Find the value of your expression.**

Whole Numbers

In this lesson, you will

- determine which operation to perform.
- divide multi-digit numbers.

a. What is the total amount of trash collected from 2010 to 2013?

b. How many more pounds of recyclables were collected in 2013 than in 2010?

c. How many times more recyclables were collected in 2012 than in 2010?

d. The amount of trash collected in 2014 is estimated to be twice the amount collected in 2011. What is that amount?

Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP1a Make Sense of Problems** and **MP3a Construct Viable Arguments:** From the first day, you want to establish a norm in your classroom that each student will discuss mathematical problems with a partner and learn to form arguments based upon stated assumptions, definitions, and previously established results. Students need time to think, discuss, and evaluate their reasoning. Assure students that this will be the case in math this year.

Motivate

- Share some recycling facts with students in the form of, did you know. . .
  - An average American uses 465 trees worth of paper during his or her lifetime.
  - Each person creates about 4.7 pounds of waste every single day.
  - Paper plus cardboard combined make up 73% of the materials in landfills.
  - Over 25 billion styrofoam cups are thrown away in the United States each year.
  - Approximately 350,000 aluminum cans are made in a minute.
- If there is a recycling program at your school, take time to discuss the importance of the program.

Activity Notes

Activity 1

- As a warm-up, you may want to have students briefly describe the amount of debris collected in the city for the last four years to make sure students understand how to read the double bar graph.
- In this activity, students need to read carefully, looking for words that suggest an operation to be performed.
- **Common Error:** The word "times" does not immediately imply that multiplication is to be done. In part (c), "times" is used, and it is a division problem. You could point out that in part (d), the problem could be rephrased as "two times the amount of trash..." to help students see the difference.
- As you circulate, do an informal assessment of whole number operations. Are there students unsure about the process?
- Ask volunteers to share their work at the board.
- **Extension:** Discuss recycling efforts in your town.

Common Core State Standards

6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.

Previous Learning

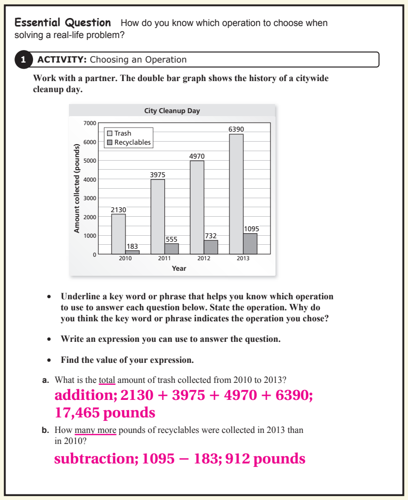
Students need to be familiar with basic computation facts involving whole numbers.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

1.1 Record and Practice Journal



Essential Question How do you know which operation to choose when solving a real-life problem?

1 ACTIVITY: Choosing an Operation

Work with a partner. The double bar graph shows the history of a citywide cleanup day.

- **Underline a key word or phrase that helps you know which operation to use to answer each question below. State the operation. Why do you think the key word or phrase indicates the operation you chose?**
- **Write an expression you can use to answer the question.**
- **Find the value of your expression.**

a. What is the total amount of trash collected from 2010 to 2013?

addition; 2130 + 3975 + 4970 + 6390; 17,465 pounds

b. How many more pounds of recyclables were collected in 2013 than in 2010?

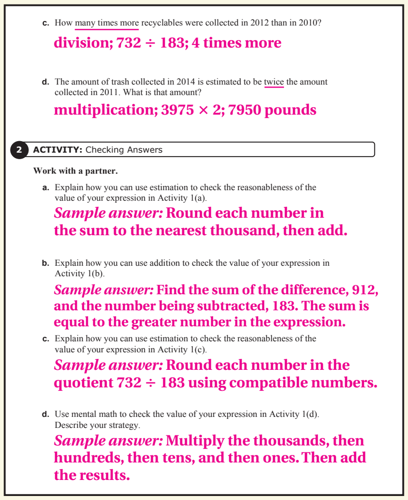
subtraction; 1095 − 183; 912 pounds

Differentiated Instruction

Kinesthetic

Provide counters to students to use with the magic square in Question 5. Students determine the number of counters needed by modeling the numbers in the completed diagonal. Next, they find a row or column with two given numbers and model those numbers with the counters. The remaining counters represent the missing number in that row or column. Students continue the process until all the numbers are found.

1.1 Record and Practice Journal



c. How many times more recyclables were collected in 2012 than in 2010?

division; 732 ÷ 183; 4 times more

d. The amount of trash collected in 2014 is estimated to be twice the amount collected in 2011. What is that amount?

multiplication; 3975 × 2; 7950 pounds

2 ACTIVITY: Checking Answers

Work with a partner.

a. Explain how you can use estimation to check the reasonableness of the value of your expression in Activity 1(a).

Sample answer: **Round each number in the sum to the nearest thousand, then add.**

b. Explain how you can use addition to check the value of your expression in Activity 1(b).

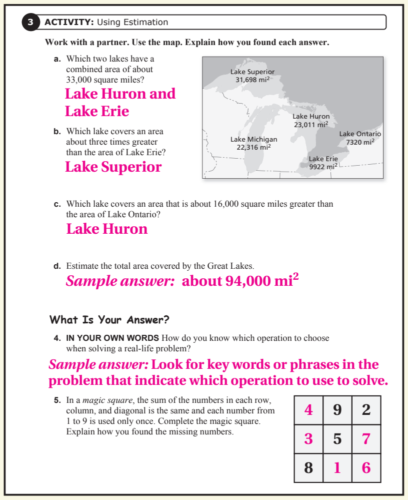
Sample answer: **Find the sum of the difference, 912, and the number being subtracted, 183. The sum is equal to the greater number in the expression.**

c. Explain how you can use estimation to check the reasonableness of the value of your expression in Activity 1(c).

Sample answer: **Round each number in the quotient 732 ÷ 183 using compatible numbers.**

d. Use mental math to check the value of your expression in Activity 1(d). Describe your strategy.

Sample answer: **Multiply the thousands, then hundreds, then tens, and then ones. Then add the results.**



3 ACTIVITY: Using Estimation

Work with a partner. Use the map. Explain how you found each answer.

a. Which two lakes have a combined area of about 33,000 square miles?

Lake Huron and Lake Erie

b. Which lake covers an area about three times greater than the area of Lake Erie?

Lake Superior

c. Which lake covers an area that is about 16,000 square miles greater than the area of Lake Ontario?

Lake Huron

d. Estimate the total area covered by the Great Lakes.

Sample answer: **about 94,000 mi^2**

What Is Your Answer?

4. IN YOUR OWN WORDS How do you know which operation to choose when solving a real-life problem?

Sample answer: **Look for key words or phrases in the problem that indicate which operation to use to solve.**

5. In a *magic square*, the sum of the numbers in each row, column, and diagonal is the same and each number from 1 to 9 is used only once. Complete the magic square. Explain how you found the missing numbers.

4	9	2
3	5	7
8	1	6

Laurie's Notes

Activity 2

- **MP8 Look for and Express Regularity in Repeated Reasoning:** As students work through problems, they should always be asking themselves if the results they are getting seem reasonable.
- In this activity, students look back at Activity 1. They use estimation and mental math to check the reasonableness of their answers. Throughout the year, encourage your students to check their answers in all of their work.
- Ask volunteers to share their answers.

Activity 3

? "Can anyone name the Great Lakes?" The acronym *HOMES* can help students remember. Huron, Ontario, Michigan, Erie, Superior

? "Have any of you visited one or more of the Great Lakes?

? "Do you know which is the smallest? Ontario the largest?" Superior

- In this activity, students will use the information provided on the map to answer questions. Remind students they should begin by using estimation.
- When students share their answers, listen for their explanations of how they found each answer. For instance, in part (a), if the combined area is 33,000 square miles, Lake Superior, with an area of almost 32,000 square miles, is not going to be one of the lakes. Students might then explain that they compared the total areas of Erie and Michigan with the total areas of Erie and Huron.

What Is Your Answer?

- **Big Idea:** In Question 4, listen for the big idea, namely that key words or phrases, along with the context, help in deciding which operation is needed to answer the question.
- In Question 5, students should be able to describe their processes of finding the missing numbers, not just what the missing numbers are.

Closure

- **Exit Ticket:**
  - Forests are being cut at a rate of 100 acres per minute. How many acres per hour is this? 6000 acres per hour
  - Recycling 15 trees worth of paper reduces air pollutants by about 177,000 tons. This is about how many tons per tree? about 11,800 tons per tree

2 ACTIVITY: Checking Answers

Math Practice

Communicate Precisely

What key words should you use so that your partner understands your explanation?

Work with a partner.

a. Explain how you can use estimation to check the reasonableness of the value of your expression in Activity 1(a).

b. Explain how you can use addition to check the value of your expression in Activity 1(b).

c. Explain how you can use estimation to check the reasonableness of the value of your expression in Activity 1(c).

d. Use mental math to check the value of your expression in Activity 1(d). Describe your strategy.

3 ACTIVITY: Using Estimation

Work with a partner. Use the map. Explain how you found each answer.

a. Which two lakes have a combined area of about 33,000 square miles?

b. Which lake covers an area about three times greater than the area of Lake Erie?

c. Which lake covers an area that is about 16,000 square miles greater than the area of Lake Ontario?

d. Estimate the total area covered by the Great Lakes.

What Is Your Answer?

4. **IN YOUR OWN WORDS** How do you know which operation to choose when solving a real-life problem?

5. In a *magic square*, the sum of the numbers in each row, column, and diagonal is the same and each number from 1 to 9 is used only once. Complete the magic square. Explain how you found the missing numbers.

?	9	2
?	5	?
8	?	?

Practice Use what you learned about choosing operations to complete Exercises 8–11 on page 7.

1.1 Lesson

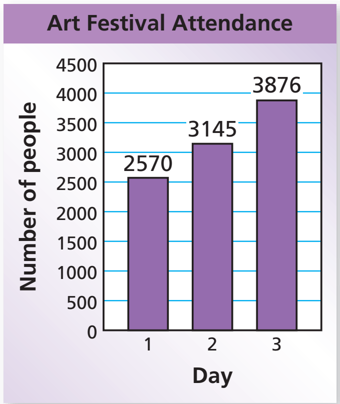
Check It Out
Lesson Tutorials
BigIdeasMath.com

Recall the four basic operations: addition, subtraction, multiplication, and division.

Operation	Words	Algebra
Addition	the *sum* of	$a + b$
Subtraction	the *difference* of	$a - b$
Multiplication	the *product* of	$a \times b$ $\quad a \cdot b$
Division	the *quotient* of	$a \div b$ $\quad \frac{a}{b}$ $\quad b\overline{)a}$

EXAMPLE 1 Adding and Subtracting Whole Numbers

The bar graph shows the attendance at a three-day art festival.



a. What is the total attendance for the art festival?

You want to find the total attendance for the three days. In this case, the phrase *total attendance* indicates you need to find the sum of the daily attendances.

Line up the numbers by their place values, then add.

$$\begin{array}{r} {\scriptstyle 111} \\ 2570 \\ 3145 \\ +\ 3876 \\ \hline 9591 \end{array}$$

The total attendance is 9591 people.

b. What is the increase in attendance from Day 1 to Day 2?

You want to find how many more people attended on Day 2 than on Day 1. In this case, the phrase *how many more* indicates you need to find the difference of the attendances on Day 2 and Day 1.

Line up the numbers by their place values, then subtract.

$$\begin{array}{r} {\scriptstyle 10} \\ {\scriptstyle 2\,\cancel{0}\,14} \\ \cancel{3}\cancel{1}\cancel{4}5 \\ -\ 2570 \\ \hline 575 \end{array}$$

The increase in attendance from Day 1 to Day 2 is 575 people.

EXAMPLE 2 Multiplying Whole Numbers

A school lunch contains 12 chicken nuggets. Ninety-five students buy the lunch. What is the total number of chicken nuggets served?

You want to find the total number of chicken nuggets in 95 groups of 12 chicken nuggets. The phrase *95 groups of 12* indicates you need to find the product of 95 and 12.

$$\begin{array}{rl} 12 & \\ \times\ 95 & \\ \hline 60 & \text{Multiply 12 by the ones digit, 5.} \\ 108 & \text{Multiply 12 by the tens digit, 9.} \\ \hline 1140 & \text{Add.} \end{array}$$

There were 1140 chicken nuggets served.

Study Tip

In Example 2, you can use estimation to check the reasonableness of your answer.

$12 \times 95 \approx 12 \times 100$
$ = 1200$

Because $1200 \approx 1140$, the answer is reasonable.

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students worked with partners to solve problems involving operations with whole numbers. (MP1a, MP3a, MP8)
- **Today:** Students will perform computations with whole numbers and review how to check solutions.

Motivate

- ❓ "Who likes roller coasters?"
- If possible, search the Internet for a video of a roller coaster ride (Skyrush at Hershey Park, PA) and show it to your students.
- Explain that today they will be doing a problem about an amusement park ride. There is a lot of math used at an amusement park.

Lesson Notes

Discuss

- Review the key words and phrases from yesterday that students suggested to help them recognize the operations needed to answer the questions.
- Discuss the different representations of operations using variables, as shown in the table.
- ❓ "It is less common to use the × to represent multiplication when using variables. Can you guess why?" The × symbol may be confused with the variable x.

Example 1

- Yesterday students read a double bar graph. Today a single bar graph provides information. Reviewing how to read a bar graph helps all students.
- ❓ "How can we find the total attendance for three days? Explain." Add the daily attendances together. The phrase "total attendance for three days" means you need to add to find the total.
- ❓ "About how much did the attendance increase from Day 1 to Day 2? Listen for about 500 to 600 people.
- When part (b) is done, ask students how to check their answers. They should be familiar with adding 575 to 2570 upward to get 3145.
- ❓ **Extension:** "What was the trend in attendance at the art show?" *Sample answer:* The increase in the number of people increased each day.

Example 2

- ❓ "What is a reasonable estimate for this problem? Explain." 1200; 12 × 95 is going to be a little less than 12 × 100 = 1200.
- ❓ "Does it matter which number is written on top?" no
- Talk through the multiplication problem. If time permits, repeat the problem by putting the 95 on top and 12 below.
- ❓ "Is our answer reasonable? Explain." Yes; 1140 is close to the estimate of 1200.

Goal Today's lesson is performing operations with whole numbers.

Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

Use the graph in Example 1.

a. What is the total attendance for Day 2 and Day 3 of the art festival? 7021 people

b. What is the increase in attendance from Day 2 to Day 3? 731 people

Extra Example 2

Construction paper packs from Paper Company A have 85 sheets of paper per pack. Twenty-three teachers each decide to buy a pack. What is the total number of sheets purchased? 1955 sheets of paper

On Your Own

1. 2427 **2.** 113

3. 4956

Extra Example 3

You make 18 equal payments for a video game system with games. You pay a total of $468. How much is each payment? $26

English Language Learners

Vocabulary
Encourage English language learners to keep a vocabulary notebook. They should include key vocabulary words as well as any other words or phrases with which they are not familiar. Accompany each word or phrase with its definition or a description.

Laurie's Notes

On Your Own

- **Neighbor Check:** Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.
- Explain to students that working independently and then checking with a neighbor will occur frequently this year.

? "Did any of you use mental math to do Question 2? Explain." Listen for an approach of subtracting 800 and then adding 1.
$912 - 799 = 912 - 800 + 1 = 112 + 1 = 113$

Example 3

- The last two examples review whole number division, an operation that some students may still need work on.
- **Teaching Tip:** When you write the problem $24\overline{)840}$ on the board, use one hand to cover the 40 in the dividend and ask, "Can 24 be divided into 8?" no Now move your hand to reveal 84 in the dividend and ask, "Can 24 be divided into 84?" yes This technique helps students to focus on the process.
- Ask students about each step in the division process. Note the downward arrow, which my students find helpful in working through the steps.

? "How do you check an answer to a division problem?" Multiply the answer (quotient) by the divisor and you should get the dividend.

On Your Own

Now You're Ready
Exercises 12–20

Find the value of the expression. Use estimation to check your answer.

1. 1745 + 682 **2.** 912 − 799 **3.** 42 × 118

EXAMPLE 3 Dividing Whole Numbers: No Remainder

You make 24 equal payments for a go-kart. You pay a total of $840. How much is each payment?

You want to find the number of groups of 24 in $840. The phrase *groups of 24 in $840* indicates you need to find the quotient of 840 and 24.

Use long division to find the quotient.
Decide where to write the first digit of the quotient.

$$\begin{array}{r} ? \\ 24\overline{)840} \end{array}$$

Do not use the hundreds place because 24 is greater than 8.

$$\begin{array}{r} ? \\ 24\overline{)840} \end{array}$$

Use the tens place because 24 is less than 84.

So, divide the tens and write the first digit of the quotient in the tens place.

$$\begin{array}{r} 3 \\ 24\overline{)840} \\ -\,72 \\ \hline 12 \end{array}$$

Divide 84 by 24: There are three groups of 24 in 84.
Multiply 3 and 24.
Subtract 72 from 84.

Next, bring down the 0 and divide the ones.

$$\begin{array}{r} 35 \\ 24\overline{)840} \\ -\,72\downarrow \\ \hline 120 \\ -\,120 \\ \hline 0 \end{array}$$

Divide 120 by 24: There are five groups of 24 in 120.

Multiply 5 and 24.
Subtract 120 from 120.

Remember

$\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$

So, quotient × divisor
= dividend.

The quotient of 840 and 24 is 35.

So, each payment is $35.

Check Find the product of the quotient and the divisor.

$$\begin{array}{rl} 35 & \text{quotient} \\ \times\ 24 & \text{divisor} \\ \hline 140 & \\ 70 & \\ \hline 840 & \text{dividend} \end{array}$$ ✓

On Your Own

Now You're Ready
Exercises 21–23

Find the value of the expression. Use estimation to check your answer.

4. $234 \div 9$ **5.** $\frac{986}{58}$ **6.** $840 \div 105$

7. Find the quotient of 9920 and 320.

When you use long division to divide whole numbers and you obtain a remainder, you can write the quotient as a mixed number using the rule

$$\text{dividend} \div \text{divisor} = \text{quotient} + \frac{\text{remainder}}{\text{divisor}}.$$

EXAMPLE 4 Real-Life Application

A 301-foot-high swing at an amusement park can take 64 people on each ride. A total of 8983 people ride the swing today. All the rides are full except for the last ride. How many rides are given? How many people are on the last ride?

To find the number of rides given, you need to find the number of groups of 64 people in 8983 people. The phrase *groups of 64 people in 8983 people* indicates you need to find the quotient of 8983 and 64.

Divide the place-value positions from left to right.

```
      140 R23
  64)8983          There is one group of 64 in 89.
    - 64
      258          There are four groups of 64 in 258.
    - 256
       23          There are no groups of 64 in 23.
      - 0
       23          The remainder is 23.
```

Do not stop here. You must write a 0 in the ones place of the quotient.

The quotient is $140\frac{23}{64}$. This indicates 140 groups of 64, with 23 remaining.

So, 141 rides are given, with 23 people on the last ride.

On Your Own

Now You're Ready
Exercises 24–26

Find the value of the expression. Use estimation to check your answer.

8. $\frac{6096}{30}$ **9.** $45,691 \div 28$ **10.** $3215 \div 430$

11. **WHAT IF?** In Example 4, 9038 people ride the swing. What is the least number of rides possible?

Laurie's Notes

On Your Own

- **Neighbor Check:** Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.
- Note that Question 5 is represented as a fraction. Students should be comfortable with different representations of division.

Example 4

- The last example involves whole number division with a remainder.
- Review how to write a quotient with a remainder as a mixed number. Use a simple problem such as $14 \div 4 = 3 + \frac{2}{4} = 3\frac{1}{2}$.
- Ask a volunteer to read the problem.
- Probe student understanding by asking how they could solve the problem if there were a few times when the line was empty, and rides occurred that were partially full. Can they find the total number of rides? Students would need to realize that long division will not help answer the question.

? "How can we find the number of rides given?" Divide total number of riders by 64 because there are 64 riders on each ride.

- **MP6 Attend to Precision:** Be sure to discuss the 0 needed in the ones place of the quotient. If students have made an estimate to begin with, they will recognize that 140 makes more sense than 14.

? "23 is less than a whole group of 64. What does the remainder mean and how can we write the remainder?" The remainder of 23 is the number of people on the last ride. Write 23 out of 64, which is $\frac{23}{64}$.

On Your Own

- Notice that Question 8 has a 0 in the quotient. Be sure to have students work this problem.

Closure

- The theoretical ride capacity for Skyrush (at Hershey Park, PA) is 1350 riders per hour. There are 2 trains with 32 riders each. How many rides is this per hour? about 21 rides per train per hour

On Your Own

4. 26

5. 17

6. 8

7. 31

Extra Example 4

A record-breaking rollercoaster at an amusement park can take 28 people on each ride. A total of 24,539 people ride the rollercoaster today. All of the rides are full except the last ride. How many rides are given? How many people are on the last ride? 877 rides; 11 people

On Your Own

8. $203\frac{1}{5}$

9. $1631\frac{23}{28}$

10. $7\frac{41}{86}$

11. 142 rides

Vocabulary and Concept Check

1. addition
2. multiplication
3. division
4. subtraction
5. addition
6. subtraction
7. **a.** dividend
   b. quotient
   c. divisor

Practice and Problem Solving

8. 2118 + 3391 + 4785 + 6354; 16,648 people

9. 4785 – 3391; 1394 more people

10. 6354 ÷ 2118; 3 times more people

11. 4785 × 2; 9570 people

12. 3091 **13.** 7081

14. 5847 **15.** 2462

16. 4436 **17.** 433

18. 3108 **19.** 6944

20. 98,884 **21.** 31

22. 7 **23.** 60

24. $105\frac{4}{61}$ **25.** $47\frac{110}{173}$

26. $209\frac{13}{32}$

Assignment Guide and Homework Check

Level	Day 1 Activity Assignment	Day 2 Lesson Assignment	Homework Check
Basic	8–11, 47–51	1–7, 13–23 odd, 27, 38	13, 15, 19, 21, 38
Average	8–11, 47–51	1–7, 13, 15, 19, 21, 28–38 even	13, 15, 19, 21, 38
Advanced	8–11, 47–51	1–7, 28–46 even	30, 36, 38, 42, 44

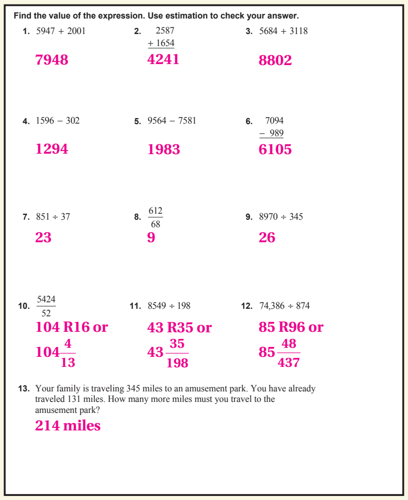
For Your Information

- **Exercise 42** Remind students that there are 128 fluid ounces in a gallon.

Common Errors

- **Exercises 12–17** Students may set up the problem incorrectly using the vertical method of adding or subtracting. Remind them to line up the numbers by place value, then add or subtract.
- **Exercises 18–20** Students may incorrectly line up the multiplication of the tens digit. Remind them that the right-most digit should line up in the tens-digit place.
- **Exercises 23, 24, and 26** Students may forget to use 0 as a place holder in the quotient. Remind them to estimate the answers to check their results.

1.1 Record and Practice Journal



Find the value of the expression. Use estimation to check your answer.

1. 5947 + 2001
7948

2. $\begin{array}{r} 2587 \\ +\ 1654 \\ \hline \end{array}$
4241

3. 5684 + 3118
8802

4. 1596 − 302
1294

5. 9564 − 7581
1983

6. $\begin{array}{r} 7094 \\ -\ 989 \\ \hline \end{array}$
6105

7. 851 ÷ 37
23

8. $\frac{612}{68}$
9

9. 8970 ÷ 345
26

10. $\frac{5424}{52}$
104 R16 or $104\frac{4}{13}$

11. 8549 ÷ 198
43 R35 or $43\frac{35}{198}$

12. 74,386 ÷ 874
85 R96 or $85\frac{48}{437}$

13. Your family is traveling 345 miles to an amusement park. You have already traveled 131 miles. How many more miles must you travel to the amusement park?
214 miles

1.1 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

VOCABULARY **Determine which operation the word or phrase represents.**

1. sum
2. times
3. the quotient of
4. decreased by
5. total of
6. minus

7. **VOCABULARY** Use the division problem shown to tell whether the number is the divisor, dividend, or quotient.

$$\begin{array}{r} 26 \\ 34\overline{)884} \end{array}$$

a. 884
b. 26
c. 34

Practice and Problem Solving

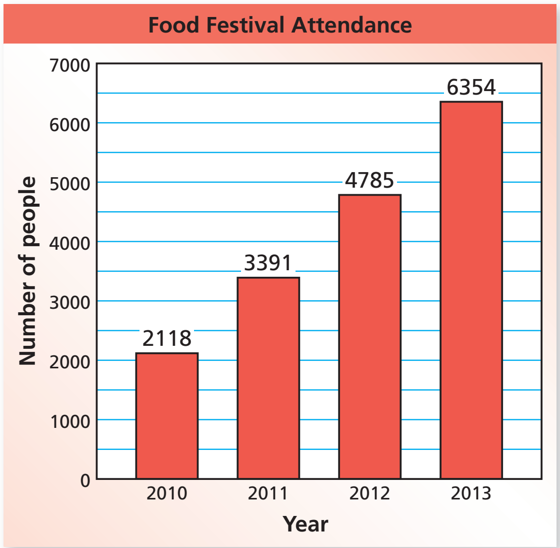
The bar graph shows the attendance at a food festival. Write an expression you can use to answer the question. Then find the value of your expression.

8. What is the total attendance at the food festival from 2010 to 2013?

9. How many more people attended the food festival in 2012 than in 2011?

10. How many times more people attended the food festival in 2013 than in 2010?

11. The festival projects that the total attendance for 2014 will be twice the attendance in 2012. What is the projected attendance for 2014?



Find the value of the expression. Use estimation to check your answer.

① **12.** 2219 + 872

13. $\begin{array}{r} 5351 \\ +\ 1730 \\ \hline \end{array}$

14. 3968 + 1879

15. 7694 − 5232

16. 9165 − 4729

17. $\begin{array}{r} 2416 \\ -\ 1983 \\ \hline \end{array}$

② **18.** $\begin{array}{r} 84 \\ \times\ 37 \\ \hline \end{array}$

19. 124 × 56

20. 419 × 236

③ **21.** 837 ÷ 27

22. $\dfrac{588}{84}$

23. 7440 ÷ 124

④ **24.** 6409 ÷ 61

25. 8241 ÷ 173

26. $\dfrac{33{,}505}{160}$

ERROR ANALYSIS **Describe and correct the error in finding the value of the expression.**

27.

$$\begin{array}{r} 39 \\ \times\ 17 \\ \hline 273 \\ 39 \\ \hline 312 \end{array}$$

28.

$$\begin{array}{r} 19 \\ 12\overline{)1308} \\ -\ 12 \\ \hline 108 \\ -\ 108 \\ \hline 0 \end{array}$$

Determine the operation you would use to solve the problem. Do not answer the question.

29. Gymnastic lessons cost $30 per week. How much will 18 weeks of gymnastic lessons cost?

30. The scores on your first two tests were 82 and 93. By how many points did your score improve?

31. You are setting up tables for a banquet for 150 guests. Each table seats 12 people. What is the minimum number of tables you will need?

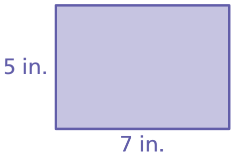
32. A store has 15 boxes of peaches. Each box contains 45 peaches. How many peaches does the store have?

33. Two shirts cost $18 and $25. What is the total cost of the shirts?

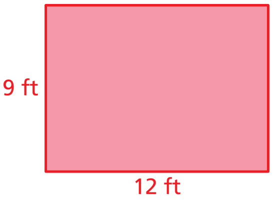
34. A gardener works for 14 hours during a week and charges $168. How much does the gardener charge for each hour?

Find the perimeter and area of the rectangle.

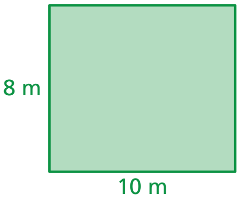
35.



36.



37.



38. **BOX OFFICE** The number of tickets sold for the opening weekend of a movie is 879,575. The movie was shown in 755 theaters across the nation. What was the average number of tickets sold at each theater?

39. **LOGIC** You find that the product of 93 and 6 is 558. How can you use addition to check your answer? How can you use division to check your answer?

40. **NUMBER SENSE** Without calculating, decide which is greater: 3999 ÷ 129 or 3834 ÷ 142. Explain.

Common Errors

- **Exercises 29–34** Students may struggle with identifying the operation to use. Have them review their notes from the activity and lesson on which phrases indicate which operations.
- **Exercises 35–36** Students may not give a complete answer. Remind them the units of measure should be included with the numerical answer.

Practice and Problem Solving

27. The partial product 39 should be moved to the left so that the 3 is under the 2 and the 9 is under the 7. The answer should be 663.

28. Two digits were brought down instead of one after subtracting 12 from 13. The answer should be 109.

29. multiplication

30. subtraction

31. division

32. multiplication

33. addition

34. division

35. 24 in.; 35 in.^{2}

36. 42 ft; 108 ft^2

37. 36 m; 80 m^2

38. 1165 tickets

39. You can use addition to check your answer by adding 93 to itself 6 times. You can use division to check your answer by dividing 558 by 93 or by 6.

40. 3999 ÷ 129; *Sample answer:* The first quotient 3999 ÷ 129 has a greater dividend and a lesser divisor than the second quotient 3834 ÷ 142. So, the first quotient will be greater than the second quotient.

English Language Learners

Vocabulary

Have English language learners dedicate four pages of their notebooks to the four basic math operations. On each page, students list words or phrases that indicate the operation. For instance:

Addition

the sum of
total attendance
total cost
combined

As students work through the problems in the book, they should add any new phrases to the pages of their notebooks.

Practice and Problem Solving

41. no; If the remainder is greater than the divisor, then the quotient should be increased until the remainder is less than the divisor or equal to zero.

42. 64 c

43. 46 tokens

44. See *Taking Math Deeper.*

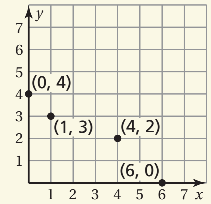
45. **a.** $424

b. $\frac{3}{4}$ qt, or $\frac{3}{16}$ gal

46. $36,000 \div 900 = 40$

Fair Game Review

47–50.



51. A

Mini-Assessment

Find the value of the expression. Check your answer using estimation.

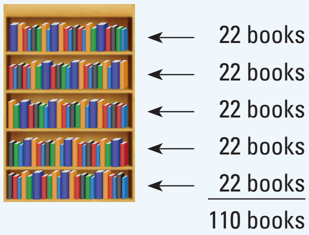
1. $1347 + 914$ 2261
2. $2538 - 1979$ 559
3. 17×223 3791
4. $374 \div 17$ 22
5. There are 240 students and 16 rooms. How many students will be in each class if there is an equal number of students in each class? 15 students

Taking Math Deeper

Exercise 44

This problem involves several calculations. One way to keep track of the calculations is to draw diagrams.

① Begin by finding the number of books you can display in one bookcase.

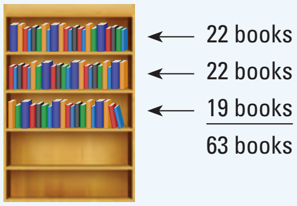


One bookcase can display up to 110 books.

② Next, find the number of bookcases needed to display 943 books.

a. Eight bookcases can display up to 880 books and 9 bookcases can display up to 990 books. So, you need to borrow 9 bookcases.

③ The first 8 bookcases are full and the 9th bookcase will display $943 - 880 = 63$ books. Use a diagram to show how you fill the 9th bookcase, starting at the top.



b. There are 19 books on the third shelf.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Start the next section

41. **REASONING** In a division problem, can the remainder be greater than the divisor? Explain.

42. **WATER COOLER** You change the water jug on the water cooler. How many cups can be completely filled before you need to change the water jug again?

43. **ARCADE** You have $9, one of your friends has $10, and two of your other friends each have $13. You combine your money to buy arcade tokens. You use a coupon to buy 8 tokens for $1. The cost of the remaining tokens is four for $1. You and your friends share the tokens evenly. How many tokens does each person get?

44. **BOOK SALE** You borrow bookcases like the one shown to display 943 books at a book sale. You plan to put 22 books on each shelf. No books will be on top of the bookcases.

a. How many bookcases must you borrow to display all the books?

b. You fill the shelves of each bookcase in order, starting with the top shelf. How many books are on the third shelf of the last bookcase?

45. **MODELING** The siding of a house is 2250 square feet. The siding needs two coats of paint. The table shows information about the paint.

Can Size	Cost	Coverage
1 quart	$18	80 square feet
1 gallon	$29	320 square feet

a. What is the minimum cost of the paint needed to complete the job?

b. How much paint is left over?

46. **Critical Thinking** Use the digits 3, 4, 6, and 9 to complete the division problem. Use each digit once.

□□,000 ÷ □00 = □0

Fair Game Review What you learned in previous grades & lessons

Plot the ordered pair in a coordinate plane. *(Skills Review Handbook)*

47. (1, 3) **48.** (0, 4) **49.** (6, 0) **50.** (4, 2)

51. **MULTIPLE CHOICE** Which of the following numbers is *not* prime? *(Skills Review Handbook)*

Ⓐ 1 Ⓑ 2 Ⓒ 3 Ⓓ 5

1.2 Powers and Exponents

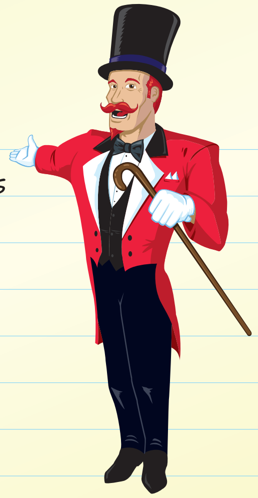
Essential Question How can you use repeated factors in real-life situations?

As I was going to St. Ives
I met a man with seven wives
Each wife had seven sacks
Each sack had seven cats
Each cat had seven kits
Kits, cats, sacks, wives
How many were going to St. Ives? Nursery Rhyme, 1730

1 ACTIVITY: Analyzing a Math Poem

Work with a partner. Here is a "St. Ives" poem written by two students. Answer the question in the poem.

As I was walking into town
I met a ringmaster with five clowns
Each clown had five magicians
Each magician had five bunnies
Each bunny had five fleas
Fleas, bunnies, magicians, clowns
How many were going into town?



Numerical Expressions
In this lesson, you will
- write expressions as powers.
- find values of powers.

Number of clowns: $5 =$

Number of magicians: $5 \times 5 =$

Number of bunnies: $5 \times 5 \times 5 =$

Number of fleas: $5 \times 5 \times 5 \times 5 =$

So, the number of fleas, bunnies, magicians, and clowns is .
Explain how you found your answer.

Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP5 Use Appropriate Tools Strategically:** When students are asked to evaluate a power, they should recognize which tool (mental math, paper and pencil, or a calculator) is appropriate.

Motivate

"Have you heard the St. Ives nursery rhyme?" St. Ives is an old village on the southwest tip of Cornwall, England. Another old village, Jamestown, VA, was settled in 1607. "By how many years did the settling of Jamestown pre-date the nursery rhyme?" 123 years

- Tell students that today's activity connects repeated patterns in a nursery rhyme to a repeated mathematical operation.

Demonstrate

- Ask volunteers to act out the rhyme. You can substitute "person" for "man" and "friends" for "wives" if necessary. Each wife (friend) should hold 7 pieces of paper. This helps to build a visual model of $1 \times 7 \times 7$, or 49.

"If 7 triangles are drawn on each piece of paper, how many triangles are there?" $1 \times 7 \times 7 \times 7 = 343$

- Discuss the idea that this pattern could continue. For instance, there could be 7 dots in every triangle and you want to know how many dots there are. The activity today looks at a way to record repeated multiplication.

Activity Notes

Activity 1

- The poem in this activity is similar in style to the St. Ives poem. The careful reader might suggest that only one person for sure is going to town and the rest (clowns, magicians, bunnies, and fleas) were along the road and not headed to town. Moving or stationary, ask the students to determine how many there were.
- Ask a volunteer to share how they found the answer to the question of how many clowns, magicians, bunnies, and fleas there were.

"What would a visual model for this problem look like?" Answers vary, but listen for a tree diagram model or perhaps 5 circles all containing 5 squares with 5 triangles within each square, and so on.

"How is this student poem similar to the St. Ives poem?" *Sample answer:* You can be sure that one is going to town.

"Does anyone recall what the word *factor* means? Can you give an example?" Students may say factors are numbers multiplied together. In the problem $3 \times 4 = 12$, 3 and 4 are factors, and 12 is the product.

"Could a problem have more than two factors? Explain." yes; $3 \times 4 \times 5 = 60$; There are 3 factors.

"How could this student poem be expanded to keep the pattern going?" Answers vary; *Sample answer:* Each flea could have bitten 5 people, or each magician could have had 5 assistants.

Common Core State Standards

6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.

Previous Learning

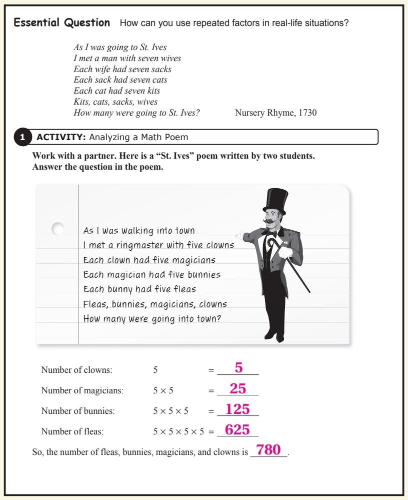
Students need to be familiar with basic computation facts as well as the meanings of *factor* and *product*.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

1.2 Record and Practice Journal



Essential Question How can you use repeated factors in real-life situations?

As I was going to St. Ives
I met a man with seven wives
Each wife had seven sacks
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Each cat had seven kits
Kits, cats, sacks, wives
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Each clown had five magicians
Each magician had five bunnies
Each bunny had five fleas
Fleas, bunnies, magicians, clowns
How many were going into town?

Number of clowns: 5 = 5
Number of magicians: 5×5 = 25
Number of bunnies: $5 \times 5 \times 5$ = 125
Number of fleas: $5 \times 5 \times 5 \times 5$ = 625

So, the number of fleas, bunnies, magicians, and clowns is 780.

Differentiated Instruction

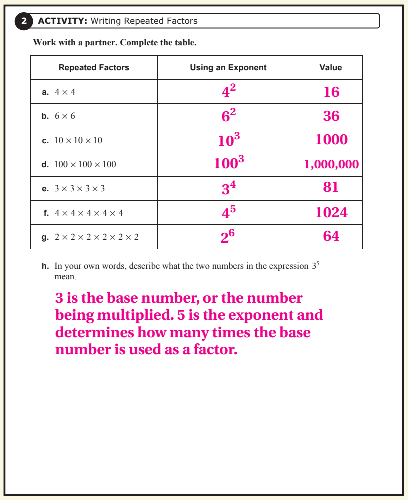
Visual

Write the first three rows of power pyramids for 2, 4, 5, and 10 on the board.

2	**4**
2×2	4×4
$2 \times 2 \times 2$	$4 \times 4 \times 4$
$2 \times 2 \times 2 \times 2$	$4 \times 4 \times 4 \times 4$
5	**10**
5×5	10×10
$5 \times 5 \times 5$	$10 \times 10 \times 10$
$5 \times 5 \times 5 \times 5$	$10 \times 10 \times 10 \times 10$

Have students write each product as a power and then find the value of each power. Ask, "What is the product of 4 factors of 2? What is the product of 2 factors of 4? What is the product of 3 factors of 5? What is the product of 4 factors of 10?" 16; 16; 125; 10,000

1.2 Record and Practice Journal



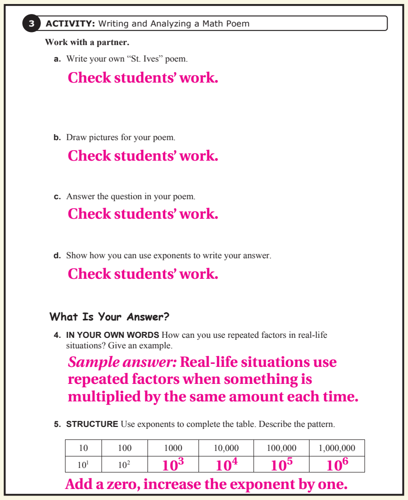
2 ACTIVITY: Writing Repeated Factors

Work with a partner. Complete the table.

Repeated Factors	Using an Exponent	Value
a. 4×4	4^2	16
b. 6×6	6^2	36
c. $10 \times 10 \times 10$	10^3	1000
d. $100 \times 100 \times 100$	100^3	1,000,000
e. $3 \times 3 \times 3 \times 3$	3^4	81
f. $4 \times 4 \times 4 \times 4 \times 4$	4^5	1024
g. $2 \times 2 \times 2 \times 2 \times 2 \times 2$	2^6	64

h. In your own words, describe what the two numbers in the expression 3^5 mean.

3 is the base number, or the number being multiplied. 5 is the exponent and determines how many times the base number is used as a factor.



3 ACTIVITY: Writing and Analyzing a Math Poem

Work with a partner.

a. Write your own "St. Ives" poem.

Check students' work.

b. Draw pictures for your poem.

Check students' work.

c. Answer the question in your poem.

Check students' work.

d. Show how you can use exponents to write your answer.

Check students' work.

What Is Your Answer?

4. IN YOUR OWN WORDS How can you use repeated factors in real-life situations? Give an example.

***Sample answer:* Real-life situations use repeated factors when something is multiplied by the same amount each time.**

5. STRUCTURE Use exponents to complete the table. Describe the pattern.

10	100	1000	10,000	100,000	1,000,000
10^1	10^2	10^3	10^4	10^5	10^6

Add a zero, increase the exponent by one.

Laurie's Notes

Activity 2

- Explain that in the notation 4^2, the 2 is called an *exponent*. Some students may be familiar with the notation and vocabulary. It will be made formal in the lesson. Discuss the sample with the class before students work with their partners.
- Observe strategies students use for finding the values in the last column. Mental math strategies are important. For instance, in finding the product $3 \times 3 \times 3 \times 3$, many students will find 3×3, then 9×3, and then use paper and pencil to find 27×3. Show students that they can also do the following:

$$\begin{gathered} 3 \times 3 \times 3 \times 3 \\ 9 \times 9 \\ 81 \end{gathered}$$

- **MP7 Look for and Make Use of Structure:** Understanding how to represent repeated factors using an exponent requires students to recognize the pattern or structure of the expression. Taking the next step of evaluating the expression also involves understanding the structure and the underlying algebraic properties that allow 3^4 to be evaluated as described above.
- Ask students to describe their strategies for performing the computations.
- In part (h), students may not have precise language to describe what 3^5 means, which is perfectly acceptable at this stage.
- **Common Error:** In describing what 3^5 means, you are not multiplying 3 five times. There are actually only four multiplications performed. The number 3 is written five times, meaning there are five *factors* of 3.

Activity 3

- Give students sufficient time to write and illustrate their poems.
- Have students share their poems aloud. If possible, use a document camera to share their illustrations.

What Is Your Answer?

- In Question 5, point out that for a number with an exponent of 1, the value is the number. There is only 1 factor, which is the number 10.
- In Question 5, students should recognize that the exponent and the number of zeros in the answer are the same. These numbers represent the base 10 place values.

Closure

- You have a dozen boxes of a dozen donuts each, and each donut has a dozen chocolate chips. Use an exponent to write an expression for the total number of chocolate chips. 12^3

2 ACTIVITY: Writing Repeated Factors

Math Practice

Repeat Calculations

What patterns do you notice with each problem? How does this help you write exponents?

Work with a partner. Copy and complete the table.

Repeated Factors	Using an Exponent	Value
a. 4×4		
b. 6×6		
c. $10 \times 10 \times 10$		
d. $100 \times 100 \times 100$		
e. $3 \times 3 \times 3 \times 3$		
f. $4 \times 4 \times 4 \times 4 \times 4$		
g. $2 \times 2 \times 2 \times 2 \times 2 \times 2$		

h. In your own words, describe what the two numbers in the expression 3^5 mean.

3 ACTIVITY: Writing and Analyzing a Math Poem

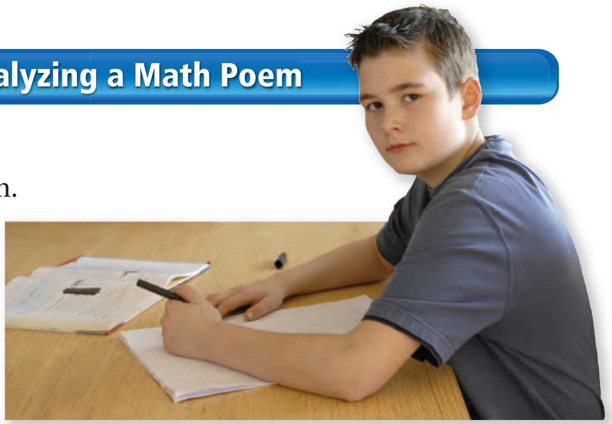
Work with a partner.

a. Write your own “St. Ives” poem.

b. Draw pictures for your poem.

c. Answer the question in your poem.

d. Show how you can use exponents to write your answer.



What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use repeated factors in real-life situations? Give an example.

5. **STRUCTURE** Use exponents to complete the table. Describe the pattern.

10	100	1000	10,000	100,000	1,000,000
10^1	10^2				

Practice Use what you learned about exponents to complete Exercises 4–6 on page 14.

1.2 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
power, *p. 12*
base, *p. 12*
exponent, *p. 12*
perfect square, *p. 13*

A **power** is a product of repeated factors. The **base** of a power is the repeated factor. The **exponent** of a power indicates the number of times the base is used as a factor.

$$\underbrace{\overset{\text{base}}{3}^{\overset{\text{exponent}}{4}}}_{\text{power}} = \underbrace{3 \cdot 3 \cdot 3 \cdot 3}_{3 \text{ is used as a factor } 4 \text{ times.}}$$

Power	Words
3^2	Three *squared*, or three to the second
3^3	Three *cubed*, or three to the third
3^4	Three to the fourth

EXAMPLE 1 Writing Expressions as Powers

Write each product as a power.

a. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

Because 4 is used as a factor 5 times, its exponent is 5.

So, $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^5$.

b. $12 \times 12 \times 12$

Because 12 is used as a factor 3 times, its exponent is 3.

So, $12 \times 12 \times 12 = 12^3$.

Math Practice

Choose Tools
Why are calculators more efficient when finding the values of expressions involving exponents?

On Your Own

Now You're Ready
Exercises 4–12

Write the product as a power.

1. $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$

2. $15 \times 15 \times 15 \times 15$

EXAMPLE 2 Finding Values of Powers

Find the value of each power.

a. 7^2

$7^2 = 7 \cdot 7$ Write as repeated multiplication.

$= 49$ Simplify.

b. 5^3

$5^3 = 5 \cdot 5 \cdot 5$

$= 125$

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students explored how to write a product of repeated factors using an exponent. (MP5, MP7)
- **Today:** Students will use formal language to describe a power and look at the specific case of perfect squares.

Motivate

- Hold up a chess or checkers board.

? "How many small squares are on each edge?" 8 "How many of each color are on each edge?" 4 "How many small squares are there in all?" 64 "How many are there of each color?" 32 "If you placed two paper clips on each small square, how many paper clips would you need?" 128

- Ask students to think about all of the answers: 4, 8, 32, 64, and 128. Do they have any observations? If necessary, ask them to think about repeatedly multiplying 2 by itself.
- Introduce the vocabulary words *power*, *base*, and *exponent*.
- Explain that the more common way to refer to a power such as 3^2 is "three squared," although "three to the second" is also correct.
- **Connection:** Discuss with students the common attribute shared by the two-dimensional *square* and the three dimensional *cube*. All sides/edges are the same length. In the power 5^3, all of the factors are the same, namely a factor of 5.

Words of Wisdom

- A common error many students make is to multiply the exponent by the base. For example, $3^2 = 9$, not 6. The simpler the problem, the more often they seem to make this error.

Lesson Notes

Example 1

- Note that two different representations are used for multiplication (• and ×).
- Discuss with students the need to be careful. The dot may be mistaken as a decimal point (2.2 versus $2 \cdot 2$) and the × may be mistaken as a variable.

? "Are there other representations of multiplication you are familiar with?" Students may mention the use of parentheses. $3(4) = 12$

On Your Own

- **Neighbor Check:** Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.

Example 2

- Students should be able to evaluate some powers using mental math.
- In order to practice vocabulary, ask a volunteer to read the problem and the answer. You should hear, "7 squared is 49" and "5 cubed is 125."

Goal Today's lesson is writing and finding values of **powers.**

Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

Write each product as a power.

a. $6 \cdot 6 \cdot 6 \cdot 6$ 6^4

b. $14 \times 14 \times 14 \times 14 \times 14$ 14^5

On Your Own

1. 6^6 **2.** 15^4

Extra Example 2

Find the value of each power.

a. 8^3 512

b. 5^4 625

Extra Example 3

Determine whether each number is a perfect square.

a. 50 not a perfect square

b. 9 perfect square

On Your Own

3. 216
4. 81
5. 81
6. 324
7. perfect square
8. not a perfect square
9. not a perfect square
10. perfect square

Extra Example 4

A baseball diamond is a square with a side length of 90 feet. What is the area of a baseball diamond? 8100 ft^2

On Your Own

11. 576 $in.^2$, 4 ft^2

English Language Learners

Vocabulary

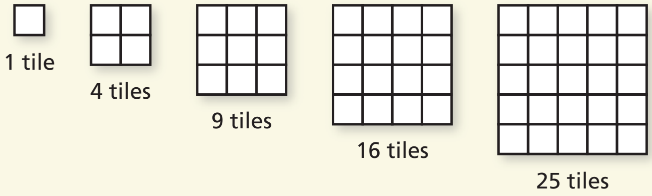
English language learners may stumble over the ordinal number of the exponent. Have students write this table in their notebooks for reference.

Number	Ordinal
1	first
2	second
3	third
4	fourth
5	fifth
6	sixth
7	seventh
8	eighth
9	ninth
10	tenth

Laurie's Notes

Example 3

- As an introduction to the definition of a perfect square, write the following sequence on the board: 1, 4, 9, 16, 25, 36, . . .
- "What are the next three numbers in the sequence?" 49, 64, 81 "What is the pattern?" Square the whole numbers in order. Students will often answer that you add 3, add 5, add 7, and so on. This is also a correct pattern, so you may need to probe further to get students to recognize that the numbers in the sequence are squares of whole numbers.
- **MP5 Use Appropriate Tools Strategically:** Define *perfect square*. Give students a pile of square tiles and ask them to make a larger square. How many tiles were used? The number will always be a perfect square. The square tiles are a tool to help students visualize square numbers.



- If you give students 12 tiles and ask them to use all of the tiles to make a square, they will not be able to do so because 12 is not a perfect square.
- Work through each problem.

On Your Own

- Check to see if students used a calculator, paper and pencil, or mental math to answer Questions 3–6. Some students may find 18^2 (= 324) by finding the value of $(20 \times 18) - (2 \times 18)$.

Example 4

- Explain that a verbal model is an equation. It states in words the formula or process that will be used to solve a problem.
- "How do you find the area of a square?" square the side length "Could you find the area of the chess or checkers board? Explain." yes; If you know, or can measure, a side length, square it to find the area.

On Your Own

- If you have another square object in your classroom, have students measure it and find its area.

Closure

- "What powers of 2 are perfect squares?" those with an even exponent: $2^2 = 4$, $2^4 = 16$, $2^6 = 64$, etc.

The square of a whole number is a **perfect square**.

EXAMPLE 3 Identifying Perfect Squares

Determine whether each number is a perfect square.

a. 64

Because $8^2 = 64$, 64 is a perfect square.

b. 20

No whole number squared equals 20. So, 20 is not a perfect square.

On Your Own

Now You're Ready
Exercises 14–21
and 25–32

Find the value of the power.

3. 6^3 **4.** 9^2 **5.** 3^4 **6.** 18^2

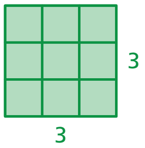
Determine whether the number is a perfect square.

7. 25 **8.** 2 **9.** 99 **10.** 100

Remember

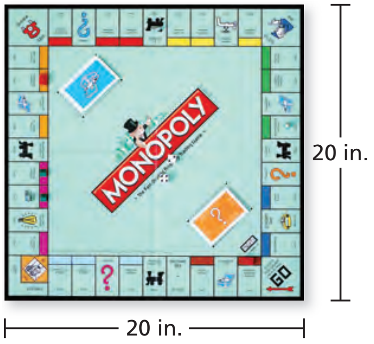
The *area* of a figure is the amount of surface it covers. Area is measured in square units.

The area of a square is equal to its side length squared.



Area $= 3^2 = 9$ square units

EXAMPLE 4 Real-Life Application



A MONOPOLY® game board is a square with a side length of 20 inches. What is the area of the game board?

Use a verbal model to solve the problem.

$$\text{area of game board} = (\text{side length})^2$$

$$= 20^2 \quad \text{Substitute 20 for side length.}$$

$$= 400 \quad \text{Multiply.}$$

The area of the game board is 400 square inches.

On Your Own

11. What is the area of the square traffic sign in square inches? in square feet?

1.2 Exercises

Vocabulary and Concept Check

1. **VOCABULARY** How are exponents and powers different?

2. **VOCABULARY** Is 10 a perfect square? Is 100 a perfect square? Explain.

3. **WHICH ONE DOESN'T BELONG?** Which one does *not* belong with the other three? Explain your reasoning.

$2^4 = 2 \times 2 \times 2 \times 2$

$3 + 3 + 3 + 3 = 3(4)$

$3^2 = 3 \times 3$

$5 \cdot 5 \cdot 5 = 5^3$

Practice and Problem Solving

Write the product as a power.

① **4.** 9×9

5. 13×13

6. $15 \times 15 \times 15$

7. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

8. $14 \times 14 \times 14$

9. $8 \cdot 8 \cdot 8 \cdot 8$

10. $11 \times 11 \times 11 \times 11 \times 11$

11. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

12. $16 \cdot 16 \cdot 16 \cdot 16$

13. **ERROR ANALYSIS** Describe and correct the error in writing the product as a power.

✗ $4 \cdot 4 \cdot 4 = 3^4$

Find the value of the power.

② **14.** 5^2

15. 4^3

16. 2^5

17. 14^2

Use a calculator to find the value of the power.

18. 7^6

19. 4^8

20. 12^4

21. 17^5

22. **ERROR ANALYSIS** Describe and correct the error in finding the value of the power.

✗ $8^3 = 8 \cdot 3 = 24$

23. **POPULATION** The population of Virginia is about 8×10^6. About how many people live in Virginia?

24. **FIGURINES** The smallest figurine in a gift shop is 2 inches tall. The height of each figurine is twice the height of the previous figurine. Write a power to represent the height of the tallest figurine. Then find the height.

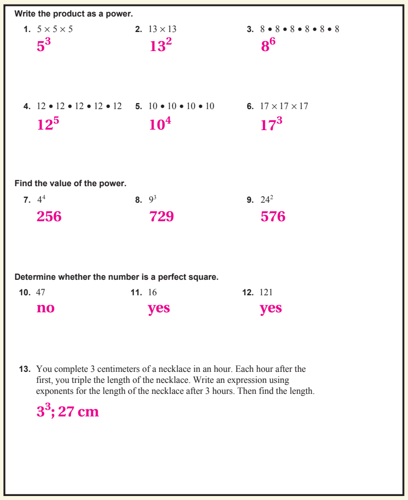
Assignment Guide and Homework Check

Level	Day 1 Activity Assignment	Day 2 Lesson Assignment	Homework Check
Basic	4–6, 40–44	1–3, 7, 13–21 odd, 22, 23, 27, 31	7, 15, 21, 22, 31
Average	4–6, 40–44	1–3, 7–13 odd, 14–24 even, 25–39 odd	7, 16, 22, 31, 37
Advanced	4–6, 40–44	1–3, 8–12 even, 13, 14–24 even, 25–33 odd, 34–39	10, 16, 22, 31, 37

Common Errors

- **Exercises 4–12** Students may miscount the number of factors. Remind them to be careful when counting the number of factors, and that the number of factors is the exponent.
- **Exercises 14–21** Students may make the same mistake that is illustrated in Exercise 22, that is, they may write the exponent as a factor. Again, remind them that the exponent is the number of times the base is used as a factor. You may want to demonstrate this point with a couple of quick examples. For instance, using Exercise 14, point out that $5^2 = 5 \times 5 = 25$ but $5 \times 2 = 10$.
- **Exercise 23** Students may write 8×10^6 as $(8 \times 10)^6$ and come up with a population of 262,144,000,000. Remind students to check the reasonableness of their answers.

1.2 Record and Practice Journal



Write the product as a power.

1. $5 \times 5 \times 5$ 5^3

2. 13×13 13^2

3. $8 \bullet 8 \bullet 8 \bullet 8 \bullet 8 \bullet 8$ 8^6

4. $12 \bullet 12 \bullet 12 \bullet 12 \bullet 12$ 12^5

5. $10 \bullet 10 \bullet 10 \bullet 10$ 10^4

6. $17 \times 17 \times 17$ 17^3

Find the value of the power.

7. 4^4 256

8. 9^3 729

9. 24^2 576

Determine whether the number is a perfect square.

10. 47 no

11. 16 yes

12. 121 yes

13. You complete 3 centimeters of a necklace in an hour. Each hour after the first, you triple the length of the necklace. Write an expression using exponents for the length of the necklace after 3 hours. Then find the length.

3^3; 27 cm

Vocabulary and Concept Check

1. An exponent indicates the number of times the base is used as a factor. A power is the entire expression (base and exponent). A power is a product of repeated factors.
2. no; yes; 10 is not the square of a whole number, so it is not a perfect square. $100 = 10^2$, so it is a perfect square.
3. $3 + 3 + 3 + 3 = 3(4)$ does not belong because it shows a product as a sum of repeated addends, whereas the other three show powers as products of repeated factors.

Practice and Problem Solving

4. 9^2
5. 13^2
6. 15^3
7. 2^5
8. 14^3
9. 8^4
10. 11^5
11. 7^6
12. 16^4
13. The base is written as the exponent and the exponent is written as the base.
    $4 \bullet 4 \bullet 4 = 4^3$
14. 25
15. 64
16. 32
17. 196
18. 117,649
19. 65,536
20. 20,736
21. 1,419,857
22. The exponent is written as a factor, but it should have been used to indicate the number of times the base is used as a factor.
    $8^3 = 8 \bullet 8 \bullet 8 = 512$
23. 8,000,000 people
24. 2^4; 16 in.

Practice and Problem Solving

25. not a perfect square

26. perfect square

27. perfect square

28. not a perfect square

29. perfect square

30. not a perfect square

31. not a perfect square

32. perfect square

33. 40,000 cm^2

34. See *Taking Math Deeper.*

35. 8 squares

36. **a.** 9 by 9 tile arrangement, 10 by 10 tile arrangement, or 11 by 11 tile arrangement

b. in the 9 by 9 arrangement: 44 tiles; in the 10 by 10 arrangement: 25 tiles; in the 11 by 11 arrangement: 4 tiles

37. See Additional Answers.

38. 7 and 8

39. 13 blocks; add $7^2 - 6^2$ blocks; 19 blocks; add $10^2 - 9^2$ blocks; 39 blocks; add $20^2 - 19^2$ blocks

Fair Game Review

40. 84
41. 165
42. 8
43. 7
44. C

Mini-Assessment

Write the product as a power. Then, find the value of the power.

1. $3 \times 3 \times 3 \times 3 \times 3$ 3^5; 243

2. 2×2 2^2; 4

3. $5 \cdot 5 \cdot 5 \cdot 5$ 5^4; 625

4. $16 \cdot 16$ 16^2; 256

5. The number of students in a school is about 11^3. About how many students are in the school? 1331 students

Taking Math Deeper

Exercise 34

This open-ended question provides an opportunity for answers at different levels.

① Whole-Number Bases (Use *Guess, Check, and Revise.*)

Exponent 2: Students can discover that $11^2 = 121$.
Exponent 3: Students can discover that $5^3 = 125$.
Exponents 4, 5, and 6: Students can discover that none of the powers fall between 120 and 130.
Exponent 7: Students can discover that $2^7 = 128$.

② Decimal Bases

Although the lesson is restricted to whole-number bases, it is still reasonable that some students will wonder about the following. "If $11^2 = 121$, isn't it reasonable that 11.1^2 is just slightly more than 121?" This type of investigative thinking can introduce the concept of square roots, which students will study in Grade 8.

③ Extension

After students have found the three answers with whole-number bases, ask them whether there are more than three. They can justify their answer using a table.

Base

Exponent	2	3	4	5	6	7	8	9	10	11	12
2	4	9	16	25	36	49	64	81	100	121	144
3	8	27	64	125	216						
4	16	81	256								
5	32	243									
6	64	729									
7	128										
8	256										

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Start the next section

Determine whether the number is a perfect square.

3 **25.** 8	**26.** 4	**27.** 81	**28.** 44
29. 49	**30.** 125	**31.** 150	**32.** 144

33. **PAINTING** A square painting measures 2 meters on each side. What is the area of the painting in square centimeters?

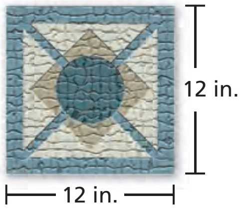
34. **NUMBER SENSE** Write three powers that have values greater than 120 and less than 130.

35. **CHECKERS** A checkers board has 64 squares. How many squares are in each row?

36. **PATIO** A landscaper has 125 tiles to build a square patio. The patio must have an area of at least 80 square feet.

a. What are the possible arrangements for the patio?

b. How many tiles are not used in each arrangement?

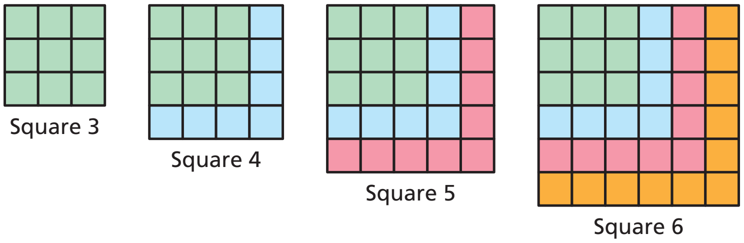


37. **PATTERNS** Copy and complete the table. Describe what happens to the value of the power as the exponent decreases. Use this pattern to find the value of 4^0.

Power	4^6	4^5	4^4	4^3	4^2	4^1
Value	4096	1024				

38. **REASONING** Consider the equation $56 = \square^2$. The missing number is between what two whole numbers?

39. Repeated Reasoning How many blocks do you need to add to Square 6 to get Square 7? to Square 9 to get Square 10? to Square 19 to get Square 20? Explain.



Fair Game Review What you learned in previous grades & lessons

Find the value of the expression. *(Skills Review Handbook)*

40. 6 × 14	**41.** 11 × 15	**42.** 56 ÷ 7	**43.** 112 ÷ 16

44. **MULTIPLE CHOICE** You buy a box of gum that has 12 packs. Each pack has 5 pieces. Which expression represents the total number of pieces of gum? *(Skills Review Handbook)*

Ⓐ 12 + 5 Ⓑ 12 − 5 Ⓒ 12 × 5 Ⓓ 12 ÷ 5

1.3 Order of Operations

Essential Question What is the effect of inserting parentheses into a numerical expression?

1 ACTIVITY: Comparing Different Orders

Work with a partner. Find the value of the expression by using different orders of operations. Are your answers the same? (Circle *yes* or *no.*)

a.	Add, then multiply. $3 + 4 \times 2 =$	Multiply, then add. $3 + 4 \times 2 =$	Same? Yes No
b.	Add, then subtract. $5 + 3 - 1 =$	Subtract, then add. $5 + 3 - 1 =$	Same? Yes No
c.	Divide, then multiply. $12 \div 3 \cdot 2 =$	Multiply, then divide. $12 \div 3 \cdot 2 =$	Same? Yes No
d.	Divide, then add. $16 \div 4 + 4 =$	Add, then divide. $16 \div 4 + 4 =$	Same? Yes No
e.	Multiply, then subtract. $8 \times 4 - 2 =$	Subtract, then multiply. $8 \times 4 - 2 =$	Same? Yes No
f.	Multiply, then divide. $8 \cdot 4 \div 2 =$	Divide, then multiply. $8 \cdot 4 \div 2 =$	Same? Yes No
g.	Subtract, then add. $13 - 4 + 6 =$	Add, then subtract. $13 - 4 + 6 =$	Same? Yes No
h.	Multiply, then add. $1 \times 2 + 3 =$	Add, then multiply. $1 \times 2 + 3 =$	Same? Yes No

Numerical Expressions

In this lesson, you will

- evaluate numerical expressions with whole-number exponents.

Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP6 Attend to Precision:** When working with even simple expressions such as $8 - 2 + 4$, the order of operations must be used. Students may incorrectly reason that the order in which operations are performed does not matter when it is a *simple* problem.

Motivate

- **Model:** Place two calculators, one scientific and one non-scientific, under a document camera or give the calculators to a student. Press the same sequence of keys, $20 - 8 \times 2$ and then enter (or $=$), on each calculator.

❓ "What is the answer on each calculator?" The scientific calculator should display 4 and the non-scientific calculator should display 24.

❓ "Is it okay to have different answers to the same question?" No. The correct answer is 4. To obtain the correct answer on the non-scientific calculator, perform the multiplication separately and then subtract the result from 20.

Discuss

- The non-scientific calculator performs the operations in order from left to right. The scientific calculator uses the agreed upon *order of operations* and performs the multiplication before the subtraction.

Activity Notes

Activity 1

- Introduce the activity and have students work with partners to complete it.

❓ When students have finished, ask, "Did the order in which you performed the operations matter?" Order mattered in all but parts (b), (f), and (h).

- Ask students whether they were able to predict when the order was going to matter and when it was not. They may not be able to predict at this point.
- If time permits, have students write expressions involving two different operations where the order matters and where the order does not matter. Students will recognize that it is more challenging to write expressions where the order does not matter.

Common Core State Standards

6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.

Previous Learning

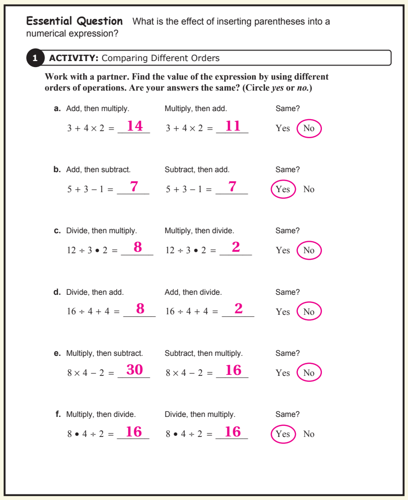
In Grade 5, students evaluated numerical expressions using the order of operations (no exponents).

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

1.3 Record and Practice Journal



Essential Question What is the effect of inserting parentheses into a numerical expression?

1 ACTIVITY: Comparing Different Orders

Work with a partner. Find the value of the expression by using different orders of operations. Are your answers the same? (Circle *yes* or *no*.)

a. Add, then multiply. Multiply, then add. Same?
$3 + 4 \times 2 =$ 14 $3 + 4 \times 2 =$ 11 Yes [✓ No]

b. Add, then subtract. Subtract, then add. Same?
$5 + 3 - 1 =$ 7 $5 + 3 - 1 =$ 7 [✓ Yes] No

c. Divide, then multiply. Multiply, then divide. Same?
$12 \div 3 \bullet 2 =$ 8 $12 \div 3 \bullet 2 =$ 2 Yes [✓ No]

d. Divide, then add. Add, then divide. Same?
$16 \div 4 + 4 =$ 8 $16 \div 4 + 4 =$ 2 Yes [✓ No]

e. Multiply, then subtract. Subtract, then multiply. Same?
$8 \times 4 - 2 =$ 30 $8 \times 4 - 2 =$ 16 Yes [✓ No]

f. Multiply, then divide. Divide, then multiply. Same?
$8 \bullet 4 \div 2 =$ 16 $8 \bullet 4 \div 2 =$ 16 [✓ Yes] No

Differentiated Instruction

Kinesthetic

Students should understand the order of operations before parentheses (symbols of grouping) are introduced. Write

$5 + 10 \div 5 \times 3 - 1$

on the board and have students evaluate the expression.

$5 + 10 \div 5 \times 3 - 1 = 10$

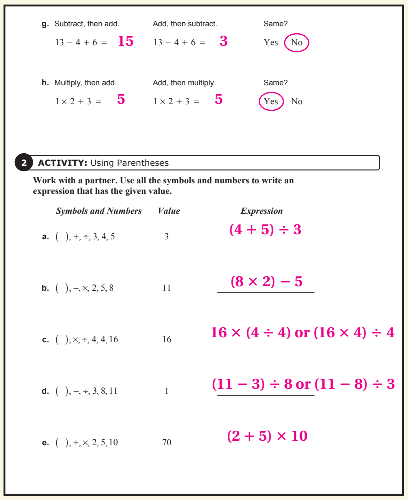
Then ask a student to add parentheses so that addition is performed first and have students evaluate the new expression.

$(5 + 10) \div 5 \times 3 - 1 = 8$

Next ask a student to add parentheses so that addition and multiplication are performed first and have students evaluate the new expression.

$(5 + 10) \div (5 \times 3) - 1 = 0$

1.3 Record and Practice Journal



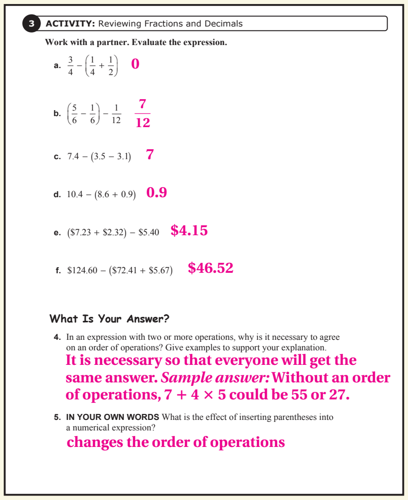
g. Subtract, then add. $13 - 4 + 6 =$ 15 Add, then subtract. $13 - 4 + 6 =$ 3 Same? Yes [✓ No]

h. Multiply, then add. $1 \times 2 + 3 =$ 5 Add, then multiply. $1 \times 2 + 3 =$ 5 Same? [✓ Yes] No

2 ACTIVITY: Using Parentheses

Work with a partner. Use all the symbols and numbers to write an expression that has the given value.

	Symbols and Numbers	*Value*	*Expression*
a.	( ), +, ÷, 3, 4, 5	3	$(4 + 5) \div 3$
b.	( ), −, ×, 2, 5, 8	11	$(8 \times 2) - 5$
c.	( ), ×, ÷, 4, 4, 16	16	$16 \times (4 \div 4)$ or $(16 \times 4) \div 4$
d.	( ), −, ÷, 3, 8, 11	1	$(11 - 3) \div 8$ or $(11 - 8) \div 3$
e.	( ), +, ×, 2, 5, 10	70	$(2 + 5) \times 10$



3 ACTIVITY: Reviewing Fractions and Decimals

Work with a partner. Evaluate the expression.

a. $\frac{3}{4} - \left(\frac{1}{4} + \frac{1}{2}\right)$ 0

b. $\left(\frac{5}{6} - \frac{1}{6}\right) - \frac{1}{12}$ $\frac{7}{12}$

c. $7.4 - (3.5 - 3.1)$ 7

d. $10.4 - (8.6 + 0.9)$ 0.9

e. ($7.23 + $2.32) − $5.40 $4.15

f. $124.60 − ($72.41 + $5.67) $46.52

What Is Your Answer?

4. In an expression with two or more operations, why is it necessary to agree on an order of operations? Give examples to support your explanation.

It is necessary so that everyone will get the same answer. *Sample answer:* Without an order of operations, $7 + 4 \times 5$ could be 55 or 27.

5. IN YOUR OWN WORDS What is the effect of inserting parentheses into a numerical expression?

changes the order of operations

Laurie's Notes

Activity 2

- Explain that when parentheses are used in an expression, the operation(s) within the parentheses are performed first.
- **MP2 Reason Abstractly and Quantitatively:** As students work these problems, they need to reason quantitatively about the ending value and consider what numbers and operations they have to work with. For example, the first problem involves the operations of addition and division. Students should reason that the sum of two of the numbers must be divisible by the third number. Trial and error is not the first strategy—reasoning is.
- Explain the logic puzzle activity. The numbers may be used in any order.
- Have students share their results when all have finished.

Activity 3

- This activity gives students an opportunity to review prior work with fractions and decimals.
- Circulate as students are working these problems. Listen for student understanding and recall of finding sums and differences of fractions and decimals. Make note of which students may need a review of these skills.
- Have students share their results when all have finished.

What Is Your Answer?

- Students should be comfortable with the idea that when evaluating an expression with two or more operations, performing operations in a particular order is necessary.

Closure

- **Writing Prompt:** To evaluate $4 - 3 \times (2 + 1)$, . . . Perform the addition in parentheses, multiply, then subtract.

2 ACTIVITY: Using Parentheses

Work with a partner. Use all the symbols and numbers to write an expression that has the given value.

	Symbols and Numbers	*Value*	*Expression*
a.	( ), +, ÷, 3, 4, 5	3	
b.	( ), −, ×, 2, 5, 8	11	
c.	( ), ×, ÷, 4, 4, 16	16	
d.	( ), −, ÷, 3, 8, 11	1	
e.	( ), +, ×, 2, 5, 10	70	

3 ACTIVITY: Reviewing Fractions and Decimals

Work with a partner. Evaluate the expression.

a. $\frac{3}{4} - \left(\frac{1}{4} + \frac{1}{2}\right) =$

b. $\left(\frac{5}{6} - \frac{1}{6}\right) - \frac{1}{12} =$

c. $7.4 - (3.5 - 3.1) =$

d. $10.4 - (8.6 + 0.9) =$

e. $(\$7.23 + \$2.32) - \$5.40 =$

f. $\$124.60 - (\$72.41 + \$5.67) =$

Math Practice

Use Operations

How do you know which operation to perform first?

What Is Your Answer?

4. In an expression with two or more operations, why is it necessary to agree on an order of operations? Give examples to support your explanation.

5. **IN YOUR OWN WORDS** What is the effect of inserting parentheses into a numerical expression?

Practice Use what you learned about the order of operations to complete Exercises 3–5 on page 20.

1.3 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
numerical expression, *p. 18*
evaluate, *p. 18*
order of operations, *p. 18*

A **numerical expression** is an expression that contains only numbers and operations. To **evaluate**, or find the value of, a numerical expression, use a set of rules called the **order of operations**.

Key Idea

Order of Operations

1. Perform operations in **P**arentheses.
2. Evaluate numbers with **E**xponents.
3. **M**ultiply or **D**ivide from left to right.
4. **A**dd or **S**ubtract from left to right.

EXAMPLE 1 Using Order of Operations

a. Evaluate $12 - 2 \times 4$.

$$\begin{aligned} 12 - 2 \times 4 &= 12 - 8 && \text{Multiply 2 and 4.} \\ &= 4 && \text{Subtract 8 from 12.} \end{aligned}$$

b. Evaluate $7 + 60 \div (3 \times 5)$.

$$\begin{aligned} 7 + 60 \div (3 \times 5) &= 7 + 60 \div 15 && \text{Perform operation in parentheses.} \\ &= 7 + 4 && \text{Divide 60 by 15.} \\ &= 11 && \text{Add 7 and 4.} \end{aligned}$$

EXAMPLE 2 Using Order of Operations with Exponents

Evaluate $30 \div (7 + 2^3) \times 6$.

Evaluate the power in parentheses first.

$$\begin{aligned} 30 \div (7 + 2^3) \times 6 &= 30 \div (7 + 8) \times 6 && \text{Evaluate } 2^3. \\ &= 30 \div 15 \times 6 && \text{Perform operation in parentheses.} \\ &= 2 \times 6 && \text{Divide 30 by 15.} \\ &= 12 && \text{Multiply 2 and 6.} \end{aligned}$$

Study Tip

Remember to multiply and divide from left to right. In Example 2, you should divide before multiplying because the division symbol comes first when reading from left to right.

On Your Own

Now You're Ready
Exercises 6–14

Evaluate the expression.

1. $7 \cdot 5 + 3$
2. $(28 - 20) \div 4$
3. $6 \times 15 - 10 \div 2$
4. $6 + 2^4 - 1$
5. $4 \cdot 3^2 + 18 - 9$
6. $16 + (5^2 - 7) \div 3$

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students explored performing the operations in a numerical expression using different orders. (MP2, MP6)
- **Today:** Students will use the order of operations to evaluate a numerical expression.

Motivate

- Write "double" on a sheet of paper and "add 10" on another sheet of paper.
- Have each student choose his or her favorite number and record it. Hold up the two sheets of paper, one at a time, and tell students to perform the operation on the number. For instance, if a student chooses 4:

  double, then add 10 $\longrightarrow$ 18
  add 10, then double $\longrightarrow$ 28
- Share with students that for expressions such as $2 \times 4 + 10$ *or* $10 + 4 \times 2$, the multiplication is performed first!

Words of Wisdom

- A common error is that students may forget the "left to right" portion of the rule. For instance, $24 - 10 + 6 = 20$ (not 8).
- You may introduce the common acronym PEMDAS as a memory tool for the order of operations. Remind them again, however, that the left-to-right rule is an important part of the order of operations.

Lesson Notes

Key Idea

- **FYI:** Students should recall the order of operations from Grade 5. It is being extended now to include the use of exponents in numerical expressions.

Example 1

"How many operations are in the expression in part (a)?" 2 "What operation should be performed first?" multiplication

"How many operations are in the expression in part (b)?" 3 "What operation should be performed first?" multiplication "Why?" Perform operations within parentheses first.

Example 2

"How many operations are in this expression?" 4

- Ask a volunteer to explain the order in which the expression should be evaluated.
- **Common Error:** Students will say that 2^3 is 6 because they multiplied the base and the exponent.
- Refer to the Study Tip to reinforce the left-to-right rule.

On Your Own

- Each student should work independently before checking his or her work with a neighbor. Encourage students to show their work instead of trying to evaluate the expressions in their heads.

Goal Today's lesson is using the **order of operations** to **evaluate numerical expressions.**

Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

English Language Learners

Pair Activity

Encourage English language learners to verbalize the process of evaluating using the order of operations. Give each student an expression to evaluate. Include expressions containing exponents and parentheses. After evaluating the expression, the student should explain his or her solution to a partner.

Extra Example 1

a. Evaluate $16 + 5 \times 2$. 26
b. Evaluate $47 - 5 \times (32 \div 4)$. 7

Extra Example 2

Evaluate $15 \times (12 - 3^2) \div 9$. 5

On Your Own

1. 38 **2.** 2
3. 85 **4.** 21
5. 45 **6.** 22

Extra Example 3

a. Evaluate $10 - 2(1 + 3)$. 2

b. Evaluate $7 + 5(8 - 6) \times 2^3$. 87

Extra Example 4

A group of people visit a museum.

Age	Number of People	Admission Price per Person
65 and older	1	\$8
13–64	2	\$12
12 and under	4	\$4

What is the total admission price? \$48

On Your Own

7. 4 **8.** 0

9. 8

10. $\boxed{\text{Cost of 10 spheres}} +$

$\boxed{\text{Cost of 6 paint bottles}}$

$+ \boxed{\text{Cost of 9 rods}}$

$10 \cdot 2 + 6 \cdot 3 + 9 \cdot 1$
$= 20 + 18 + 9 = 47$

Your total cost increases by \$3 to \$47. You would need one more sphere which costs \$2 and one more rod which costs \$1.

Laurie's Notes

Example 3

- Discuss different ways multiplication can be represented. Share with students that a new way to represent multiplication is to use parentheses. All of the following represent 3 times 4:
  $3 \times 4 \qquad 3 \cdot 4 \qquad 3(4) \qquad (3)(4)$
- Explain that $3(2 + 7)$ is the same as $3 \times (2 + 7)$. A number written next to the parentheses implies multiplication.
- Work through each part of the example. Before you begin each part, ask a volunteer to identify the operations in the expression.

Example 4

- Ask a volunteer to read the problem.
- Do not skip the step of writing the verbal model because it explains in words how to solve the problem.

? "Would the answer be different if the operations were performed from left to right?" yes

- **FYI:** In this contextual problem, students understand almost intuitively that the 3 products must be found before adding. Strip away the context and write the expression $9 \cdot 2 + 6 \cdot 3 + 8 \cdot 1$ on the board and students are apt to perform the operations left to right without regard to the order of operations.
- **MP6 Attend to Precision:** Attending to precision, with or without a context involved, is what we want to develop in all students.

On Your Own

- Have students identify aloud to a partner the different operations they see in each exercise.
- As they write the solution, have students say the steps aloud.
- Ask volunteers to show their solutions on the board.

Closure

Exit Ticket: Evaluate each expression.

a. $18 + 4 \times 10$ 58 **b.** $12 \div 6 \times 2$ 4 **c.** $5^2 - 20 + 3(24 - 18)$ 23

The symbols × and • are used to indicate multiplication. You can also use parentheses to indicate multiplication. For example, $3(2 + 7)$ is the same as $3 \times (2 + 7)$.

EXAMPLE 3 Using Order of Operations

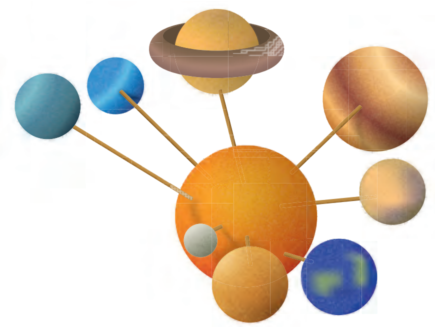
a. Evaluate $9 + 7(5 - 2)$.

$$\begin{aligned} 9 + 7(5 - 2) &= 9 + 7(3) && \text{Perform operation in parentheses.} \\ &= 9 + 21 && \text{Multiply 7 and 3.} \\ &= 30 && \text{Add 9 and 21.} \end{aligned}$$

b. Evaluate $15 - 4(6 + 1) \div 2^2$.

$$\begin{aligned} 15 - 4(6 + 1) \div 2^2 &= 15 - 4(7) \div 2^2 && \text{Perform operation in parentheses.} \\ &= 15 - 4(7) \div 4 && \text{Evaluate } 2^2. \\ &= 15 - 28 \div 4 && \text{Multiply 4 and 7.} \\ &= 15 - 7 && \text{Divide 28 by 4.} \\ &= 8 && \text{Subtract 7 from 15.} \end{aligned}$$

EXAMPLE 4 Real-Life Application



You buy foam spheres, paint bottles, and wooden rods to construct a model of our solar system. What is your total cost?

Item	Quantity	Cost per Item
Spheres	9	\$2
Paint	6	\$3
Rods	8	\$1

Use a verbal model to solve the problem.

cost of 9 spheres + cost of 6 paint bottles + cost of 8 rods

$9 \cdot 2 \quad + \quad 6 \cdot 3 \quad + \quad 8 \cdot 1$

$$\begin{aligned} 9 \cdot 2 + 6 \cdot 3 + 8 \cdot 1 &= 18 + 18 + 8 && \text{Multiply.} \\ &= 44 && \text{Add.} \end{aligned}$$

Your total cost is \$44.

On Your Own

Now You're Ready
Exercises 18–23

Evaluate the expression.

7. $50 + 6(12 \div 4) - 8^2$ **8.** $5^2 - 5(10 - 5)$ **9.** $\frac{8(3 + 4)}{7}$

10. WHAT IF? In Example 4, you add the dwarf planet Pluto to your model. Use a verbal model to find your total cost assuming you do not need more paint. Explain.

1.3 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **WRITING** Why does $12 - 8 \div 2 = 8$, but $(12 - 8) \div 2 = 2$?

2. **REASONING** Describe the steps in evaluating the expression $8 \div (6 - 4) + 3^2$.

Practice and Problem Solving

Find the value of the expression.

3. $(4 \times 15) - 3$
4. $10 - (7 + 1)$
5. $18 \div (6 + 3)$

Evaluate the expression.

① ② 6. $5 + 18 \div 6$
7. $(11 - 3) \div 2 + 1$
8. $45 \div 9 \times 12$
9. $6^2 - 3 \cdot 4$
10. $42 \div \left(15 - 2^3\right)$
11. $4^2 \cdot 2 + 8 \cdot 7$
12. $3^2 + 12 \div (6 - 3) \times 8$
13. $(10 + 4) \div (26 - 19)$
14. $\left(5^2 - 4\right) \cdot 2 - 18$

ERROR ANALYSIS **Describe and correct the error in evaluating the expression.**

15. ✗ $9 + 2 \times 3 = 11 \times 3$
$= 33$

16. ✗ $19 - 6 + 12 = 19 - 18$
$= 1$

17. **POETRY** You need to read 20 poems in 5 days for an English project. Each poem is 2 pages long. Evaluate the expression $20 \times 2 \div 5$ to find how many pages you need to read each day.

Evaluate the expression.

③ 18. $9^2 - 8(6 + 2)$
19. $(3 - 1)^3 + 7(6) - 5^2$
20. $8\left(1\frac{1}{6} + \frac{5}{6}\right) \div 4$
21. $7^2 - 2\left(\frac{11}{8} - \frac{3}{8}\right)$
22. $8(7.3 + 3.7) - 14 \div 2$
23. $2^4(5.2 - 3.2) \div 4$

24. **MONEY** You have four \$10 bills and eighteen \$5 bills in your piggy bank. How much money do you have?

25. **THEATER** Before a show, there are 8 people in a theater. Five groups of 4 people enter, and then three groups of 2 people leave. Evaluate the expression $8 + 5(4) - 3(2)$ to find how many people are in the theater.

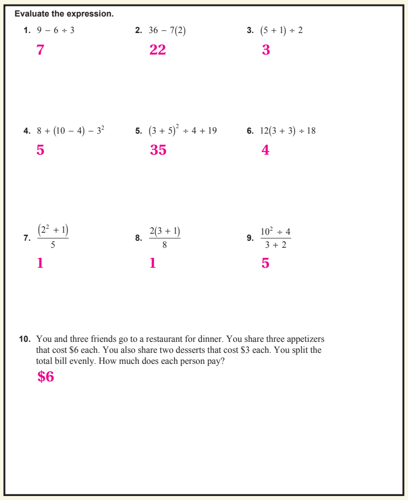
Assignment Guide and Homework Check

Level	Day 1 Activity Assignment	Day 2 Lesson Assignment	Homework Check
Basic	3–5, 34–38	1, 2, 7–23 odd, 24–28 even	9, 15, 21, 24, 26
Average	3–5, 34–38	1, 2, 7–15 odd, 19–25 odd, 26–32 even	9, 15, 21, 26, 32
Advanced	3–5, 34–38	1, 2, 6–14 even, 15–25 odd, 26, 28–33	12, 15, 19, 28, 32

Common Errors

- **Exercises 8 and 12** Students may want to evaluate multiplication before division in an expression (or a part of an expression) that contains division before multiplication from left to right. Remind students that multiplication and division are performed from left to right.
- **Exercises 18–23** Students may forget that parentheses can be used to indicate multiplication. Remind them of the various ways to indicate multiplication.
- **Exercises 26–28** Students may be unsure of how to evaluate these expressions. Remind them that the fraction bar means division. If necessary, have students rewrite the expressions using division symbols in place of the fraction bars. Be sure they insert additional parentheses if necessary.
- **Exercise 32** Students may not see the implication that the two groups are working *simultaneously* and they may end up with a time of 88 minutes instead of 44 minutes. Point out that the two groups are working at the same time, so together they are able to clean up 400 yards in 5 minutes.

1.3 Record and Practice Journal



Evaluate the expression.

1. $9 - 6 \div 3$
7

2. $36 - 7(2)$
22

3. $(5 + 1) \div 2$
3

4. $8 + (10 - 4) - 3^2$
5

5. $(3 + 5)^2 \div 4 + 19$
35

6. $12(3 + 3) \div 18$
4

7. $\frac{(2^2 + 1)}{5}$
1

8. $\frac{2(3 + 1)}{8}$
1

9. $\frac{10^2 \div 4}{3 + 2}$
5

10. You and three friends go to a restaurant for dinner. You share three appetizers that cost \$6 each. You also share two desserts that cost \$3 each. You split the total bill evenly. How much does each person pay?
\$6

Vocabulary and Concept Check

1. Using the order of operations for $12 - 8 \div 2$, you divide 8 by 2 and then subtract the result from 12. Using the order of operations for $(12 - 8) \div 2$, you subtract 8 from 12 and then divide by 2.

2. As illustrated below, perform the operation in parentheses, evaluate 3^2, divide 8 by 2, add 4 and 9.

$$\begin{aligned} &8 \div (6 - 4) + 3^2 \\ &= 8 \div 2 + 3^2 \\ &= 8 \div 2 + 9 \\ &= 4 + 9 \\ &= 13 \end{aligned}$$

Practice and Problem Solving

3. 57
4. 2
5. 2
6. 8
7. 5
8. 60
9. 24
10. 6
11. 88
12. 41
13. 2
14. 24

15. Addition was performed before multiplication.
$9 + 2 \times 3 = 9 + 6 = 15$

16. Addition was performed before subtraction. These operations should be performed in order from left to right.

$$\begin{aligned} 19 - 6 + 12 &= 13 + 12 \\ &= 25 \end{aligned}$$

17. 8 pages
18. 17
19. 25
20. 4
21. 47
22. 81
23. 8
24. \$130
25. 22 people

Practice and Problem Solving

26. 12

27. 1

28. 3

29. $34

30. See *Taking Math Deeper.*

31. $23; Add the prices of the items you buy. Then subtract the amount of the gift card from the total.

32. 44 min; Two miles is equivalent to 3520 yards. Each group can clean $200 \div 5 = 40$ yards each minute, so together the two groups can clean 80 yards each minute. So, it takes $3520 \div 80 = 44$ minutes to clean 2 miles.

33. **a.** $27 \div 3 + 5 \times 2 = 19$

b. *Sample answer:* $9^2 + 11 - 8 \times 4 \div 1 = 60$

c. $5 \times 6 - 15 + 9 = 24$

d. $14 \times 2 \div 7 - 3 + 9 = 10$

Fair Game Review

34. 5.7	**35.** 6.1
36. 9.6	**37.** 0.9
38. D	

Mini-Assessment

Evaluate the expression.

1. $4 + 12 \div 3$ 8
2. $20 - 4 \cdot 2^2$ 4
3. $(6^2 - 3) \times (2 + 8)$ 330
4. $4^3 \div 2 - (7 - 5)^2$ 28
5. You have four $1 bills, three $5 bills and two $10 bills in your wallet. How much money do you have in your wallet? $39

Taking Math Deeper

Exercise 30

As it is, this problem is straightforward. It has several simple solutions. For instance, $1 \times 100 + 1 - 1 \div 1 = 100$. You can make the problem more challenging by restricting the numbers that can be used. Here are some examples.

① Use only fours.

Restrict the problem so that students can only use the digit 4. With this restriction, can you write an expression that is equal to 100 and still use all four operations (and no parentheses)?

$$\frac{444}{4} - \frac{44}{4} + 4 \times 4 - 4 \times 4 = 111 - 11 + 16 - 16 = 100$$

② Use only threes.

Restrict the problem so that students can only use the digit 3. Notice that the pattern shown in part (1) can be used for *any* digit.

$$\frac{333}{3} - \frac{33}{3} + 3 \times 3 - 3 \times 3 = 111 - 11 + 9 - 9 = 100$$

③ Here is another way to make the problem more challenging.

(1) Use only 3's.

(2) Use each operation at most once.

$$33 \times 3 + 3 \div 3 = 99 + 1 = 100$$

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Start the next section

Evaluate the expression.

26. $\frac{6(3+5)}{4}$ **27.** $\frac{12^2 - 4(6) + 1}{11^2}$ **28.** $\frac{26 \div 2 + 5}{3^2 - 3}$

29. **FIELD TRIP** Eighty students are going on a field trip to a history museum. The total cost includes

- 2 bus rentals and
- $10 per student for lunch.

What is the total cost per student?

30. **OPEN-ENDED** Use all four operations without parentheses to write an expression that has a value of 100.

31. **SHOPPING** You buy 6 notebooks, 10 folders, 1 pack of pencils, and 1 lunch box for school. After using a $10 gift card, how much do you owe? Explain how you solved the problem.

32. **LITTER CLEANUP** Two groups collect litter along the side of a road. It takes each group 5 minutes to clean up a 200-yard section. How long does it take to clean up 2 *miles*? Explain how you solved the problem.

33. Number Sense Copy each statement. Insert +, −, ×, or ÷ symbols to make each statement true.

a. 27 ☐ 3 ☐ 5 ☐ 2 = 19 **b.** 9^2 ☐ 11 ☐ 8 ☐ 4 ☐ 1 = 60

c. 5 ☐ 6 ☐ 15 ☐ 9 = 24 **d.** 14 ☐ 2 ☐ 7 ☐ 3 ☐ 9 = 10

Fair Game Review What you learned in previous grades & lessons

Add or subtract. *(Skills Review Handbook)*

34. 5.2 + 0.5 **35.** 8 − 1.9 **36.** 12.6 − 3 **37.** 0.7 + 0.2

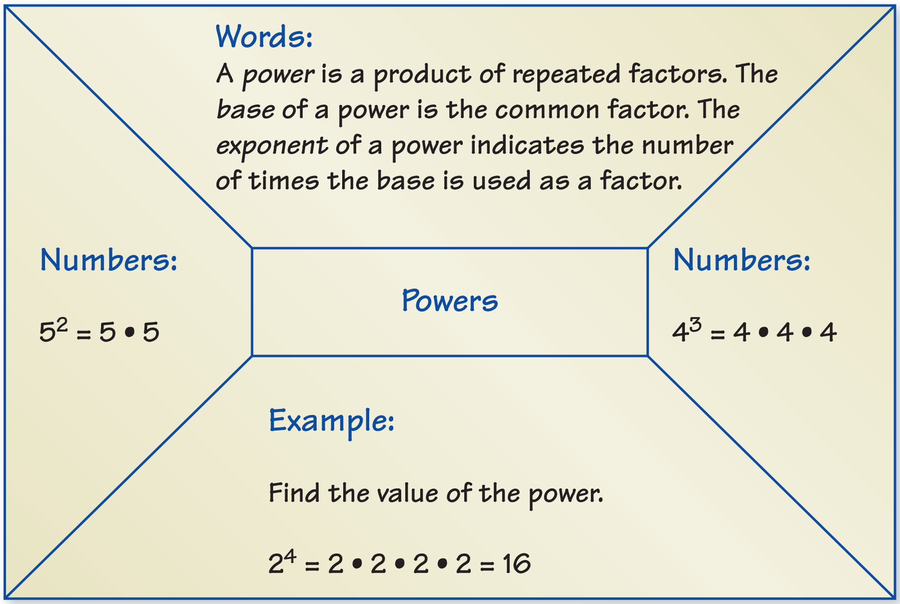
38. **MULTIPLE CHOICE** You are making two recipes. One recipe calls for $2\frac{1}{3}$ cups of flour. The other recipe calls for $1\frac{1}{4}$ cups of flour. How much flour do you need to make both recipes? *(Skills Review Handbook)*

Ⓐ $1\frac{1}{12}$ cups Ⓑ $3\frac{1}{12}$ cups Ⓒ $3\frac{2}{7}$ cups Ⓓ $3\frac{7}{12}$ cups

1 Study Help

Check It Out
Graphic Organizer
BigIdeasMath.com

You can use an **information frame** to help you organize and remember concepts. Here is an example of an information frame for powers.



Words:

A *power* is a product of repeated factors. The *base* of a power is the common factor. The *exponent* of a power indicates the number of times the base is used as a factor.

Numbers:

$5^2 = 5 \cdot 5$

Powers

Numbers:

$4^3 = 4 \cdot 4 \cdot 4$

Example:

Find the value of the power.

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$

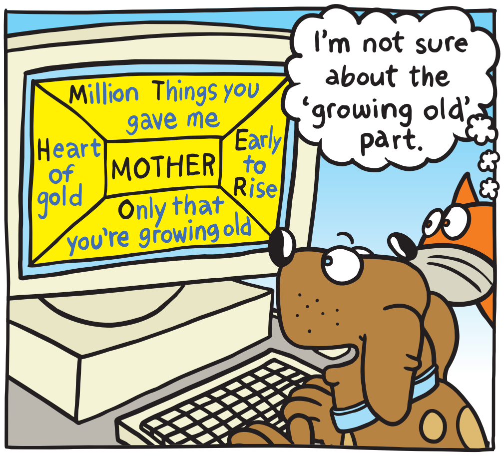
On Your Own

Make information frames to help you study these topics.

1. adding whole numbers
2. subtracting whole numbers
3. multiplying whole numbers
4. dividing whole numbers
5. order of operations

After you complete this chapter, make information frames for the following topics.

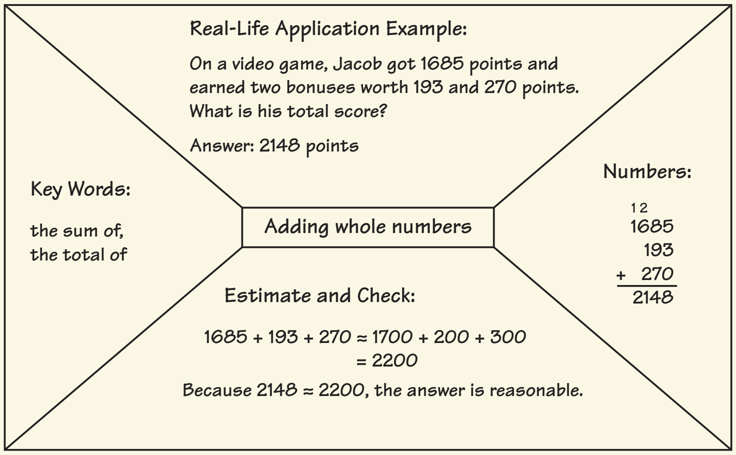
6. prime factorization
7. greatest common factor (GCF)
8. least common multiple (LCM)
9. least common denominator (LCD)



"Dear Mom, I am sending you an information frame card for Mother's Day!"

Sample Answers

1.



Real-Life Application Example:

On a video game, Jacob got 1685 points and earned two bonuses worth 193 and 270 points. What is his total score?

Answer: 2148 points

Key Words:

the sum of,
the total of

Adding whole numbers

Numbers:

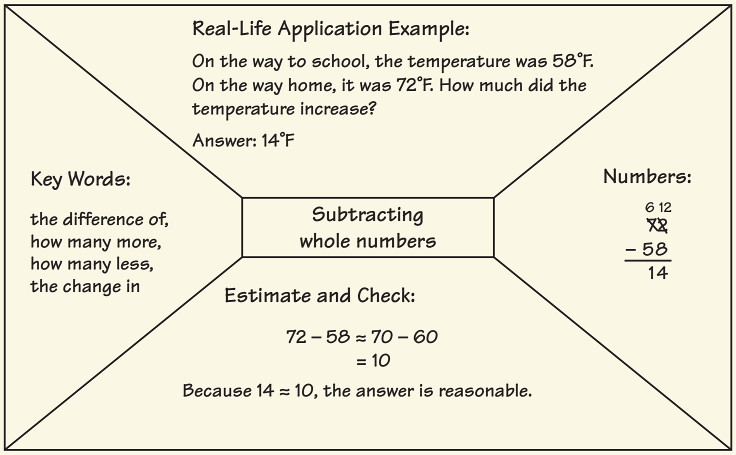
$$\begin{array}{r} {}^{1\,2} \\ 1685 \\ 193 \\ +\ \ 270 \\ \hline 2148 \end{array}$$

Estimate and Check:

$1685 + 193 + 270 \approx 1700 + 200 + 300$
$= 2200$

Because $2148 \approx 2200$, the answer is reasonable.

2.



Real-Life Application Example:

On the way to school, the temperature was 58°F. On the way home, it was 72°F. How much did the temperature increase?

Answer: 14°F

Key Words:

the difference of,
how many more,
how many less,
the change in

Subtracting whole numbers

Numbers:

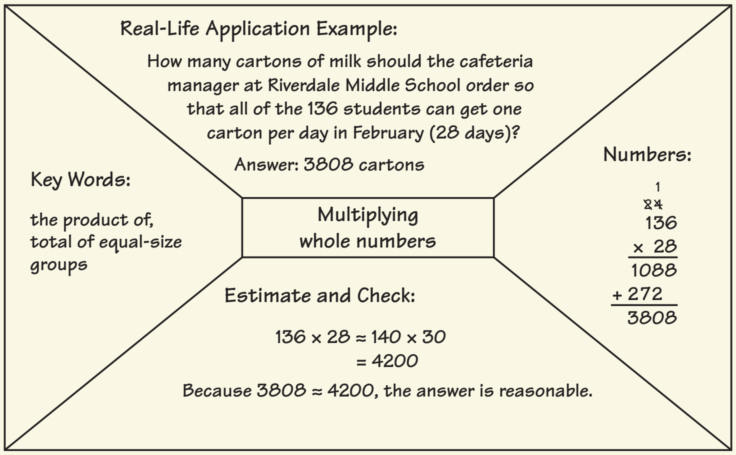
$$\begin{array}{r} {}^{6\ 12} \\ \cancel{7}\cancel{2} \\ -\ 58 \\ \hline 14 \end{array}$$

Estimate and Check:

$72 - 58 \approx 70 - 60$
$= 10$

Because $14 \approx 10$, the answer is reasonable.

3.



Real-Life Application Example:

How many cartons of milk should the cafeteria manager at Riverdale Middle School order so that all of the 136 students can get one carton per day in February (28 days)?

Answer: 3808 cartons

Key Words:

the product of,
total of equal-size groups

Multiplying whole numbers

Numbers:

$$\begin{array}{r} {}^{1} \\ \cancel{2}\cancel{4} \\ 136 \\ \times\ 28 \\ \hline 1088 \\ +\,272 \\ \hline 3808 \end{array}$$

Estimate and Check:

$136 \times 28 \approx 140 \times 30$
$= 4200$

Because $3808 \approx 4200$, the answer is reasonable.

4–5. Available at *BigIdeasMath.com.*

List of Organizers

Available at *BigIdeasMath.com*

- Comparison Chart
- Concept Circle
- Definition (Idea) and Example Chart
- Example and Non-Example Chart
- Formula Triangle
- Four Square
- **Information Frame**
- Information Wheel
- Notetaking Organizer
- Process Diagram
- Summary Triangle
- Word Magnet
- Y Chart

About this Organizer

An **Information Frame** can be used to help students organize and remember concepts. Students write the topic in the middle rectangle. Then students write related concepts in the spaces around the rectangle. Related concepts can include *Words, Numbers, Algebra, Example, Definition, Non-Example, Visual, Procedure, Details,* and *Vocabulary*. Students can place their information frames on note cards to use as a quick study reference.

Technology for the Teacher

Editable Graphic Organizer

Answers

1. 8161

2. 2703

3. 13,524

4. 9

5. 27

6. 121

7. perfect square

8. not a perfect square

9. 9

10. 3

11. 6

12. 4

13. 16 seats

14. 225 in.^2

15. 3 mi

16. $38

Technology for the **Teacher**

Online Assessment
Assessment Book
ExamView® Assessment Suite

Alternative Quiz Ideas

100% Quiz
Error Notebook
Group Quiz
Homework Quiz
Math Log
Notebook Quiz
Partner Quiz
Pass the Paper

Partner Quiz

- Partner quizzes are to be completed by students working in pairs. Student pairs can be selected by the teacher, by students, through a random process, or any way that works for your class.
- Students are permitted to use their notebooks and other appropriate materials.
- Each pair submits a draft of the quiz for teacher feedback. Then they revise their work, and turn it in for a grade.
- When the pair is finished they can submit one paper, or each can submit their own.
- Teachers can give feedback in a variety of ways. It is important that the teacher does not reteach or provide the solution. The teacher can tell students which questions they have answered correctly, if they are on the right track, or if they need to rethink a problem.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Lesson Tutorials *BigIdeasMath.com*	Resources by Chapter • Enrichment and Extension • Technology Connection Game Closet at *BigIdeasMath.com* Start the next section

1.1–1.3 Quiz

Check It Out
Progress Check
BigIdeasMath.com

Find the value of the expression. Use estimation to check your answer. *(Section 1.1)*

1. $4265 + 3896$

2. $5327 - 2624$

3. 276×49

4. $648 \div 72$

Find the value of the power. *(Section 1.2)*

5. 3^3

6. 11^2

Determine whether the number is a perfect square. *(Section 1.2)*

7. 36

8. 15

Evaluate the expression. *(Section 1.3)*

9. $6 + 21 \div 7$

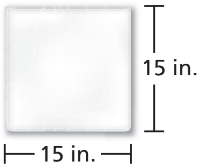
10. $\dfrac{4(12 - 3)}{12}$

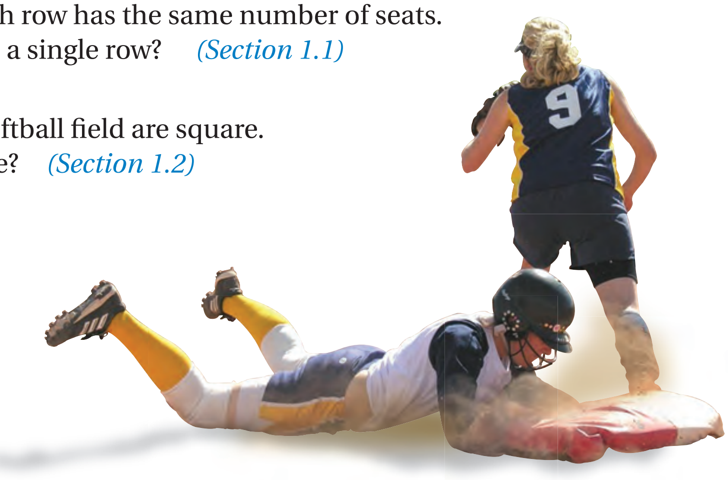
11. $16 \div 2^3 + 6 - 2$

12. $2 \times 14 \div (3^2 - 2)$

13. **AUDITORIUM** An auditorium has a total of 592 seats. There are 37 rows of seats, and each row has the same number of seats. How many seats are there in a single row? *(Section 1.1)*

14. **SOFTBALL** The bases on a softball field are square. What is the area of each base? *(Section 1.2)*





15. **DUATHLON** In an 18-mile duathlon, you run, then bike 12 miles, and then run again. The two runs are the same distance. Find the distance of each run. *(Section 1.3)*

16. **AMUSEMENT PARK** Tickets for an amusement park cost \$10 for adults and \$6 for children. Find the total cost for 2 adults and 3 children. *(Section 1.3)*

1.4 Prime Factorization

Essential Question Without dividing, how can you tell when a number is divisible by another number?

1 ACTIVITY: Finding Divisibility Rules for 2, 3, 5, and 10

Work with a partner. Copy the set of numbers (1–50) as shown.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

a. Highlight all the numbers that are divisible by 2.

b. Put a box around the numbers that are divisible by 3.

c. Underline the numbers that are divisible by 5.

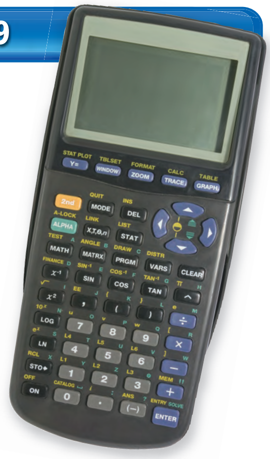
d. Circle the numbers that are divisible by 10.

e. **STRUCTURE** In parts (a)–(d), what patterns do you notice? Write four rules to determine when a number is divisible by 2, 3, 5, and 10.

Common Factors and Multiples

In this lesson, you will
- use divisibility rules to find prime factorizations of numbers.

2 ACTIVITY: Finding Divisibility Rules for 6 and 9



Work with a partner.

a. List ten numbers that are divisible by 6. Write a rule to determine when a number is divisible by 6. Use a calculator to check your rule with large numbers.

b. List ten numbers that are divisible by 9. Write a rule to determine when a number is divisible by 9. Use a calculator to check your rule with large numbers.

Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP4 Model with Mathematics** and **MP2 Reason Abstractly and Quantitatively:** Writing a factor pair is the first step in finding the prime factorization of a number. The possible factor pairs are related to the dimensions of the possible rectangles with a fixed area.

Motivate

- ? Write the number 2520 on the board. "What do you think is special about 2520?" Students may state that it is even, or it ends in 0.
- ? "Because 2520 ends in 0, what number divides into it evenly?" 10
- ? "Because 2520 is an even number, what number divides into it evenly?" 2
- ? "What other numbers do you think divide into 2520 evenly?"
- Students may guess correctly that 2520 is the least number divisible by all of the numbers 1–10.

Discuss

- Review definition of *divisible*. A number is divisible by another number if the second number is a factor of the first number.

Activity Notes

Activity 1

- Tell students that they will be looking for patterns today. If they do not have a highlighter, suggest that they lightly shade over the number.
- When students have finished, ask volunteers to share their observations.
- Students will likely note that all of the numbers divisible by 2 appear in columns, as do the numbers divisible by 5 and by 10. This is a function of how the table was set up. This would not be true if the width of the first row was 9. In this case, the numbers divisible by 3 would be in column.
- You want students to focus on the numbers and not the position of the numbers in the table.
- The divisibility rule for 3 will likely be the most challenging. Students will see that the boxed numbers appear on a diagonal. Prompt with leading questions as needed.
- Check to see that all four rules are stated correctly.

Activity 2

- Help students to think about rules for 6 and 9. You might ask, "If a number is divisible by 10, what other numbers is it also divisible by? Explain."
- **MP3a Construct Viable Arguments:** Listen for students to give viable arguments of why a number divisible by 10 is also divisible by 2 and 5. This will help them think about the divisibility rule for 6.
- Check to see that the rules are stated correctly.

Common Core State Standards

6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

Previous Learning

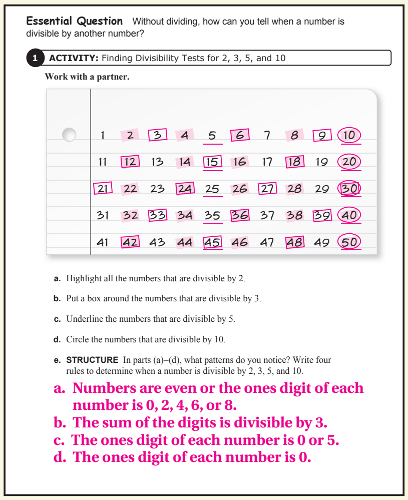
Students need to be familiar with dividing whole numbers and with the meanings of *prime* and *composite numbers*.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

1.4 Record and Practice Journal



Essential Question Without dividing, how can you tell when a number is divisible by another number?

① **ACTIVITY:** Finding Divisibility Tests for 2, 3, 5, and 10

Work with a partner.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

a. Highlight all the numbers that are divisible by 2.

b. Put a box around the numbers that are divisible by 3.

c. Underline the numbers that are divisible by 5.

d. Circle the numbers that are divisible by 10.

e. **STRUCTURE** In parts (a)–(d), what patterns do you notice? Write four rules to determine when a number is divisible by 2, 3, 5, and 10.

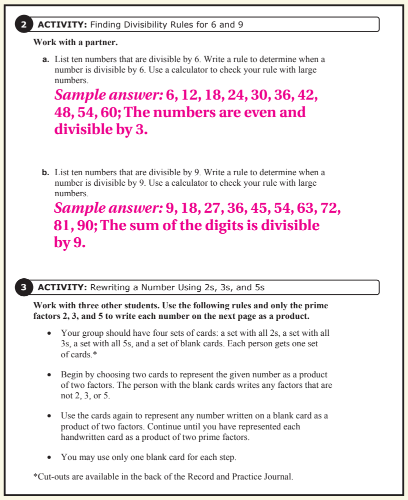
a. Numbers are even or the ones digit of each number is 0, 2, 4, 6, or 8.
b. The sum of the digits is divisible by 3.
c. The ones digit of each number is 0 or 5.
d. The ones digit of each number is 0.

English Language Learners

Graphic Organizer

Have students organize the divisibility rules in a table. In the first column, list the divisor. In the second column, list the rule. In the third column, list examples of numbers that are divisible by the divisor. Being organized saves time, allowing for more time to work on language skills.

1.4 Record and Practice Journal



2 ACTIVITY: Finding Divisibility Rules for 6 and 9

Work with a partner.

a. List ten numbers that are divisible by 6. Write a rule to determine when a number is divisible by 6. Use a calculator to check your rule with large numbers.

Sample answer: **6, 12, 18, 24, 30, 36, 42, 48, 54, 60; The numbers are even and divisible by 3.**

b. List ten numbers that are divisible by 9. Write a rule to determine when a number is divisible by 9. Use a calculator to check your rule with large numbers.

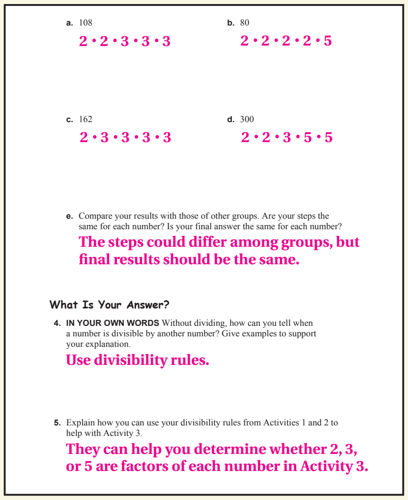
Sample answer: **9, 18, 27, 36, 45, 54, 63, 72, 81, 90; The sum of the digits is divisible by 9.**

3 ACTIVITY: Rewriting a Number Using 2s, 3s, and 5s

Work with three other students. Use the following rules and only the prime factors 2, 3, and 5 to write each number on the next page as a product.

- Your group should have four sets of cards: a set with all 2s, a set with all 3s, a set with all 5s, and a set of blank cards. Each person gets one set of cards.*
- Begin by choosing two cards to represent the given number as a product of two factors. The person with the blank cards writes any factors that are not 2, 3, or 5.
- Use the cards again to represent any number written on a blank card as a product of two factors. Continue until you have represented each handwritten card as a product of two prime factors.
- You may use only one blank card for each step.

*Cut-outs are available in the back of the Record and Practice Journal.



a. 108

$2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

b. 80

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$

c. 162

$2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

d. 300

$2 \cdot 2 \cdot 3 \cdot 5 \cdot 5$

e. Compare your results with those of other groups. Are your steps the same for each number? Is your final answer the same for each number?

The steps could differ among groups, but final results should be the same.

What Is Your Answer?

4. IN YOUR OWN WORDS Without dividing, how can you tell when a number is divisible by another number? Give examples to support your explanation.

Use divisibility rules.

5. Explain how you can use your divisibility rules from Activities 1 and 2 to help with Activity 3.

They can help you determine whether 2, 3, or 5 are factors of each number in Activity 3.

Laurie's Notes

Activity 3

- Give time for students to form groups of 4.
- Hand out the cards and ask a volunteer to read the directions. Be clear that the cards students have are all 2s, all 3s, all 5s, or all blank cards. No student should have a mix of numbers or blank cards. In the example shown, note that the number 5 is not used. Students should not expect to use all of their cards.
- The last step is important, writing the number as the product of all the factors they found.
- If time is a constraint, you might consider doing this activity at the board by taping numbers to the board.
- Discuss the results of the activity. The results should be the same for all groups, but the steps they took in getting there likely will vary. For instance, in part (d) the first step might be 2 × 150, 3 × 100, or 5 × 60.
- You could suggest additional numbers for students to factor if time allows.

What Is Your Answer?

- Listen for an answer involving the divisibility rules.
- **Extension:** Students may ask about divisibility rules for other numbers such as 4 and 8. A number is divisible by 4 if the last two digits are divisible by 4. A number is divisible by 8 if the last three digits are divisible by 8.

Closure

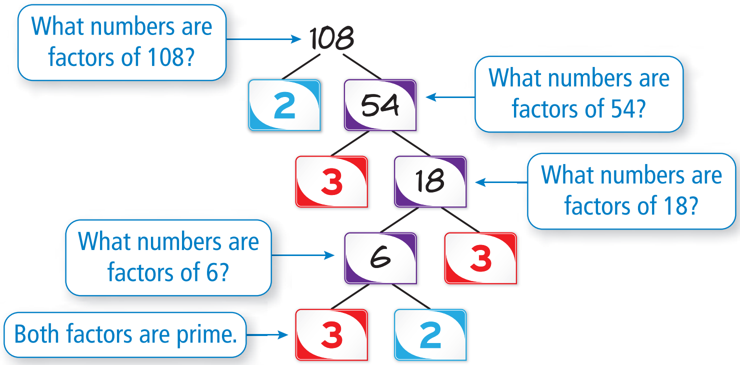
- Write a 3-digit number that is divisible by 3 and 10. Explain why you are correct. *Sample answer:* 510; The sum of the digits is divisible by 3, and the number ends in 0.
- Write a 3-digit number that is divisible by 2 and 3 but not 5. Explain why you are correct. *Sample answer:* 768; The number is even, the sum of the digits is divisible by 3, and the number does not end in 5 or 0.

3 ACTIVITY: Rewriting a Number Using 2s, 3s, and 5s

Work with three other students. Use the following rules and only the prime factors 2, 3, and 5 to write each number below as a product.

- Your group should have four sets of cards: a set with all 2s, a set with all 3s, a set with all 5s, and a set of blank cards. Each person gets one set of cards.
- Begin by choosing two cards to represent the given number as a product of two factors. The person with the blank cards writes any factors that are not 2, 3, or 5.
- Use the cards again to represent any number written on a blank card as a product of two factors. Continue until you have represented each handwritten card as a product of two prime factors.
- You may use only one blank card for each step.

a. Sample: 108



What numbers are factors of 108?

What numbers are factors of 54?

What numbers are factors of 18?

What numbers are factors of 6?

Both factors are prime.

$108 = 2 \cdot 3 \cdot 3 \cdot 3 \cdot 2$

Math Practice

Interpret Results

How do you know your answer makes sense?

b. 80 **c.** 162 **d.** 300

e. Compare your results with those of other groups. Are your steps the same for each number? Is your final answer the same for each number?

What Is Your Answer?

4. **IN YOUR OWN WORDS** Without dividing, how can you tell when a number is divisible by another number? Give examples to support your explanation.

5. Explain how you can use your divisibility rules from Activities 1 and 2 to help with Activity 3.

Practice Use what you learned about divisibility rules to complete Exercises 4–7 on page 28.

1.4 Lesson

Because 2 is factor of 10 and $2 \cdot 5 = 10$, 5 is also a factor of 10. The pair 2, 5 is called a **factor pair** of 10.

Key Vocabulary
factor pair, *p. 26*
prime factorization, *p. 26*
factor tree, *p. 26*

EXAMPLE 1 Finding Factor Pairs

The brass section of a marching band has 30 members. The band director arranges the brass section in rows. Each row has the same number of members. How many possible arrangements are there?

Use the factor pairs of 30 to find the number of arrangements.

$30 = 1 \cdot 30$	There could be 1 row of 30 or 30 rows of 1.
$30 = 2 \cdot 15$	There could be 2 rows of 15 or 15 rows of 2.
$30 = 3 \cdot 10$	There could be 3 rows of 10 or 10 rows of 3.
$30 = 5 \cdot 6$	There could be 5 rows of 6 or 6 rows of 5.
$30 = 6 \cdot 5$	The factors 5 and 6 are already listed.

∴ There are 8 possible arrangements: 1 row of 30, 30 rows of 1, 2 rows of 15, 15 rows of 2, 3 rows of 10, 10 rows of 3, 5 rows of 6, or 6 rows of 5.

Study Tip

When making an organized list of factor pairs, stop finding pairs when the factors begin to repeat.

On Your Own

Now You're Ready
Exercises 8–15

List the factor pairs of the number.

1. 18 **2.** 24 **3.** 51

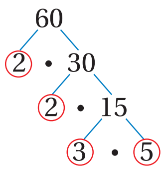
4. **WHAT IF?** The woodwinds section of the marching band has 38 members. Which has more possible arrangements, the brass section or the woodwinds section? Explain.

Key Idea

Prime Factorization

The **prime factorization** of a composite number is the number written as a product of its prime factors.

You can use factor pairs and a **factor tree** to help find the prime factorization of a number. The factor tree is complete when only prime factors appear in the product. A factor tree for 60 is shown.



$60 = 2 \cdot 2 \cdot 3 \cdot 5$, or $2^2 \cdot 3 \cdot 5$

Remember

A *prime number* is a whole number greater than 1 with exactly two factors, 1 and itself. A *composite number* is a whole number greater than 1 with factors other than 1 and itself.

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students wrote divisibility rules for 2, 3, 5, 6, 9, and 10. (MP2, MP3a, MP4)
- **Today:** Students will use the divisibility rules to help them write the prime factorization of numbers.

Motivate

? "Do you know someone who has ordered an item over the Internet?" Most students will say yes.

- In order to make the transactions safe, credit card numbers are encrypted.
- To encrypt a number, a company makes available (in a public key) a large number (400 digits) that is the product of two very large prime numbers.
- The sender uses that number to encrypt the message to the company.
- Only the company knows the factors needed to decrypt the message.
- While it is easy to multiply two very large prime numbers, it is nearly impossible to find the two original primes in a timely manner. There is no fast algorithm for prime factorization.
- Today students will learn how to find the prime factorization of a number.

Lesson Notes

Example 1

- Define factor pairs and give an example.
- Work through the example as shown, pointing out that when the factors begin to repeat, you are done, as explained in the Study Tip.
- **Visual model:** You could use 30 square tiles and ask how to form a rectangle using all of the tiles. The dimensions of the rectangles are the factor pairs found in this example.
- In the context of the problem, 5 • 6 is a factor pair and it can be interpreted two ways: 5 rows of 6 or 6 rows of 5. It is still one factor pair.

On Your Own

- **Neighbor Check:** Have students work independently, and then have their neighbors check their work. Have students discuss any discrepancies.
- **MP2 Reason Abstractly and Quantitatively:** Note even though 38 > 30, 38 has fewer factor pairs than 30. Students should reason that the size of the number is not what determines the number of factor pairs.

Key Idea

? "What is a composite number?" a number greater than one that has more than two factors

- Write the definition of prime factorization.
- Work through the example shown, using the vocabulary: factors, prime, and composite.

Goal Today's lesson is writing the **prime factorization** of a number.

Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

There are 40 members in the book club at school. Each member sits at a desk. Each row needs to have the same number of desks. How many possible arrangements are there? 8

On Your Own

1. 1, 18; 2, 9; 3, 6

2. 1, 24; 2, 12; 3, 8; 4, 6

3. 1, 51; 3, 17

4. brass section; The number 38 has only two factor pairs: 1, 38 and 2, 19. So, there are only 4 possible arrangements with the woodwinds section: 1 row of 38, 38 rows of 1, 2 rows of 19, or 19 rows of 2.

Extra Example 2

Write the prime factorization of 45.
$3^2 \cdot 5$

Extra Example 3

What is the greatest perfect square that is a factor of 675? 225

On Your Own

5. $2^2 \cdot 5$
6. $2^3 \cdot 11$
7. $2 \cdot 3^2 \cdot 5$
8. $2 \cdot 3 \cdot 7 \cdot 11$
9. 36; $396 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 11$ and $2 \cdot 3 \cdot 2 \cdot 3 = 6 \cdot 6 = 36$

Differentiated Instruction

Kinesthetic
Have students work in groups to find the prime factorization of 1575 (Example 3) using a factor tree. Give each group a different pair of factors, such as 3 × 525, 5 × 315, 7 × 225, 9 × 175, and so on. Have students compare their results. They should see that the prime factorization is the same, even when starting with different factor pairs.

Laurie's Notes

Example 2

- Ask students to name a factor pair for the number 48. This should give at least two different ways in which the problem can be started.
- Work through two different versions of a prime factor tree for 48. As the Study Tip states, it is important for students to understand that the steps may be different, but the final factorization will be the same.
- ? "Why is $2 \cdot 2 \cdot 3 \cdot 2 \cdot 2$ the same as $3 \cdot 2 \cdot 2 \cdot 2 \cdot 2$?" Commutative Property of Multiplication
- ? "Why isn't 1 listed in the prime factorization of a number?" 1 is neither prime nor composite.

Example 3

- Students will need to find the prime factorization of 1575 in order to answer the question.

On Your Own

- **Common Error:** In Question 5, students may begin to write a factor pair of 10 and 10. It is easy for them to forget that factor pairs must multiply to give the answer, not add.
- In Question 8, students should be encouraged to use their divisibility rules to find the largest factor they can, using mental math, versus starting with $2 \cdot 231$.

Closure

- **Exit Ticket:** The class ended and a student didn't finish finding the prime factorization. The student's first step was 8 × 24.
  a) What was the original number? 192
  b) Finish finding the prime factorization. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$

EXAMPLE 2 Writing a Prime Factorization

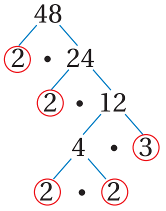
Write the prime factorization of 48.

Choose any factor pair of 48 to begin the factor tree.

Study Tip

Notice that beginning with different factor pairs results in the same prime factorization. Every composite number has only one prime factorization.

Tree 1

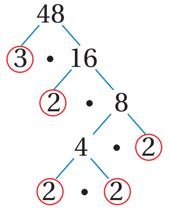


Find a factor pair and draw "branches."

Circle the prime factors as you find them.

Find factors until each branch ends at a prime factor.

Tree 2



$48 = 2 \cdot 2 \cdot 3 \cdot 2 \cdot 2$

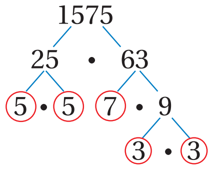
$48 = 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

The prime factorization of 48 is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$, or $2^4 \cdot 3$.

EXAMPLE 3 Using a Prime Factorization

What is the greatest perfect square that is a factor of 1575?

Because 1575 has many factors, it is not efficient to list all of its factors and check for perfect squares. Use the prime factorization of 1575 to find any perfect squares that are factors.



$1575 = 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7$

The prime factorization shows that 1575 has three factors other than 1 that are perfect squares.

$3 \cdot 3 = 9$ $\quad 5 \cdot 5 = 25$ $\quad (3 \cdot 5) \cdot (3 \cdot 5) = 15 \cdot 15 = 225$

So, the greatest perfect square that is a factor of 1575 is 225.

On Your Own

Now You're Ready
Exercises 16–23 and 29–32

Write the prime factorization of the number.

5. 20 **6.** 88 **7.** 90 **8.** 462

9. What is the greatest perfect square that is a factor of 396? Explain.

1.4 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** What is the prime factorization of a number?

2. **VOCABULARY** How can you use a factor tree to help you write the prime factorization of a number?

3. **WHICH ONE DOESN'T BELONG?** Which factor pair does not belong with the other three? Explain your reasoning.

2, 28　　4, 14　　6, 9　　7, 8

Practice and Problem Solving

Use divisibility rules to determine whether the number is divisible by 2, 3, 5, 6, 9, and 10. Use a calculator to check your answer.

4. 1044　　**5.** 1485　　**6.** 1620　　**7.** 1709

List the factor pairs of the number.

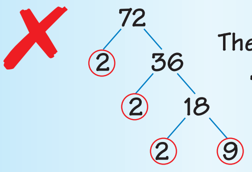
① **8.** 15　　**9.** 22　　**10.** 34　　**11.** 39

12. 45　　**13.** 54　　**14.** 59　　**15.** 61

Write the prime factorization of the number.

② **16.** 16　　**17.** 25　　**18.** 30　　**19.** 26

20. 84　　**21.** 54　　**22.** 65　　**23.** 77



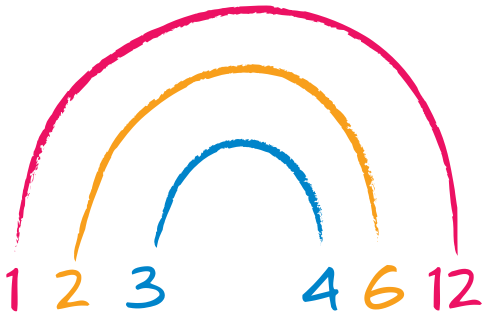
The prime factorization of

$72 = 2 \cdot 2 \cdot 2 \cdot 9$

$= 2^3 \cdot 9.$

24. **ERROR ANALYSIS** Describe and correct the error in writing the prime factorization.

25. **FACTOR RAINBOW** You can use a factor rainbow to check whether a list of factors is correct. To create a factor rainbow, list the factors of a number in order from least to greatest. Then draw arches that link the factor pairs. For perfect squares, there is no connecting arch in the middle. So, just circle the middle number. A factor rainbow for 12 is shown. Create factor rainbows for 6, 24, 36, and 48.



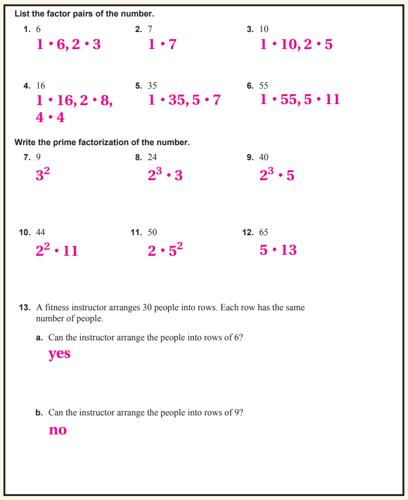
Assignment Guide and Homework Check

Level	Day 1 Activity Assignment	Day 2 Lesson Assignment	Homework Check
Basic	4–7, 40–44	1–3, 9–21 odd, 24, 25	9, 13, 17, 19, 21
Average	4–7, 40–44	1–3, 11–23 odd, 24, 26, 27, 29, 33, 35	11, 19, 21, 27, 35
Advanced	4–7, 40–44	1–3, 14–32 even, 33–36, 39	14, 22, 26, 35, 36

Common Errors

- **Exercises 16–23** Students may think that smaller numbers have fewer factors. Remind students to keep finding factors until all factors are prime numbers.
- **Exercises 26–28** Students may incorrectly find the value of a power. Remind them that the exponent is the number of times the base is a factor. That is, $3^2 = 3 \cdot 3$, not $3 \cdot 2$.

1.4 Record and Practice Journal



List the factor pairs of the number.

1. 6 $1 \cdot 6, 2 \cdot 3$

2. 7 $1 \cdot 7$

3. 10 $1 \cdot 10, 2 \cdot 5$

4. 16 $1 \cdot 16, 2 \cdot 8, 4 \cdot 4$

5. 35 $1 \cdot 35, 5 \cdot 7$

6. 55 $1 \cdot 55, 5 \cdot 11$

Write the prime factorization of the number.

7. 9 3^2

8. 24 $2^3 \cdot 3$

9. 40 $2^3 \cdot 5$

10. 44 $2^2 \cdot 11$

11. 50 $2 \cdot 5^2$

12. 65 $5 \cdot 13$

13. A fitness instructor arranges 30 people into rows. Each row has the same number of people.

a. Can the instructor arrange the people into rows of 6?

yes

b. Can the instructor arrange the people into rows of 9?

no

Vocabulary and Concept Check

1. The prime factorization of a composite number is the number written as a product of its prime factors.

2. First, find a factor pair and draw "branches." Next, circle the prime factors as you find them. Then, find factors until each branch ends at a prime factor.

3. 6, 9 does not belong because it is a factor pair of 54 and the others are factor pairs of 56.

Practice and Problem Solving

4. 2, 3, 6, 9

5. 3, 5, 9

6. 2, 3, 5, 6, 9, 10

7. None, 1709 is a prime number.

8. 1, 15; 3, 5

9. 1, 22; 2, 11

10. 1, 34; 2, 17

11. 1, 39; 3, 13

12. 1, 45; 3, 15; 5, 9

13. 1, 54; 2, 27; 3, 18; 6, 9

14. 1, 59

15. 1, 61

16. $2 \cdot 2 \cdot 2 \cdot 2$ or 2^4

17. $5 \cdot 5$ or 5^2

18. $2 \cdot 3 \cdot 5$

19. $2 \cdot 13$

20. $2 \cdot 2 \cdot 3 \cdot 7$ or $2^2 \cdot 3 \cdot 7$

21. $2 \cdot 3 \cdot 3 \cdot 3$ or $2 \cdot 3^3$

22. $5 \cdot 13$

23. $7 \cdot 11$

24. 9 is not prime, it is equal to $3 \cdot 3$.
$72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 2^3 \cdot 3^2$

25. See Additional Answers.

Practice and Problem Solving

26. 180
27. 1575
28. 12,584
29. 4
30. 25
31. 36
32. 1
33. yes; 2 is a prime number because it only has 1 and itself as factors. The rest of the even whole numbers have 2 as a factor.
34. composite; The total number of players on the baseball team is equal to the number in each group times the number of groups, so it must be composite.
35. See *Taking Math Deeper*.
36. 6
37. cupcake table; Because 60 has more factors than 75, there are more rectangular arrangements.
38. 26 yd
39. See Additional Answers.

Fair Game Review

40. 145
41. 357
42. 2395
43. 1248
44. B

Mini-Assessment

1. List the factor pairs of 36.
   1, 36; 2, 18; 3, 12; 4, 9; 6, 6
2. Write the prime factorization of 42.
   $2 \cdot 3 \cdot 7$
3. Write the prime factorization of 56.
   $2^3 \cdot 7$
4. What is the greatest perfect square that is a factor of 192? 64

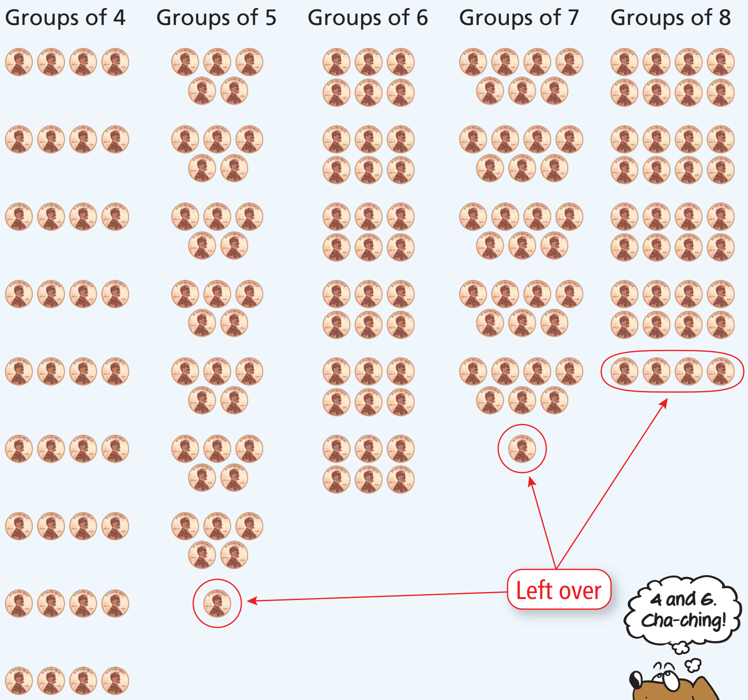
Taking Math Deeper

Exercise 35

One way to solve this problem is to try separating manipulatives into equal groups, and see if any are left over.

① Find 36 objects to represent each of the 36 students. You can use pennies.

② Each group should have *at least* 4 students but *no more than* 8 students. So, see if you can separate the 36 pennies into equal groups of 4, 5, 6, 7, and 8 with none left over.



③ So, the possible group sizes are 9 groups of 4 students and 6 groups of 6 students.

Project

Research the meaning of the word *simulation*. Explain how the solution above is a simulation. Explain why simulations are useful.

Reteaching and Enrichment Strategies

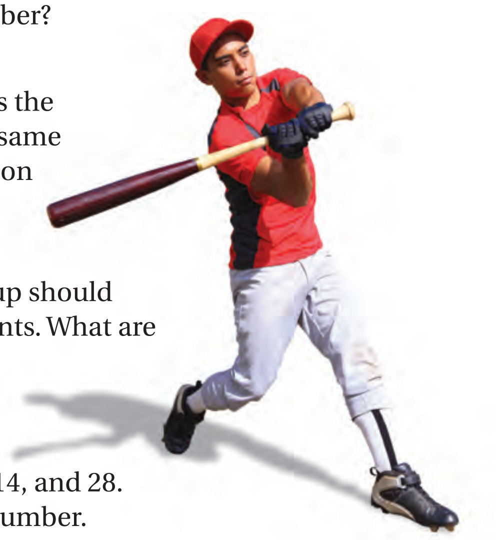
If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Start the next section

Find the number represented by the prime factorization.

26. $2^2 \cdot 3^2 \cdot 5$ **27.** $3^2 \cdot 5^2 \cdot 7$ **28.** $2^3 \cdot 11^2 \cdot 13$

Find the greatest perfect square that is a factor of the number.

③ **29.** 244 **30.** 650 **31.** 756 **32.** 1290



33. CRITICAL THINKING Is 2 the only even prime number? Explain.

34. BASEBALL The coach of a baseball team separates the players into groups for drills. Each group has the same number of players. Is the total number of players on the baseball team *prime* or *composite*? Explain.

35. SCAVENGER HUNT A teacher divides 36 students into equal groups for a scavenger hunt. Each group should have at least 4 students but no more than 8 students. What are the possible group sizes?

36. PERFECT NUMBERS A *perfect number* is a number that equals the sum of its factors, not including itself. For example, the factors of 28 are 1, 2, 4, 7, 14, and 28. Because $1 + 2 + 4 + 7 + 14 = 28$, 28 is a perfect number. What are the perfect numbers between 1 and 28?

37. BAKE SALE One table at a bake sale has 75 cookies. Another table has 60 cupcakes. Which table allows for more rectangular arrangements when all the cookies and cupcakes are displayed? Explain.

38. MODELING The stage manager of a school play creates a rectangular acting area of 42 square yards. String lights will outline the acting area. To the nearest whole number, how many yards of string lights does the manager need to enclose this area?

Rectangular Prism

Volume = 40 cubic inches

39. **Volume** The volume of a rectangular prism can be found using the formula *volume* = *length* × *width* × *height*. Using only whole number dimensions, how many different prisms are possible? Explain.

Fair Game Review What you learned in previous grades & lessons

Find the difference. *(Skills Review Handbook)*

40. 192 − 47 **41.** 451 − 94 **42.** 3210 − 815 **43.** 4752 − 3504

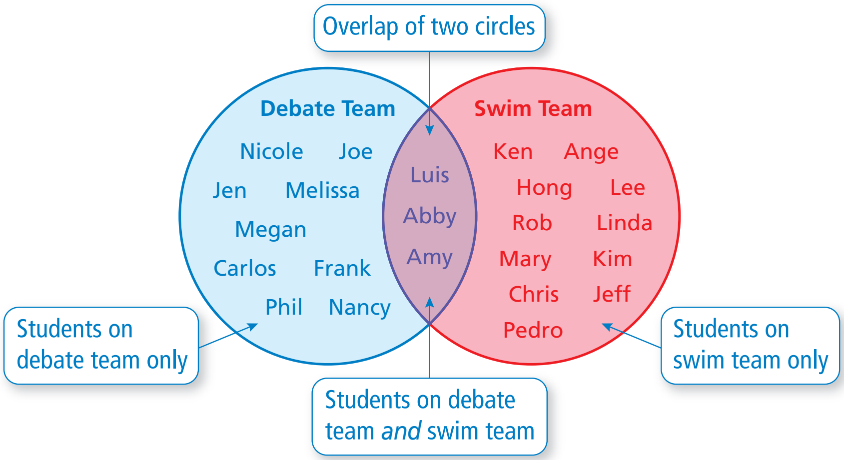
44. MULTIPLE CHOICE You buy 168 pears. There are 28 pears in each bag. How many bags of pears do you buy? *(Skills Review Handbook)*

Ⓐ 5 Ⓑ 6 Ⓒ 7 Ⓓ 28

1.5 Greatest Common Factor

Essential Question How can you find the greatest common factor of two numbers?

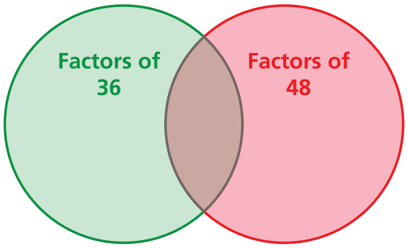
A **Venn diagram** uses circles to describe relationships between two or more sets. The Venn diagram shows the names of students enrolled in two activities. Students enrolled in both activities are represented by the overlap of the two circles.



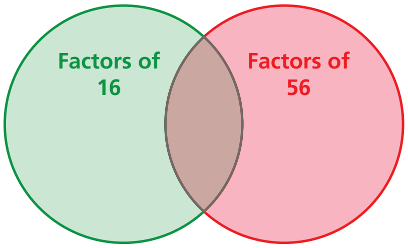
1 ACTIVITY: Identifying Common Factors

Work with a partner. Copy and complete the Venn diagram. Identify the *common factors* of the two numbers.

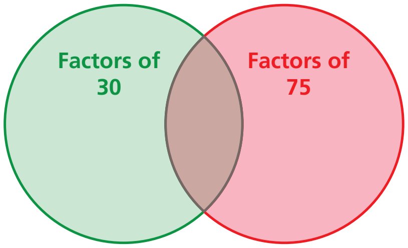
a. 36 and 48



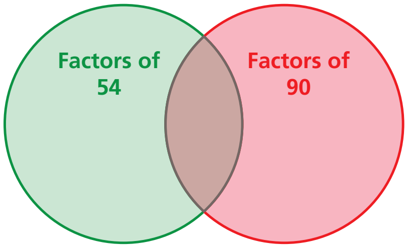
b. 16 and 56



c. 30 and 75



d. 54 and 90



e. Look at the Venn diagrams in parts (a)–(d). Explain how to identify the *greatest common factor* of each pair of numbers. Then circle it in each diagram.

Common Factors

In this lesson, you will
- use diagrams to identify common factors.
- find greatest common factors.

Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP5 Use Appropriate Tools Strategically:** Oftentimes there are different strategies or methods for solving a problem. A mathematically proficient student is able to assess which method will be more efficient and why. The listing method is more efficient for the problem GCF(16, 20) while the prime factorization method is more efficient for the problem GCF(120, 32, 48).

Motivate

- Before class, make 10 index cards where 5 of them have a red square drawn, 3 have a green circle, and 2 have a red circle. Distribute the cards to 10 volunteers.
- Place two loops of yarn on the floor in the front of the room. Begin each task with students outside of the loops.
- Task 1: Ask the students holding a square to stand in one loop and students with a circle to stand in the other loop.
- Task 2: Ask the students holding a red shape to stand in one loop and students with a green shape to stand in the other loop.
- Task 3: Ask the students holding a square to stand in one loop and students with a red shape to stand in the other loop.
- For the third task, students should see that the loops must intersect. Ask where each set of students is standing, including the green circles.

Discuss

- Discuss the definition of Venn diagrams and relate them to the Motivate activity just completed. Venn diagrams are named for the English mathematician, John Venn (1834–1923), who first used the circles to show relationships between different sets of elements.

Activity Notes

Activity 1

? "What are the factors of 24?" 1, 2, 3, 4, 6, 8, 12, 24

- Introduce the activity. Have students work in pairs while you circulate to see that they are recording the factors correctly.
- Remind students of the divisibility rules that will help them determine the factors of the given numbers.

? "Were there any problems that did not have any factors in common? Explain." No, they all have at least the number 1 in common.

- **FYI:** Two numbers that have only 1 as a common factor are called *relatively prime*.
- **Connection:** Writing the factors in a Venn diagram is similar to the list method of finding the greatest common factor. The Venn diagram method allows students to see that there can be several common factors, but only one greatest common factor.

Common Core State Standards

6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 . . ..

6.EE.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); . . .

Extending the Standards

The GCF standard specifies numbers from 1–100, so finding common factors could be handled solely by listing the factors. If you wish, the examples and exercises in this text can be worked this way. However, we chose to show the prime factorization method also.

Previous Learning

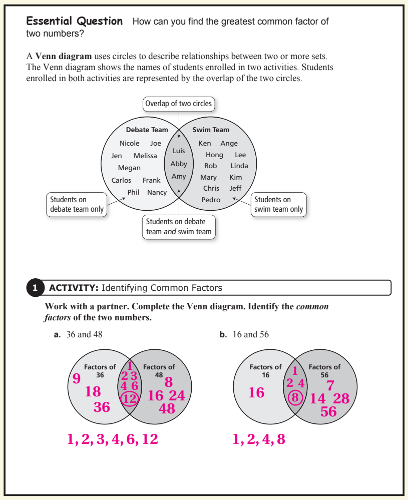
Students need to be familiar with finding factors of a number and finding the prime factorization of a number.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

1.5 Record and Practice Journal



Essential Question How can you find the greatest common factor of two numbers?

A **Venn diagram** uses circles to describe relationships between two or more sets. The Venn diagram shows the names of students enrolled in two activities. Students enrolled in both activities are represented by the overlap of the two circles.

1 ACTIVITY: Identifying Common Factors

Work with a partner. Complete the Venn diagram. Identify the *common factors* of the two numbers.

a. 36 and 48

1, 2, 3, 4, 6, 12

b. 16 and 56

1, 2, 4, 8

Differentiated Instruction

Visual

Show students the following method for finding the GCF of 18, 24, and 42.

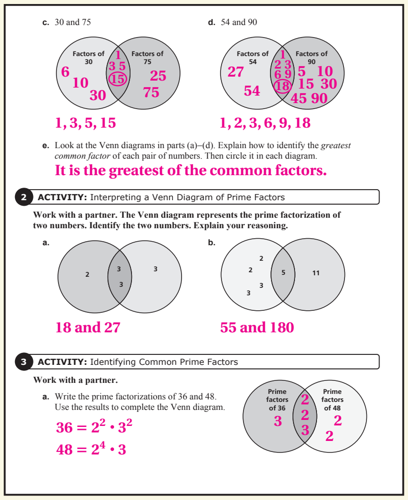
1. List the factors of each number in ascending order from left to right.
2. Look at the least number in the set, in this case 18.
3. Decide whether the greatest factor, 18, is the GCF. If not, then cross it out.
4. Look at the factor to the left, 9. Decide whether this factor is the GCF. If not, then cross it out.
5. Continue working right to left, until you find the GCF, which in this case is 6.

Factors of 18: 1, 2, 3, 6, ~~9~~, ~~18~~

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Factors of 42: 1, 2, 3, 6, 7, 14, 21, 28

1.5 Record and Practice Journal



c. 30 and 75

1, 3, 5, 15

d. 54 and 90

1, 2, 3, 6, 9, 18

e. Look at the Venn diagrams in parts (a)–(d). Explain how to identify the *greatest common factor* of each pair of numbers. Then circle it in each diagram.

It is the greatest of the common factors.

② **ACTIVITY:** Interpreting a Venn Diagram of Prime Factors

Work with a partner. The Venn diagram represents the prime factorization of two numbers. Identify the two numbers. Explain your reasoning.

a.

18 and 27

b.

55 and 180

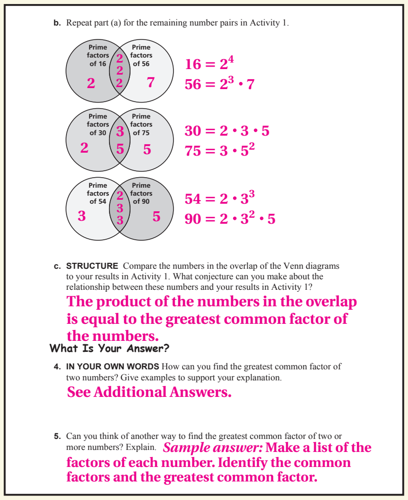
③ **ACTIVITY:** Identifying Common Prime Factors

Work with a partner.

a. Write the prime factorizations of 36 and 48. Use the results to complete the Venn diagram.

$36 = 2^2 \cdot 3^2$

$48 = 2^4 \cdot 3$



b. Repeat part (a) for the remaining number pairs in Activity 1.

$16 = 2^4$

$56 = 2^3 \cdot 7$

$30 = 2 \cdot 3 \cdot 5$

$75 = 3 \cdot 5^2$

$54 = 2 \cdot 3^3$

$90 = 2 \cdot 3^2 \cdot 5$

c. **STRUCTURE** Compare the numbers in the overlap of the Venn diagrams to your results in Activity 1. What conjecture can you make about the relationship between these numbers and your results in Activity 1?

The product of the numbers in the overlap is equal to the greatest common factor of the numbers.

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you find the greatest common factor of two numbers? Give examples to support your explanation.

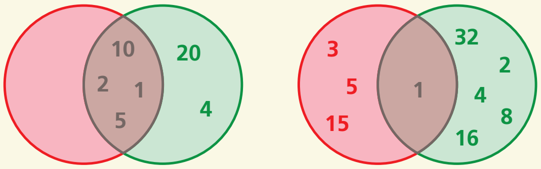
See Additional Answers.

5. Can you think of another way to find the greatest common factor of two or more numbers? Explain. ***Sample answer:*** **Make a list of the factors of each number. Identify the common factors and the greatest common factor.**

Laurie's Notes

Activity 2

- **MP1a Make Sense of Problems** and **MP5:** In this activity, students must make sense of the structure of the Venn diagram and how it is used as a visual tool to represent the factors of two numbers. Each number in the intersection is a factor of both original numbers.
- Ask volunteers to share their reasoning about each problem.
- **Extension:** Draw the Venn diagrams shown and ask students to explain how the two problems are different from those in the activity.



Activity 3

- Discuss the directions with students. Be sure they understand that they are using the prime factorization of each number and *not* the factors of the number. Review the definition of prime factor as needed.
- If students are having difficulty, model the first example as a whole class and then have partners work the remaining three problems.
- **MP7 Look for and Make Use of Structure:** Listen to the students' explanations to part (c). The greatest common factor can be found by determining which prime factors the numbers have in common.

What Is Your Answer?

- For Question 4, you are listening for the product of the prime factors the numbers have in common. Students should also describe the method used in Activity 1.

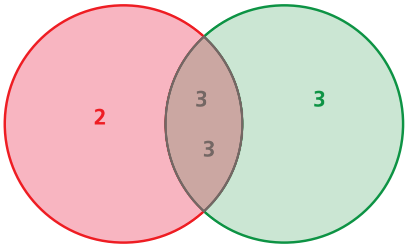
Closure

- **Exit Ticket:** What is the greatest common factor of 16 and 48? 16
  Name two numbers whose greatest common factor is 8. *Sample answer:* 32 and 56

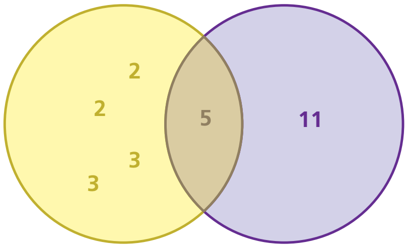
2 ACTIVITY: Interpreting a Venn Diagram of Prime Factors

Work with a partner. The Venn diagram represents the prime factorization of two numbers. Identify the two numbers. Explain your reasoning.

a.



b.



3 ACTIVITY: Identifying Common Prime Factors

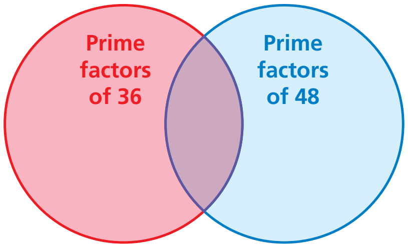
Math Practice

Interpret a Solution

What does the diagram of the resulting prime factorization mean?

Work with a partner.

a. Write the prime factorizations of 36 and 48. Use the results to complete the Venn diagram.



b. Repeat part (a) for the remaining number pairs in Activity 1.

c. **STRUCTURE** Compare the numbers in the overlap of the Venn diagrams to your results in Activity 1. What conjecture can you make about the relationship between these numbers and your results in Activity 1?

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you find the greatest common factor of two numbers? Give examples to support your explanation.

5. Can you think of another way to find the greatest common factor of two numbers? Explain.

Practice Use what you learned about greatest common factors to complete Exercises 4–6 on page 34.

1.5 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Factors that are shared by two or more numbers are called **common factors**. The greatest of the common factors is called the **greatest common factor** (GCF). One way to find the GCF of two or more numbers is by listing factors.

Key Vocabulary
Venn diagram, *p. 30*
common factors, *p. 32*
greatest common factor, *p. 32*

EXAMPLE 1 Finding the GCF Using Lists of Factors

Find the GCF of 24 and 40.

List the factors of each number.

Factors of 24: ①, ②, 3, ④, 6, ⑧, 12, 24

Factors of 40: ①, ②, ④, 5, ⑧, 10, 20, 40

Circle the common factors.

The common factors of 24 and 40 are 1, 2, 4, and 8. The greatest of these common factors is 8.

So, the GCF of 24 and 40 is 8.

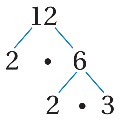
Another way to find the GCF of two or more numbers is by using prime factors. The GCF is the product of the common prime factors of the numbers.

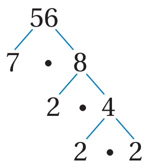
EXAMPLE 2 Finding the GCF Using Prime Factorizations

Study Tip
Examples 1 and 2 show two different methods for finding the GCF. After solving with one method, you can use the other method to check your answer.

Find the GCF of 12 and 56.

Make a factor tree for each number.





Write the prime factorization of each number.

$$\begin{array}{l} 12 = 2 \cdot 2 \cdot 3 \\ 56 = 2 \cdot 2 \cdot 2 \cdot 7 \end{array}$$

Circle the common prime factors.

$$\uparrow \quad \uparrow$$

$$2 \cdot 2 = 4$$

Find the product of the common prime factors.

So, the GCF of 12 and 56 is 4.

On Your Own

Now You're Ready
Exercises 7–18

Find the GCF of the numbers using lists of factors.

1. 8, 36
2. 18, 72
3. 14, 28, 49

Find the GCF of the numbers using prime factorizations.

4. 20, 45
5. 32, 90
6. 45, 75, 120

Multi-Language Glossary at BigIdeasMath.com

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students used Venn diagrams to find common factors of two numbers. (MP1a, MP5, MP7)
- **Today:** Students will learn two methods for finding the greatest common factor of two numbers.

Motivate

? Write LOL on the board and ask, "What does this mean?" Most will say "Laugh Out Loud" or "Laughing Out Loud." There are other interpretations.

- Write a list of common acronyms on the board. Ask students to identify as many as they can. They could work in pairs.

OJ	Orange juice	BLT	Bacon, lettuce, and tomato
YTD	Year to date	PIN	Personal identification number
TBA	To be announced	EST	Eastern Standard Time

? "What does the acronym GCF mean?" greatest common factor

Lesson Notes

Example 1

- The first method of finding the GCF is similar to the first activity yesterday.

? "What are the factors of 24?" 1, 2, 3, 4, 6, 8, 12, 24

? "What are the factors of 40?" 1, 2, 4, 5, 8, 10, 20, 40

? "What factors appear in both lists?" 1, 2, 4, 8

- **FYI:** Students should not just say, "The greatest common factor is 8." The complete answer is, "The greatest common factor *of 12 and 40* is 8."

Example 2

- The second method is similar to the third activity yesterday.

? "What prime factors do 12 and 56 have in common?" two factors of 2

- The greatest common factor of two (or more) numbers is the product of the prime factors that they have in common. The greatest common factor of 12 and 56 is 2 • 2 or 4.
- The solution can be checked by using the first method of listing factors.

? "Does it matter which method you use and why?" no; Each method will give you the correct answer.

? "How do you decide which method to use?" If the numbers are relatively small, use the listing method. If the numbers are relatively large, use the prime factorization method.

On Your Own

- Students should be encouraged to think and reason first. In Question 2, 72 is divisible by 18, so the GCF is 18.
- Ask volunteers to share their work at the board or document camera to check solutions.

Goal Today's lesson is finding the **greatest common factor (GCF)** of two or more numbers.

Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

Find the GCF of 36 and 54 using lists of factors. 18

Extra Example 2

Find the GCF of 40 and 48 using prime factorization. 8

On Your Own

1. 4
2. 18
3. 7
4. 5
5. 2
6. 15

Extra Example 3

Which pair of numbers, 24 and 36, or 36 and 54, has a GCF of 12? 24 and 36

Extra Example 4

You are arranging your collection of DVDs into stacks. You have 16 drama, 32 action, and 40 comedy DVDs. You want all of the stacks to be the same height. What is the greatest number of DVDs you can have in a stack to make them the same height, without any left over? 8

On Your Own

7. *Sample answer:* 10, 20
8. no; The prime factorization of 30 is $2 \cdot 3 \cdot 5$, so the GCF is still $2 \cdot 3 = 6$

English Language Learners

Abbreviations

The abbreviation GCF is pronounced *gee see ef*. Other languages may pronounce the letters differently causing students to become confused. When speaking or writing, use both forms, ". . . GCF, or greatest common factor…," to reinforce the pronunciation and meaning. This is also true with the abbreviations LCM and GCD.

Laurie's Notes

Example 3

- Read the problem.
- **MP2 Reason Abstractly and Quantitatively:** Asking students questions where they need to explain constraints or parameters helps them to focus on and develop their reasoning skills.
- ? "Why can't a greater number be a factor of a lesser number?" The greater number can't divide into the lesser number evenly.
- **Big Idea:** The GCF of two numbers will always be less than or equal to the lesser of the two original numbers.
- Work through the remainder of the problem as shown.

Example 4

- Ask a volunteer to read the problem. Discuss the list of gifts. Discuss what it means to have identical groups of the gifts in each piñata.
- ? "How could you distribute the 18 bottles of nail polish?" Put 1 in 18 different piñatas, put 2 in nine different piñatas, put 3 in six different piñatas, and so on.
- Repeat the question for the earrings and lollipops.
- **MP3a Construct Viable Arguments:** It is important to ask students to justify their answers and communicate their reasoning to others.
- ? "Which method for finding the GCF should be used and why?" Prime factorization, because there are 3 numbers and finding the prime factorization of each is fairly quick.
- **Extension:** Ask what will be in each of the six piñatas.

On Your Own

- **Reasoning:** In Question 7, students are given the GCF and they need to find the two original numbers. Write the various solutions on the board and look for similarities in the answers.

Closure

- **Writing prompt:** To find the GCF of 16 and 28, I would… *Sample answer:* List the factors of each number. Then identify the greatest common factor.

  Factors of 16: ①, ②, ④, 8, 16

  Factors of 28: ①, ②, ④, 7, 14, 28

  The greatest common factor of 16 and 28 is 4.

EXAMPLE ③ Finding Two Numbers with a Given GCF

Which pair of numbers has a GCF of 15?

Ⓐ 10, 15 Ⓑ 30, 60 Ⓒ 21, 45 Ⓓ 45, 75

The number 15 cannot be a factor of the lesser number 10. So, you can eliminate Statement A.

The number 15 cannot be a factor of a number that does not have a 0 or 5 in the ones place. So, you can eliminate Statement C.

List the factors for Statements B and D. Then identify the GCF for each.

Choice B: **Factors of 30:** ①, ②, ③, ⑤, ⑥, ⑩, ⑮, ㉚

Factors of 60: ①, ②, ③, 4, ⑤, ⑥, ⑩, 12, ⑮, 20, ㉚, 60

The GCF of 30 and 60 is 30.

Choice D: **Factors of 45:** ①, ③, ⑤, 9, ⑮, 45

Factors of 75: ①, ③, ⑤, ⑮, 25, 75

The GCF of 45 and 75 is 15.

The correct answer is Ⓓ.

EXAMPLE ④ Real-Life Application

You are filling piñatas for your sister's birthday party. The list shows the gifts you are putting into the piñatas. You want identical groups of gifts in each piñata with no gifts left over. What is the greatest number of piñatas you can make?

∗ 18 bottles of nail polish
∗ 24 pairs of earrings
∗ 42 lollipops

The GCF of the numbers of gifts represents the greatest number of identical groups of gifts you can make with no gifts left over. So, to find the number of piñatas, find the GCF.

$$\begin{aligned} 18 &= 2 \cdot 3 \cdot 3 \\ 24 &= 2 \cdot 3 \cdot 2 \cdot 2 \\ 42 &= 2 \cdot 3 \cdot 7 \end{aligned}$$

$$2 \cdot 3 = 6$$

Find the product of the common prime factors.

STEM VIDEO

The GCF of 18, 24, and 42 is 6.

So, you can make at most 6 piñatas.

On Your Own

Now You're Ready
Exercises 23–25

7. Write a pair of numbers whose greatest common factor is 10.
8. **WHAT IF?** In Example 4, you add 6 more pairs of earrings. Does this change your answer? Explain your reasoning.

1.5 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** What is the greatest common factor (GCF) of two numbers?

2. **WRITING** Describe how to find the GCF of two numbers by using prime factorization.

3. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

What is the greatest common factor of 24 and 32?

What is the greatest common divisor of 24 and 32?

What is the greatest common prime factor of 24 and 32?

What is the product of the common prime factors of 24 and 32?

Practice and Problem Solving

Use a Venn diagram to find the greatest common factor of the numbers.

4. 12, 30
5. 32, 54
6. 24, 108

Find the GCF of the numbers using lists of factors.

① 7. 6, 15
8. 14, 84
9. 45, 76
10. 39, 65
11. 51, 85
12. 40, 63

Find the GCF of the numbers using prime factorizations.

② 13. 45, 60
14. 27, 63
15. 36, 81
16. 72, 84
17. 61, 73
18. 189, 200

ERROR ANALYSIS **Describe and correct the error in finding the GCF.**

19. ✗

$42 = 2 \cdot 3 \cdot 7$

$154 = 2 \cdot 7 \cdot 11$

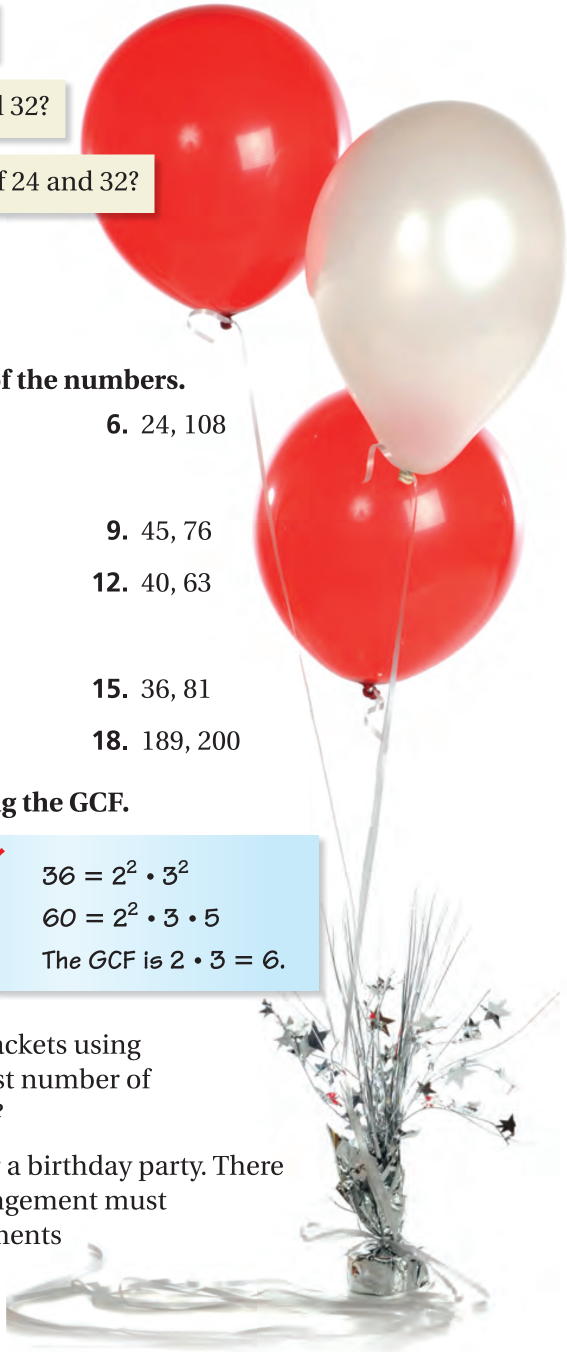
The GCF is 7.

20. ✗

$36 = 2^2 \cdot 3^2$

$60 = 2^2 \cdot 3 \cdot 5$

The GCF is $2 \cdot 3 = 6$.



21. **CLASSROOM** A teacher is making identical activity packets using 92 crayons and 23 sheets of paper. What is the greatest number of packets the teacher can make with no items left over?

22. **BALLOONS** You are making balloon arrangements for a birthday party. There are 16 white balloons and 24 red balloons. Each arrangement must be identical. What is the greatest number of arrangements you can make using every balloon?

Assignment Guide and Homework Check

Level	Day 1 Activity Assignment	Day 2 Lesson Assignment	Homework Check
Basic	4–6, 35–39	1–3, 7–21 odd, 22, 23	7, 11, 15, 21, 23
Average	4–6, 35–39	1–3, 10–15, 20, 22, 23–33 odd	10, 15, 22, 23, 31
Advanced	4–6, 35–39	1–3, 10–34 even	10, 16, 22, 24, 30

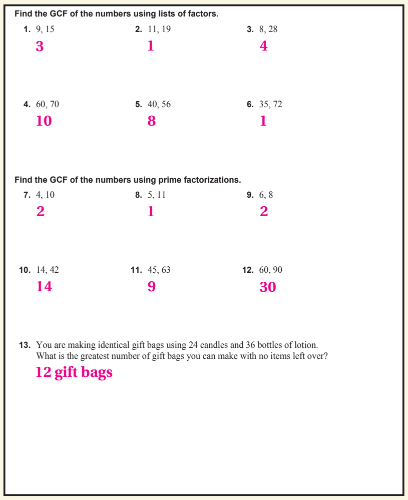
For Your Information

- **Exercise 3** In the *Different Words, Same Question* exercise, three of the four choices pose the same question using different words. The remaining choice poses a different question. So there are two answers.

Common Errors

- **Exercise 3** Students may get confused with the use of the words “divisor” and “factor.” Explain that the greatest common divisor (GCD) is another name for GCF. It is the greatest common factor, that can be divided evenly into the given numbers.
- **Exercises 4–6** Some students may struggle using a Venn diagram. Make sure they understand that the intersection of any (or all) circles can be used to find the GCF of the numbers represented by the circles.
- **Exercises 23–25** Some students may struggle if they use a Venn diagram for three sets. Tell them that the same rules apply for sets of three as with a set of two. Make sure they understand that the intersection of any (or all) circles can be used to find the GCF of the numbers represented by the circles.

1.5 Record and Practice Journal



Find the GCF of the numbers using lists of factors.

1. 9, 15
3

2. 11, 19
1

3. 8, 28
4

4. 60, 70
10

5. 40, 56
8

6. 35, 72
1

Find the GCF of the numbers using prime factorizations.

7. 4, 10
2

8. 5, 11
1

9. 6, 8
2

10. 14, 42
14

11. 45, 63
9

12. 60, 90
30

13. You are making identical gift bags using 24 candles and 36 bottles of lotion. What is the greatest number of gift bags you can make with no items left over?
12 gift bags

Vocabulary and Concept Check

1. The GCF is the greatest factor that is shared by the two numbers.

2. First, find the prime factorization of both numbers. Next, identify common prime factors. Then, find the product of the common prime factors.

3. What is the greatest common prime factor of 24 and 32?; 2; 8

Practice and Problem Solving

4. 6
5. 2
6. 12
7. 3
8. 14
9. 1
10. 13
11. 17
12. 1
13. 15
14. 9
15. 9
16. 12
17. 1
18. 1

19. 7 is the greatest common *prime* factor. The GCF is $2 \cdot 7 = 14$.

20. Not all of the common prime factors are included. The GCF is $2^2 \cdot 3 = 12$.

21. 23 packets

22. 8 arrangements

Practice and Problem Solving

23. 7 **24.** 6

25. 14

26. *Sample answer:* 16, 32, and 48; Multiply 16 by 1, 2, and 3.

27. *Sample answer:* Prime factorization because it is tedious to find all the factors of large numbers.

28. sometimes

29. always

30. never

31. 12; 6 red, 5 pink, and 4 yellow

32. See Additional Answers.

33. **a.** Because 73 is a prime number and the GCF of the three numbers is 1.

b. 18; The GCF of 54 and 36 is 18. 18 divides evenly into 72 leaving one banana left over.

34. See *Taking Math Deeper*.

Fair Game Review

35. Commutative Property of Addition

36. Associative Property of Addition

37. Commutative Property of Multiplication

38. Associative Property of Multiplication

39. B

Mini-Assessment

Find the GCF of the numbers.

1. 8, 20 4

2. 35, 56 7

3. 18, 45 9

4. 12, 54, 84 6

Taking Math Deeper

Exercise 34

This problem is conceptually challenging. *Stop and think* about what information each sentence reveals.

① *One-foot-by-one-foot tiles cover the floor of each room.*

- A whole number of one-foot-by-one-foot tiles covers each floor. There are no partial tiles.
- The number of tiles along the length and the number of tiles along the width are factors of the number of tiles in the room.

It is also helpful to connect the tiles to a room's length, width, and area.

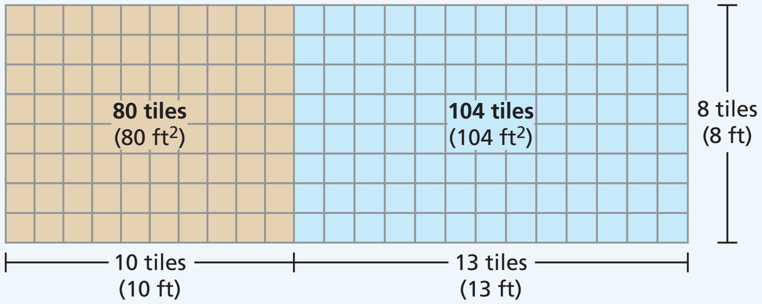
- The numbers of tiles along the length and width of each room represent the length (in feet) and width (in feet) of each room.
- The number of tiles in the room represents the area (in square feet).

② *Describe how the greatest possible length of the adjoining wall is related to the total number of tiles in each room.*

- Each room has the same number of tiles along the adjoining wall. Therefore, the numbers of tiles in the rooms must have a common factor.

So, the **greatest** possible length of the adjoining wall is the **greatest** common factor of the numbers of tiles in the rooms.

③ One possible diagram is shown using two reasonable room sizes of 80 square feet and 104 square feet. The GCF of 80 and 104 is 8, meaning that the greatest possible length of the adjoining wall is 8 feet, which is also reasonable.



Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Start the next section

Find the GCF of the numbers.

④ **23.** 35, 56, 63 **24.** 30, 60, 78 **25.** 42, 70, 84

26. **OPEN-ENDED** Write a set of three numbers that have a GCF of 16. What procedure did you use to find your answer?

27. **REASONING** You need to find the GCF of 256 and 400. Would you rather list their factors or use their prime factorizations? Explain.

CRITICAL THINKING **Tell whether the statement is *always*, *sometimes*, or *never* true.**

28. The GCF of two even numbers is 2.

29. The GCF of two prime numbers is 1.

30. When one number is a multiple of another, the GCF of the numbers is the greater of the numbers.

31. **BOUQUETS** A florist is making identical bouquets using 72 red roses, 60 pink roses, and 48 yellow roses. What is the greatest number of bouquets that the florist can make if no roses are left over? How many of each color are in each bouquet?

32. **VENN DIAGRAM** Consider the numbers 252, 270, and 300.

a. Create a Venn diagram using the prime factors of the numbers.

b. Use the Venn diagram to find the GCF of 252, 270, and 300.

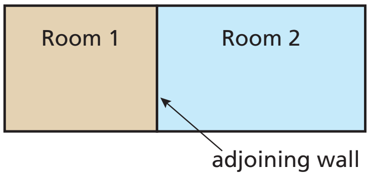
c. What is the GCF of 252 and 270? 252 and 300? Explain how you found your answer.

33. **FRUIT BASKETS** You are making fruit baskets using 54 apples, 36 oranges, and 73 bananas.

a. Explain why you cannot make identical fruit baskets without leftover fruit.

b. What is the greatest number of identical fruit baskets you can make with the least amount of fruit left over? Explain how you found your answer.

34. **Problem Solving** Two rectangular, adjacent rooms share a wall. One-foot-by-one-foot tiles cover the floor of each room. Describe how the greatest possible length of the adjoining wall is related to the total number of tiles in each room. Draw a diagram that represents one possibility.



Fair Game Review What you learned in previous grades & lessons

Tell which property is being illustrated. *(Skills Review Handbook)*

35. $13 + (29 + 7) = 13 + (7 + 29)$ **36.** $13 + (7 + 29) = (13 + 7) + 29$

37. $(6 \times 37) \times 5 = (37 \times 6) \times 5$ **38.** $(37 \times 6) \times 5 = 37 \times (6 \times 5)$

39. **MULTIPLE CHOICE** In what order should you perform the operations in the expression $4 \times 3 - 12 \div 2 + 5$? *(Section 1.3)*

Ⓐ $\times, -, \div, +$ Ⓑ $\times, \div, -, +$ Ⓒ $\times, \div, +, -$ Ⓓ $\times, +, -, \div$

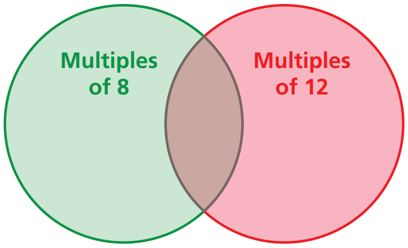
1.6 Least Common Multiple

Essential Question How can you find the least common multiple of two numbers?

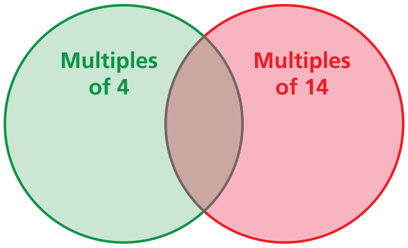
1 ACTIVITY: Identifying Common Multiples

Work with a partner. Using the first several multiples of each number, copy and complete the Venn diagram. Identify any *common multiples* of the two numbers.

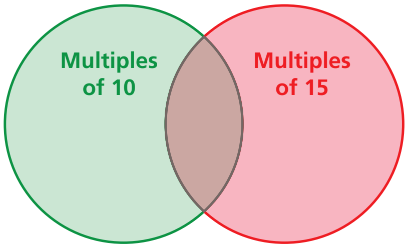
a. 8 and 12



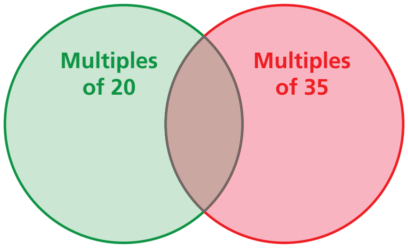
b. 4 and 14



c. 10 and 15



d. 20 and 35



e. Look at the Venn diagrams in parts (a)–(d). Explain how to identify the *least common multiple* of each pair of numbers. Then circle it in each diagram.

Common Multiples

In this lesson, you will
- use diagrams to identify common multiples.
- find least common multiples.

Laurie's Notes

Introduction

Standards for Mathematical Practice

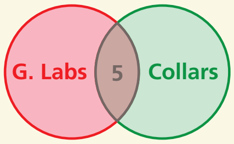
- **MP5 Use Appropriate Tools Strategically:** This section is similar to the last in that students are presented two methods for finding the least common multiple of two numbers. A mathematically proficient student is able to assess which method will be more efficient and why. The listing method is more efficient for the problem LCM(8, 12), while the prime factorization method is more efficient for the problem LCM(24, 32).

For Your Information

- A key usage of multiples in CCSS is recognizing that if a number x is a multiple of two numbers a and b, then x is a multiple of $a \cdot b$. Students will see this in Chapter 3.

Motivate

- Pose this puzzle to students: There are forty dogs. Fourteen are Golden Labs and 29 are wearing collars. If five dogs are Golden Labs and are wearing collars, how many dogs are not Golden Labs and not wearing collars? 2; Golden Labs without collars: 9, With collars and not Golden Labs: 24
- If students are stuck, ask them what tool they have used to represent the relationship between different sets of elements.
- To get students started, you could present the following Venn diagram.



- This problem will help students remember that there is an exterior of the Venn diagram.

Activity Notes

Activity 1

- ❓ "What are the first five multiples of 4?" 4, 8, 12, 16, 20
- Introduce the activity. Students may ask how many multiples they need to list. There is no set answer, however they should list at least the first 5 or 6 multiples.
- Have each student work with his or her partner while you circulate around the room to see that they are recording the multiples correctly.
- Accuracy is important. If students list a multiple incorrectly, all subsequent multiples are likely going to be incorrect.
- ❓ "Were there any problems that had more than one multiple in common? Explain." Yes Students may not have extended their lists enough to see other multiples.

Common Core State Standards

6.NS.4 Find . . . the least common multiple of two whole numbers less than or equal to 12. . . .

Extending the Standards

The LCM standard specifies numbers from 1–12, so finding common multiples could be handled solely by listing the multiples. If you wish, the examples and exercises in this text can be worked this way. However, we chose to show the prime factorization method also.

Previous Learning

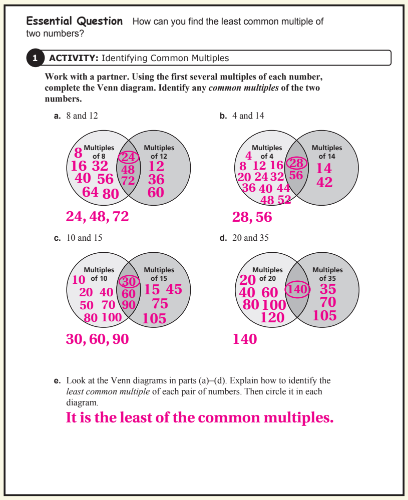
Students need to be familiar with finding multiples of a number and finding the prime factorization of a number.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

1.6 Record and Practice Journal



Essential Question How can you find the least common multiple of two numbers?

1 ACTIVITY: Identifying Common Multiples

Work with a partner. Using the first several multiples of each number, complete the Venn diagram. Identify any *common multiples* of the two numbers.

a. 8 and 12

24, 48, 72

b. 4 and 14

28, 56

c. 10 and 15

30, 60, 90

d. 20 and 35

140

e. Look at the Venn diagrams in parts (a)–(d). Explain how to identify the *least common multiple* of each pair of numbers. Then circle it in each diagram.

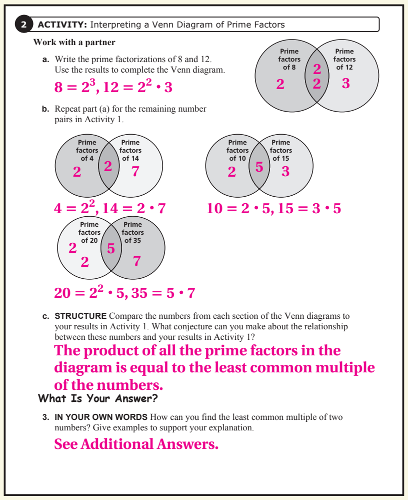
It is the least of the common multiples.

English Language Learners

Vocabulary

In the lesson, be sure that students can distinguish between *common multiple* and *least common multiple*. In the extension, students need to distinguish between *denominator, common denominator,* and *least common denominator.*

1.6 Record and Practice Journal



2 ACTIVITY: Interpreting a Venn Diagram of Prime Factors

Work with a partner

a. Write the prime factorizations of 8 and 12. Use the results to complete the Venn diagram.

$8 = 2^3, 12 = 2^2 \cdot 3$

b. Repeat part (a) for the remaining number pairs in Activity 1.

$4 = 2^2, 14 = 2 \cdot 7$

$10 = 2 \cdot 5, 15 = 3 \cdot 5$

$20 = 2^2 \cdot 5, 35 = 5 \cdot 7$

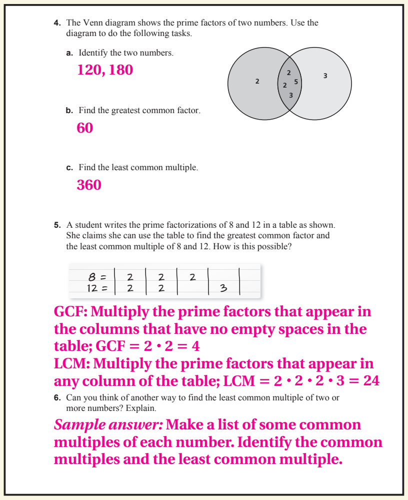
c. **STRUCTURE** Compare the numbers from each section of the Venn diagrams to your results in Activity 1. What conjecture can you make about the relationship between these numbers and your results in Activity 1?

The product of all the prime factors in the diagram is equal to the least common multiple of the numbers.

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you find the least common multiple of two numbers? Give examples to support your explanation.

See Additional Answers.



4. The Venn diagram shows the prime factors of two numbers. Use the diagram to do the following tasks.

a. Identify the two numbers.

120, 180

b. Find the greatest common factor.

60

c. Find the least common multiple.

360

5. A student writes the prime factorizations of 8 and 12 in a table as shown. She claims she can use the table to find the greatest common factor and the least common multiple of 8 and 12. How is this possible?

8 =	2	2	2	
12 =	2	2		3

GCF: Multiply the prime factors that appear in the columns that have no empty spaces in the table; GCF = $2 \cdot 2 = 4$

LCM: Multiply the prime factors that appear in any column of the table; LCM = $2 \cdot 2 \cdot 2 \cdot 3 = 24$

6. Can you think of another way to find the least common multiple of two or more numbers? Explain.

***Sample answer:* Make a list of some common multiples of each number. Identify the common multiples and the least common multiple.**

Laurie's Notes

Activity 2

- Discuss the directions with students. Be sure they understand that they are using the prime factorization of each number as they did in the previous lesson so they should be comfortable with how to proceed.
- **MP7 Look for and Make Use of Structure:** Listen to the students' explanations to part (c). The least common multiple of two numbers can be found by finding the product of their prime factors listed in the Venn diagram.
- **Big Idea:** The Venn diagram is necessary in listing the prime factorization. The factors the two numbers have in common are written only once in the Venn diagram. This is an important connection for students to understand.

? "Think of two numbers that have no prime factors in common. What would the Venn diagram look like for their prime factorizations?" There would be no factors where the two circles intersect. "Does this method still work?" yes

What Is Your Answer?

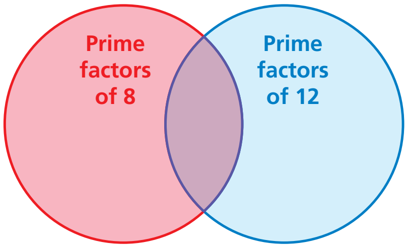
- Question 4 summarizes greatest common factor (GCF) and least common multiple (LCM). Students are often confused by the two concepts. When students have finished, ask a volunteer to share his or her work.

Closure

- **Exit Ticket:** What is the least common multiple of 16 and 20? 80 Name two numbers whose least common multiple is 36. *Sample answer:* 9 and 12

2 ACTIVITY: Interpreting a Venn Diagram of Prime Factors

Work with a partner.



a. Write the prime factorizations of 8 and 12. Use the results to complete the Venn diagram.

b. Repeat part (a) for the remaining number pairs in Activity 1.

c. **STRUCTURE** Compare the numbers from each section of the Venn diagrams to your results in Activity 1. What conjecture can you make about the relationship between these numbers and your results in Activity 1?

What Is Your Answer?

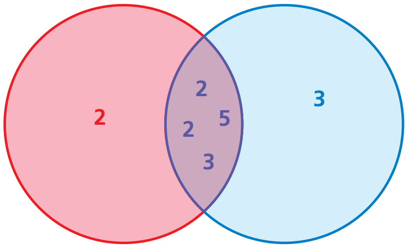
Math Practice

Construct Arguments

How can you use diagrams to support your explanation?

3. **IN YOUR OWN WORDS** How can you find the least common multiple of two numbers? Give examples to support your explanation.

4. The Venn diagram shows the prime factors of two numbers.



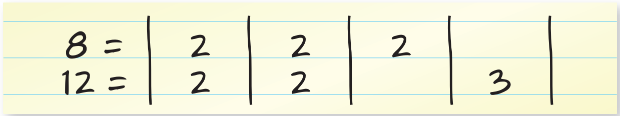
Use the diagram to do the following tasks.

a. Identify the two numbers.

b. Find the greatest common factor.

c. Find the least common multiple.

5. A student writes the prime factorizations of 8 and 12 in a table as shown. She claims she can use the table to find the greatest common factor and the least common multiple of 8 and 12. How is this possible?



8 =	2	2	2	
12 =	2	2		3

6. Can you think of another way to find the least common multiple of two or more numbers? Explain.

Practice Use what you learned about least common multiples to complete Exercises 3–5 on page 40.

1.6 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Multiples that are shared by two or more numbers are called **common multiples**. The least of the common multiples is called the **least common multiple** (LCM). You can find the LCM of two or more numbers by listing multiples or using prime factors.

Key Vocabulary
common multiples, *p. 38*
least common multiple, *p. 38*

EXAMPLE 1 Finding the LCM Using Lists of Multiples

Find the LCM of 4 and 6.

List the multiples of each number.

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, . . .

Multiples of 6: 6, 12, 18, 24, 30, 36, . . .

Circle the common multiples.

Some common multiples of 4 and 6 are 12, 24, and 36. The least of these common multiples is 12.

So, the LCM of 4 and 6 is 12.

On Your Own

Now You're Ready
Exercises 6–11

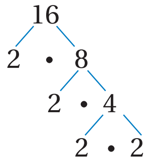
Find the LCM of the numbers using lists of multiples.

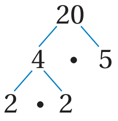
1. 3, 8
2. 9, 12
3. 6, 10

EXAMPLE 2 Finding the LCM Using Prime Factorizations

Find the LCM of 16 and 20.

Make a factor tree for each number.





Write the prime factorization of each number. Circle each different factor where it appears the greater number of times.

$16 = 2 \cdot 2 \cdot 2 \cdot 2$ 2 appears more often here, so circle all 2s.

$20 = 2 \cdot 2 \cdot 5$ 5 appears once. Do not circle the 2s again.

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 = 80$ Find the product of the circled factors.

So, the LCM of 16 and 20 is 80.

On Your Own

Now You're Ready
Exercises 12–17

Find the LCM of the numbers using prime factorizations.

4. 14, 18
5. 28, 36
6. 24, 90

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students used Venn diagrams to find common multiples of two numbers. (MP5, MP7)
- **Today:** Students will learn two methods for finding the least common multiple of two numbers.

Motivate

- **Puzzle Time:** A bell rings every 3 minutes, a dog barks every 4 minutes, and a person coughs every 5 minutes. If you just heard all 3 sounds, how long must you wait to hear
  - the bell ring and the dog bark? 12 minutes
  - the bell ring and the person cough? 15 minutes
  - the dog bark and the person cough? 20 minutes
  - all 3 sounds at the same time? 60 minutes
- Explain that there are different methods for answering these questions.

Lesson Notes

Example 1

? "What are the multiples of 4?" 4, 8, 12, 16, 20, 24, 26, 32, 36…

? "What are the multiples of 6?" 6, 12, 18, 24, 30, 36, 42…

? "What multiples appear in both lists?" 12, 24, and 36 "What do you notice about all of the common multiples?" They are the multiples of 12.

- **FYI:** Students should not just say, "The least common multiple is 12." The complete answer is, "The least common multiple *of 4 and 6* is 12."

On Your Own

- **Think-Pair-Share:** Students should read each question independently and then work in pairs to answer the questions. When they have answered the questions, the pair should compare their answers with another group and discuss any discrepancies.

Example 2

- Any factor that appears in either list is used in finding the LCM. This can be confusing. If needed, draw a Venn diagram to discuss this problem.
- The solution can be checked by using the first method of listing multiples.

? "Does it matter which method you use and why?" no; Each method will give you the correct answer.

? "How do you decide which method to use?" If the numbers are relatively small, use the listing method. If the numbers are relatively large, use the prime factorization method.

On Your Own

- Ask volunteers to share their work at the board or document camera to check solutions.

Goal Today's lesson is finding the **least common multiple (LCM)** of two or more numbers.

Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

Find the LCM of 4 and 9. 36

On Your Own

1. 24
2. 36
3. 30

Extra Example 2

Find the LCM of 16 and 24. 48

On Your Own

4. 126
5. 252
6. 360

Extra Example 3

Find the LCM of 3, 8, and 16. 48

On Your Own

7. 40 **8.** 60

9. *Sample answer:* 4, 10, 25

Extra Example 4

An advertising sign changes every 15 seconds. Another advertising sign changes every 25 seconds. Both signs just changed. After how many minutes will both signs change at the same time again? 75 seconds or $1\frac{1}{4}$ minutes

On Your Own

10. 90 seconds or $1\frac{1}{2}$ minutes

Differentiated Instruction

Visual

Show students the following method for finding the LCM of 300 and 45. This method uses exponents in the prime factorization.

$$300 = 2 \times 2 \times 3 \times 5 \times 5$$
$$= 2^2 \times 3 \times 5^2$$
$$45 = 3 \times 3 \times 5$$
$$= 3^2 \times 5$$

The greatest power of 2 is 2^2.
The greatest power of 3 is 3^2.
The greatest power of 5 is 5^2.
So the LCM is

$$2^2 \times 3^2 \times 5^2 = 900.$$

Laurie's Notes

Example 3

- ? "We want to find the LCM for 3 numbers. Which method do you think will be the most efficient and why?" prime factorization; Listing the multiples of 18 is more challenging than listing the multiples of 4 and 15. Writing the prime factorization of all 3 numbers is fairly quick.
- Work through the problem as shown.
- ? "Is it possible for the LCM of two numbers to be one of the numbers? Explain." yes; The greater of the two numbers could be the LCM if the greater number is a multiple of the lesser number.
- **Big Idea:** The LCM of two numbers will always be greater than or equal to the greater of the two original numbers.

On Your Own

- Ask students to explain which method they used to find the LCM and why they chose that method.
- **MP2 Reason Abstractly and Quantitatively** and **MP3a Construct Viable Arguments:** In Question 9, students are given the LCM and they need to find the two original numbers. This is not a trivial problem, so ask students to describe their reasoning process.

Example 4

- This problem is similar to the opening Motivate puzzle. Students should also see the connection between finding the LCM of 3 and 4 and finding the LCM of 30 and 40.
- Ask a volunteer to read the problem.
- ? "What do the multiples represent in this problem?" how many seconds pass before the light changes
- Although 120 seconds is the LCM, in the context of the problem, it is appropriate to convert 120 seconds to 2 minutes.

On Your Own

- **Neighbor Check:** Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.

Closure

- **Writing Prompt:** To find the LCM of 8 and 15, I would… *Sample answer:* Find the prime factorization of each number. Then circle each different factor where it appears the greatest number of times.
  8 = ②×②×②
  15 = ③×⑤
  The least common multiple of 8 and 15 is 2 × 2 × 2 × 3 × 5 = 120.

EXAMPLE 3 Finding the LCM of Three Numbers

Find the LCM of 4, 15, and 18.

Write the prime factorization of each number. Circle each different factor where it appears the greatest number of times.

$4 = \enclose{circle}{2} \cdot \enclose{circle}{2}$	2 appears most often here, so circle both 2s.
$15 = 3 \cdot \enclose{circle}{5}$	5 appears here only, so circle 5.
$18 = 2 \cdot \enclose{circle}{3} \cdot \enclose{circle}{3}$	3 appears most often here, so circle both 3s.
$2 \cdot 2 \cdot 5 \cdot 3 \cdot 3 = 180$	Find the product of the circled factors.

So, the LCM of 4, 15, and 18 is 180.

On Your Own

Now You're Ready
Exercises 22–27

Find the LCM of the numbers.

7. 2, 5, 8

8. 6, 10, 12

9. Write a set of numbers whose least common multiple is 100.

EXAMPLE 4 Real-Life Application

A traffic light changes every 30 seconds. Another traffic light changes every 40 seconds. Both lights just changed. After how many minutes will both lights change at the same time again?

Find the LCM of 30 and 40 by listing multiples of each number. Circle the least common multiple.

Multiples of 30: 30, 60, 90, $\enclose{circle}{120}$, . . .

Multiples of 40: 40, 80, $\enclose{circle}{120}$, 160, . . .

The LCM is 120. So, both lights will change again after 120 seconds.

Because there are 60 seconds in 1 minute, there are $120 \div 60 = 2$ minutes in 120 seconds.

Both lights will change at the same time again after 2 minutes.

On Your Own

10. **WHAT IF?** In Example 4, the traffic light that changes every 40 seconds is adjusted to change every 45 seconds. Both lights just changed. After how many minutes will both lights change at the same time again?

1.6 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** What is the least common multiple (LCM) of two numbers?
2. **WRITING** Describe how to find the LCM of two numbers by using prime factorization.

Practice and Problem Solving

Use a Venn diagram to find the least common multiple of the numbers.

3. 3, 7
4. 6, 8
5. 12, 15

Find the LCM of the numbers using lists of multiples.

① 6. 2, 9
7. 3, 4
8. 8, 9
9. 5, 8
10. 15, 20
11. 12, 18

Find the LCM of the numbers using prime factorizations.

② 12. 9, 21
13. 12, 27
14. 18, 45
15. 22, 33
16. 36, 60
17. 35, 50

18. **ERROR ANALYSIS** Describe and correct the error in finding the LCM.

✗

$6 \times 9 = 54$

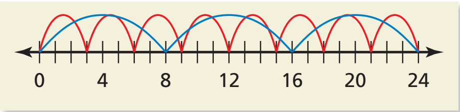
The LCM of 6 and 9 is 54.

19. **AQUATICS** You have diving lessons every fifth day and swimming lessons every third day. Today you have both lessons. In how many days will you have both lessons on the same day again?

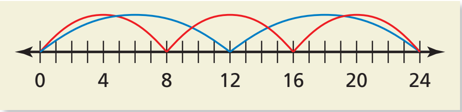
20. **HOT DOGS** Hot dogs come in packs of 10, while buns come in packs of eight. What are the least numbers of packs you should buy in order to have the same numbers of hot dogs and buns?

21. **MODELING** Which model represents an LCM that is different from the other three? Explain your reasoning.

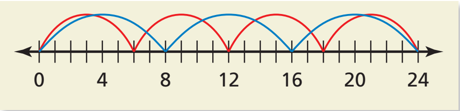
A.



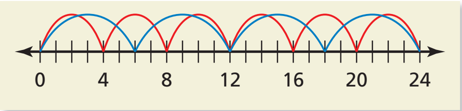
B.



C.



D.



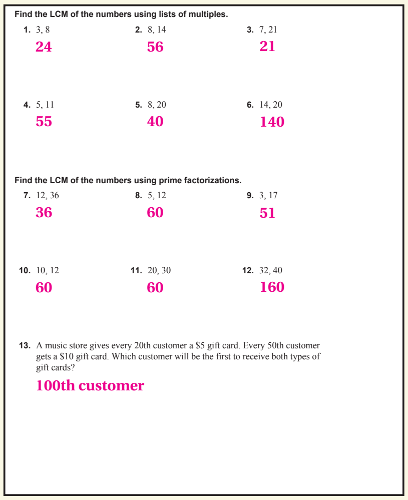
Assignment Guide and Homework Check

Level	Day 1 Activity Assignment	Day 2 Lesson Assignment	Homework Check
Basic	3–5, 37–40	1–2, 7–17 odd, 18, 19–23 odd	9, 15, 19, 23
Average	3–5, 37–40	1–2, 8–20 even, 25–35 odd	10, 14, 20, 25, 33
Advanced	3–5, 37–40	1–2, 10, 14–18 even, 22–32 even, 34–36	10, 16, 22, 26, 34

Common Errors

- **Exercises 3–5** Some students may struggle using a Venn diagram. Make sure they understand that the product of all prime factors in the circles is the LCM of the numbers represented by the circles.
- **Exercises 12–17** After writing the prime factorizations, students may struggle over which factors to use to find the LCM. If needed, students should draw a Venn diagram to clarify the problem.
- **Exercise 21** The number lines may confuse students. Explain to them that the loops of each number will intersect on common multiples.
- **Exercise 35** Some students may struggle if they use a Venn diagram for three sets. Tell them the same rules apply for sets of three as with a set of two. Make sure they understand the product of all prime factors in the circles is the LCM of the numbers represented by the circles.

1.6 Record and Practice Journal



Find the LCM of the numbers using lists of multiples.

1. 3, 8
24

2. 8, 14
56

3. 7, 21
21

4. 5, 11
55

5. 8, 20
40

6. 14, 20
140

Find the LCM of the numbers using prime factorizations.

7. 12, 36
36

8. 5, 12
60

9. 3, 17
51

10. 10, 12
60

11. 20, 30
60

12. 32, 40
160

13. A music store gives every 20th customer a $5 gift card. Every 50th customer gets a $10 gift card. Which customer will be the first to receive both types of gift cards?
100th customer

Vocabulary and Concept Check

1. The LCM of two numbers is the least of the multiples shared by the two numbers.
2. First, find the prime factorization of both numbers. Next, circle each different factor where it appears the greatest number of times. Then, find the product of the circled factors.

Practice and Problem Solving

3. 21
4. 24
5. 60
6. 18
7. 12
8. 72
9. 40
10. 60
11. 36
12. 63
13. 108
14. 90
15. 66
16. 180
17. 350
18. The product of two numbers is not necessarily the LCM. Use prime factorization to see that the LCM is $2 \cdot 3 \cdot 3 = 18$.
19. 15 days
20. 4 packs of hot dogs and 5 packs of buns
21. D; This model represents multiples of 4 and 6 which have an LCM of 12. The other models represent multiples of 3 and 8, 8 and 12, and 6 and 8, which have an LCM of 24.

Practice and Problem Solving

22. 42 **23.** 165

24. 36 **25.** 120

26. 126 **27.** 1260

28. *Sample answer:* Prime factorization because it is tedious to list all of the multiples of two numbers that do not have any common factors.

29. always

30. sometimes

31. never

32. See *Taking Math Deeper*.

33. 300th caller

34. you: 7 mi; your friend: 6 mi

35. **a.** See Additional Answers.

b. 240

c. 80;120

36. The LCM of the two numbers is equal to their product when the two numbers have no common prime factors.

Fair Game Review

37. 3^2 **38.** 5^4

39. 17^5 **40.** B

Mini-Assessment

Find the LCM of the numbers.

1. 15, 18 90

2. 8, 14 56

3. 24, 32 96

4. 5, 8, 10 40

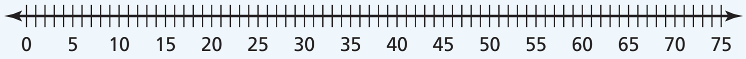
5. 6, 8, 12 48

Taking Math Deeper

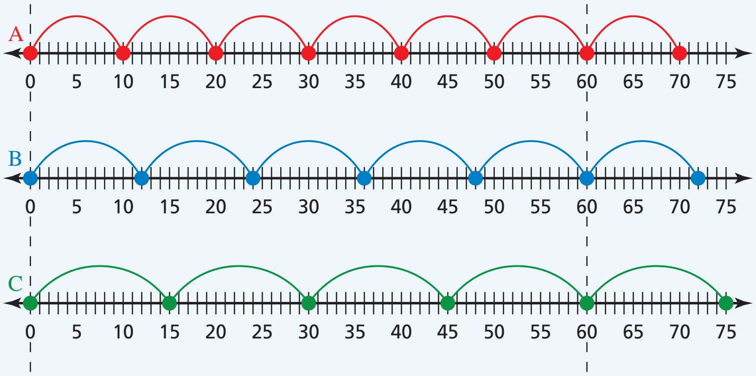
Exercise 32

One way to solve this problem is to represent the arrival times visually using number lines.

① Create three number lines that you can use to show the arrival times of the three subway lines. Make each number line large enough to show several arrivals for each subway line.



② The three subway lines just arrived at the same time. Let zero represent this time on each number line. Show the arrival times of the subway lines on the number lines.



③ After 0, the next value plotted on all three number lines is 60. So, you must wait 60 minutes until all three subway lines arrive at the same time again.

Project

Research the subway system in New York City. Choose a major station and write a real-life problem involving arrival times of subway lines.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Start the next section

Find the LCM of the numbers.

③ **22.** 2, 3, 7

23. 3, 5, 11

24. 4, 9, 12

25. 6, 8, 15

26. 7, 18, 21

27. 9, 10, 28

28. **REASONING** You need to find the LCM of 13 and 14. Would you rather list their multiples or use their prime factorizations? Explain.

CRITICAL THINKING **Tell whether the statement is *always*, *sometimes*, or *never* true.**

29. The LCM of two different prime numbers is their product.

30. The LCM of a set of numbers is equal to one of the numbers in the set.

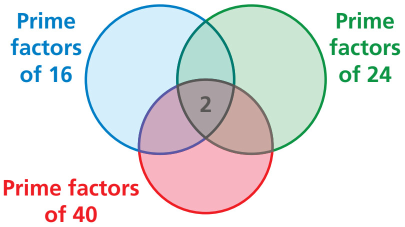
31. The GCF of two different numbers is the LCM of the numbers.

32. **SUBWAY** At Union Station, you notice that three subway lines just arrived at the same time. The table shows their arrival schedule. How long must you wait until all three lines arrive at Union Station at the same time again?

Subway Line	Arrival Time
A	every 10 min
B	every 12 min
C	every 15 min

33. **RADIO CONTEST** A radio station gives away \$15 to every 15th caller, \$25 to every 25th caller, and free concert tickets to every 100th caller. When will the station first give away *all* three prizes to one caller?

34. **TREADMILL** You and a friend are running on treadmills. You run 0.5 mile every 3 minutes, and your friend runs 2 miles every 14 minutes. You both start and stop running at the same time and run a whole number of miles. What is the least possible number of miles you and your friend can run?



35. **VENN DIAGRAM** Refer to the Venn diagram.

a. Copy and complete the Venn diagram.

b. What is the LCM of 16, 24, and 40?

c. What is the LCM of 16 and 40? 24 and 40?

36. Number Sense When is the LCM of two numbers equal to their product?

Fair Game Review What you learned in previous grades & lessons

Write the product as a power. *(Section 1.2)*

37. 3×3

38. $5 \cdot 5 \cdot 5 \cdot 5$

39. $17 \times 17 \times 17 \times 17 \times 17$

40. **MULTIPLE CHOICE** Which two powers have the same value? *(Section 1.2)*

Ⓐ 1^3 and 3^1

Ⓑ 2^4 and 4^2

Ⓒ 3^2 and 2^3

Ⓓ 4^3 and 3^4

Extension 1.6 Adding and Subtracting Fractions

Check It Out
Lesson Tutorials
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Key Vocabulary

least common denominator, *p. 42*

Recall that you can add and subtract fractions with unlike denominators by writing equivalent fractions with a common denominator. One way to do this is by multiplying the numerator and the denominator of each fraction by the denominator of the other fraction.

EXAMPLE 1 Adding Fractions Using a Common Denominator

Find $\frac{5}{8} + \frac{1}{6}$.

Rewrite the fractions with a common denominator. Use the product of the denominators as the common denominator.

$$\frac{5}{8} + \frac{1}{6} = \frac{5 \cdot 6}{8 \cdot 6} + \frac{1 \cdot 8}{6 \cdot 8}$$ Rewrite the fractions using a common denominator of $8 \cdot 6 = 48$.

$$= \frac{30}{48} + \frac{8}{48}$$ Multiply.

$$= \frac{38}{48}$$ Add the numerators.

$$= \frac{\overset{1}{\cancel{2}} \cdot 19}{\underset{1}{\cancel{2}} \cdot 24}$$ Divide out the common factor 2.

$$= \frac{19}{24}$$ Simplify.

Study Tip

A fraction is in *simplest form* when the numerator and the denominator have no common factors other than 1.

The **least common denominator** (LCD) of two or more fractions is the least common multiple (LCM) of the denominators. The LCD provides another method for adding and subtracting fractions with unlike denominators.

EXAMPLE 2 Adding Fractions Using the LCD

Find $\frac{5}{8} + \frac{1}{6}$.

Find the LCM of the denominators.

Multiples of 8: 8, 16, ㉔, 32, 40, ㊽, . . .

Multiples of 6: 6, 12, 18, ㉔, 30, 36, 42, ㊽, . . .

The LCM of 8 and 6 is 24. So, the LCD is 24.

Common Multiples

In this extension, you will

- use least common multiples to add and subtract fractions.

$$\frac{5}{8} + \frac{1}{6} = \frac{5 \cdot 3}{8 \cdot 3} + \frac{1 \cdot 4}{6 \cdot 4}$$ Rewrite the fractions using the LCD, 24.

$$= \frac{15}{24} + \frac{4}{24}$$ Multiply.

$$= \frac{19}{24}$$ Add the numerators.

Laurie's Notes

Introduction

Connect

- **Yesterday:** Students learned two methods for finding the least common multiple of two numbers. (MP2, MP3a)
- **Today:** Students will use the least common multiple as another possible method to add and subtract fractions with unlike denominators.

Motivate

? "Have you heard the phrase, 'You can't add apples and oranges'?"

- Ask them what they think the phrase means. They might talk about needing a common denominator.

Lesson Notes

Example 1

- Similar to what students have seen in prior grades, one way to get a common denominator is to use the product of the two denominators.
- **MP2 Reason Abstractly and Quantitatively:** Quantitative reasoning involves thinking about the units involved (eighths and sixths) and their meaning. Not just how to compute with them.

? "Can you add $\frac{5}{8} + \frac{1}{6}$ and get $\frac{6}{14}$? Explain." no; $\frac{6}{14}$ is less than $\frac{1}{2}$. Because $\frac{5}{8}$ is greater than $\frac{1}{2}$ and you are adding a positive amount to it, the sum must be greater than $\frac{1}{2}$.

? "What are equivalent fractions?" fractions that are equal but do not have the same denominator

- Explain that you want to rewrite $\frac{5}{8}$ and $\frac{1}{6}$ as equivalent fractions with a common denominator. The common denominator will be 8 • 6.
- In the first step of the problem, I use an alternate color to emphasize that when you multiply the numerator and denominator by the same number you're just multiplying by 1, so the value of the fraction is unchanged.

? "The sum is $\frac{38}{48}$. Can this fraction be simplified?" yes; by a factor of 2

Example 2

- Define least common denominator (LCD). Explain that using the LCD is another method to add and subtract fractions.
- This example uses the same two fractions as Example 1.

? "What is the least common multiple of 8 and 6?" 24

- Explain that to add the fractions, a common denominator of 24 is used.
- Work through the problem. The sum is the same as Example 1, and it is already in simplest form.
- Discuss the two strategies for finding equivalent fractions: the product of the denominators versus the least common multiple of the denominators.

? "Can you think of a problem where the product of the denominators and the least common multiple of the denominators would be the same?" Hopefully, students will understand that if the denominators have no common factors (relatively prime), the two methods are the same.

Common Core State Standards

6.NS.4 Find . . . the least common multiple of two whole numbers less than or equal to 12. . . .

Goal Today's lesson is using the least common multiple to add and subtract fractions with unlike denominators.

Technology for the Teacher

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

Find $\frac{1}{3} + \frac{1}{4}$. $\frac{7}{12}$

Extra Example 2

Find $\frac{1}{6} + \frac{5}{9}$. $\frac{13}{18}$

Record and Practice Journal Extension 1.6 Practice

1. $\frac{25}{30}, \frac{9}{30}$
2. $\frac{20}{36}, \frac{33}{36}$
3. $>$
4. $<$
5. $=$
6. $<$
7. $1\frac{1}{12}$
8. $\frac{7}{8}$
9. $3\frac{27}{28}$
10. $6\frac{3}{10}$
11. $\frac{1}{4}$
12. $\frac{23}{60}$
13. $4\frac{17}{28}$
14. $\frac{4}{9}$
15. $\frac{1}{12}$ c
16. $2\frac{5}{12}$ lb

Extra Example 3

Find $5\frac{2}{3} + 2\frac{1}{4}$. $7\frac{11}{12}$

Practice

1. $\frac{4}{24}, \frac{9}{24}$.
2. $\frac{40}{70}, \frac{21}{70}$
3. $\frac{15}{36}, \frac{8}{36}$
4. $\frac{30}{40}, \frac{25}{40}, \frac{4}{40}$
5. $<$
6. $<$
7. $=$
8. $>$
9. $1\frac{5}{12}$
10. $1\frac{5}{14}$
11. $\frac{17}{60}$
12. $\frac{7}{72}$
13. $5\frac{11}{18}$
14. $5\frac{23}{80}$
15. $1\frac{1}{12}$
16. $1\frac{10}{33}$
17. *Sample answer:* The LCD method uses numbers that are easier to work with, but there is extra work in finding the LCD. Using the other method, there are no preliminary steps for finding the LCD, but there may be more simplifying in the solution.

Mini-Assessment

1. Find $\frac{1}{4} + \frac{3}{8}$ using a common denominator. $\frac{5}{8}$
2. Find $\frac{3}{4} + \frac{1}{8}$ using the LCD. $\frac{7}{8}$
3. Find $5\frac{3}{5} - 1\frac{1}{7}$. $4\frac{16}{35}$

Laurie's Notes

Example 3

- Write the problem. Ask students to estimate an answer and explain their reasoning. A reasonable estimate is about $2\frac{1}{2}$.

? "How do you write $4\frac{3}{4}$ as an improper fraction?" Listen for an explanation that gives $\frac{19}{4}$ as an answer.

? "How do you write $2\frac{3}{10}$ as an improper fraction?" Listen for an explanation that gives $\frac{23}{10}$ as an answer.

- Explain that the first method uses the product of the two denominators as the common denominator. Work through the example as shown.
- Explain that the second method uses the least common multiple of the two denominators as the common denominator. Work through the example as shown.
- Take time to show students that instead of using improper fractions, the problem can be done by just renaming the fractional parts, and then subtracting the number parts and fraction parts.
- When asked which method they prefer, students will often say the second method because the numbers they work with are smaller. There is usually at least one student, however, who prefers the first method because they can skip the step of finding the LCD if it is not obvious.
- Discuss the Study Tip. Students should be familiar with $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$ from Grade 5. (CCSS 5.NF.1) Explain the symbol ± if students are unfamiliar with it. The variables used to generalize the procedure from prior grades should be familiar to students.

Practice

- **Neighbor Check:** Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.

Closure

- **Exit Ticket:** Find $3\frac{1}{6} + 4\frac{2}{3}$ using each method. $7\frac{5}{6}$

To add or subtract mixed numbers, first rewrite the numbers as improper fractions. Then find the common denominator.

EXAMPLE 3 Subtracting Mixed Numbers

Find $4\frac{3}{4} - 2\frac{3}{10}$.

Write the difference using improper fractions.

$$4\frac{3}{4} - 2\frac{3}{10} = \frac{19}{4} - \frac{23}{10}$$

Method 1: Use the product of the denominators as the common denominator.

$$\begin{aligned} \frac{19}{4} - \frac{23}{10} &= \frac{19 \cdot 10}{4 \cdot 10} - \frac{23 \cdot 4}{10 \cdot 4} && \text{Rewrite the fractions using a common denominator of } 4 \cdot 10 = 40. \\ &= \frac{190}{40} - \frac{92}{40} && \text{Multiply.} \\ &= \frac{98}{40} && \text{Subtract the numerators.} \\ &= \frac{49}{20}, \text{ or } 2\frac{9}{20} && \text{Simplify.} \end{aligned}$$

Method 2: Use the LCD. The LCM of 4 and 10 is 20.

$$\begin{aligned} \frac{19}{4} - \frac{23}{10} &= \frac{19 \cdot 5}{4 \cdot 5} - \frac{23 \cdot 2}{10 \cdot 2} && \text{Rewrite the fractions using the LCD, 20.} \\ &= \frac{95}{20} - \frac{46}{20} && \text{Multiply.} \\ &= \frac{49}{20}, \text{ or } 2\frac{9}{20} && \text{Simplify.} \end{aligned}$$

Study Tip

Notice that Method 1 uses the same procedure shown in Example 1. You can generalize the procedure using the rule

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}.$$

Practice

Use the LCD to rewrite the fractions with the same denominator.

1. $\frac{1}{6}, \frac{3}{8}$

2. $\frac{4}{7}, \frac{3}{10}$

3. $\frac{5}{12}, \frac{2}{9}$

4. $\frac{3}{4}, \frac{5}{8}, \frac{1}{10}$

Copy and complete the statement using <, >, or =.

5. $\frac{4}{5} \; \square \; \frac{5}{6}$

6. $\frac{5}{14} \; \square \; \frac{3}{8}$

7. $2\frac{2}{5} \; \square \; \frac{24}{10}$

8. $4\frac{9}{25} \; \square \; 4\frac{7}{20}$

Add or subtract. Write the answer in simplest form.

9. $\frac{2}{3} + \frac{3}{4}$

10. $\frac{6}{7} + \frac{1}{2}$

11. $\frac{7}{10} - \frac{5}{12}$

12. $\frac{13}{18} - \frac{5}{8}$

13. $2\frac{1}{6} + 3\frac{4}{9}$

14. $4\frac{3}{16} + 1\frac{1}{10}$

15. $1\frac{5}{6} - \frac{3}{4}$

16. $3\frac{2}{3} - 2\frac{4}{11}$

17. **COMPARING METHODS** List some advantages and disadvantages of each method shown in the examples. Which method do you prefer? Why?

1.4–1.6 Quiz

Check It Out
Progress Check
BigIdeasMath.com

List the factor pairs of the number. *(Section 1.4)*

1. 48

2. 56

Write the prime factorization of the number. *(Section 1.4)*

3. 60

4. 72

Find the GCF of the numbers using lists of factors. *(Section 1.5)*

5. 18, 42

6. 24, 44, 52

Find the GCF of the numbers using prime factorizations. *(Section 1.5)*

7. 38, 68

8. 68, 76, 92

Find the LCM of the numbers using lists of multiples. *(Section 1.6)*

9. 8, 14

10. 3, 6, 16

Find the LCM of the numbers using prime factorizations. *(Section 1.6)*

11. 18, 30

12. 6, 24, 32

Add or subtract. Write the answer in simplest form. *(Section 1.6)*

13. $\frac{3}{5} + \frac{2}{3}$

14. $\frac{7}{8} - \frac{3}{4}$

15. **PICNIC BASKETS** You are creating identical picnic baskets using 30 sandwiches and 42 cookies. What is the greatest number of baskets that you can fill using all of the food? *(Section 1.5)*

16. **RIBBON** You have 52 inches of yellow ribbon and 64 inches of red ribbon. You want to cut the ribbons into pieces of equal length with no leftovers. What is the greatest length of the pieces that you can make? *(Section 1.5)*

17. **MUSIC LESSONS** You have piano lessons every fourth day and guitar lessons every sixth day. Today you have both lessons. In how many days will you have both lessons on the same day again? Explain. *(Section 1.6)*

18. **HAMBURGERS** Hamburgers come in packs of 20, while buns come in packs of 12. What is the least number of packs you should buy in order to have the same numbers of hamburgers and buns? *(Section 1.6)*

Alternative Assessment Options

Math Chat	Student Reflective Focus Question
Structured Interview	Writing Prompt

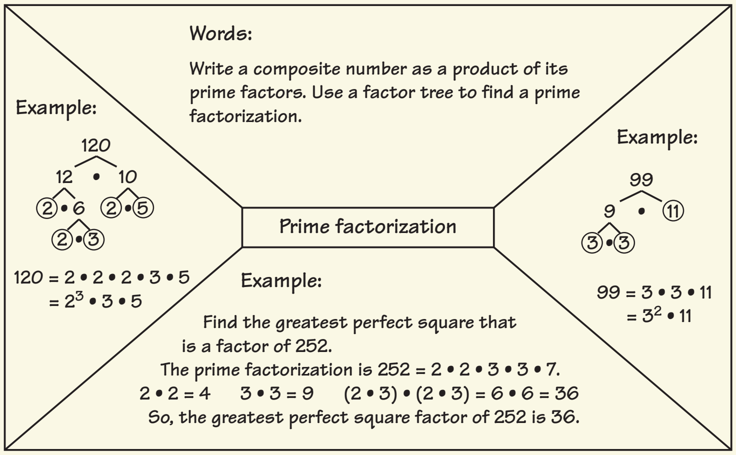
Math Chat

- Work in groups of four. Two members work together to discuss prime factorization and finding the greatest common factor. The other pair work together to discuss prime factorization and finding the least common multiple. When they are finished, they explain their findings to the other members of their group.
- The teacher should walk around the classroom listening to the groups and asking questions to ensure understanding.

Study Help Sample Answers

Remind students to complete Graphic Organizers for the rest of the chapter.

6.



7–9. Available at *BigIdeasMath.com.*

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Lesson Tutorials *BigIdeasMath.com*	Resources by Chapter • Enrichment and Extension • Technology Connection Game Closet at *BigIdeasMath.com* Start the Chapter Review

Answers

1. 1, 48; 2, 24; 3, 16; 4, 12; 6, 8

2. 1, 56; 2, 28; 4, 14; 7, 8

3. $2^2 \times 3 \times 5$

4. $2^3 \times 3^2$

5. 6

6. 4

7. 2

8. 4

9. 56

10. 48

11. 90

12. 96

13. $1\frac{4}{15}$

14. $\frac{1}{8}$

15. 6 baskets

16. 4 in.

17. 12 days; The LCM of 4 and 6 is 12. So, you will have both lessons on the same day in 12 days.

18. 3 packs of hamburgers and 5 packs of buns

Technology for the Teacher

Online Assessment
Assessment Book
ExamView® Assessment Suite

For the Teacher
Additional Review Options

- *BigIdeasMath.com*
- Online Assessment
- Game Closet at *BigIdeasMath.com*
- Vocabulary Help
- Resources by Chapter

Answers

1. 7281
2. 5156
3. 7296
4. $6\frac{30}{31}$
5. 343
6. 64
7. 256

Review of Common Errors

Exercises 1, 2

- Students may set up the problem incorrectly using the vertical method of adding or subtracting. Remind them to line up the numbers by place value, then add or subtract.

Exercise 3

- Students may incorrectly line up the multiplication of the tens digit. Remind them that the right-most digit should line up in the tens-digit place.

Exercises 5–7

- Students may make the mistake of writing the exponent as a factor. Again, remind them that the exponent is the number of times the base is used as a factor. You may want to demonstrate this point with a quick example. For instance, point out that $5^2 = 5 \times 5 = 25$ but $5 \times 2 = 10$.

Exercises 14–16

- Students may think that smaller numbers have fewer factors. Remind students to keep finding factors until all factors are prime numbers.

Exercises 26–28

- After writing the prime factorizations, students may struggle over which factors to use to find the LCM. If needed, students should draw a Venn diagram to clarify the problem.

1 Chapter Review

Check It Out
Vocabulary Help
BigIdeasMath.com

Review Key Vocabulary

power, *p. 12*
base, *p. 12*
exponent, *p. 12*
perfect square, *p. 13*
numerical expression, *p. 18*
evaluate, *p. 18*
order of operations, *p. 18*
factor pair, *p. 26*
prime factorization, *p. 26*
factor tree, *p. 26*
Venn diagram, *p. 30*
common factors, *p. 32*
greatest common factor (GCF), *p. 32*
common multiples, *p. 38*
least common multiple (LCM), *p. 38*
least common denominator (LCD), *p. 42*

Review Examples and Exercises

1.1 Whole Number Operations *(pp. 2–9)*

Use the tens place because 203 is less than 508.

$$\begin{array}{r} 2 \\ 203\overline{)\,5081} \\ -\,406 \\ \hline 102 \end{array}$$

Divide 508 by 203: There are two groups of 203 in 508.
Multiply 2 and 203.
Subtract 406 from 508.

Next, bring down the 1 and divide the ones.

$$\begin{array}{r} 25 \text{ R}6 \\ 203\overline{)\,5081} \\ -\,406\downarrow \\ \hline 1021 \\ -\,1015 \\ \hline 6 \end{array}$$

Divide 1021 by 203: There are five groups of 203 in 1021.
Multiply 5 and 203.
Subtract 1015 from 1021.

The quotient of 5081 and 203 is $25\frac{6}{203}$.

Exercises

Find the value of the expression. Use estimation to check your answer.

1. $4382 + 2899$
2. $8724 - 3568$
3. 192×38
4. $216 \div 31$

1.2 Powers and Exponents *(pp. 10–15)*

Evaluate 6^2.

$6^2 = 6 \cdot 6 = 36$ Write as repeated multiplication and simplify.

Exercises

Find the value of the power.

5. 7^3
6. 2^6
7. 4^4

1.3 Order of Operations (pp. 16–21)

Evaluate $4^3 - 15 \div 5$.

$4^3 - 15 \div 5 = 64 - 15 \div 5$ Evaluate 4^3.

$= 64 - 3$ Divide 15 by 5.

$= 61$ Subtract 3 from 64.

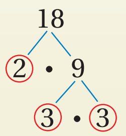
Exercises

Evaluate the expression.

8. $3 \times 6 - 12 \div 6$ **9.** $20 \times (3^2 - 4) \div 50$ **10.** $5 + (4^2 + 2) \div 6$

1.4 Prime Factorization (pp. 24–29)

Write the prime factorization of 18.



Find a factor pair and draw "branches."

Circle the prime factors as you find them.

Continue until each branch ends at a prime factor.

∴ The prime factorization of 18 is 2 • 3 • 3, or $2 \cdot 3^2$.

Exercises

List the factor pairs of the number.

11. 28 **12.** 44 **13.** 63

Write the prime factorization of the number.

14. 42 **15.** 50 **16.** 66

1.5 Greatest Common Factor (pp. 30–35)

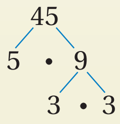
a. Find the GCF of 32 and 76.

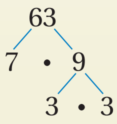
Factors of 32: ①,②,④, 8, 16, 32
Factors of 76: ①,②,④, 19, 38, 76

The greatest of the common factors is 4.

∴ So, the GCF of 32 and 76 is 4.

b. Find the GCF of 45 and 63.





45 = 3 • 3 • 5

63 = 3 • 3 • 7

3 • 3 = 9

∴ So, the GCF of 45 and 63 is 9.

Review Game

Numerical Expressions and Factors

Materials

- copy of game
- pencil
- paper

Players: 2

Directions

- Play in pairs. There are five rounds, each with four parts.
- In each round, players take turns choosing a part and determining the answer. Each part may be used only once.
- Players add their answers in the round. The winner of the round is determined by the *Who Wins?* statement of the round.
- The winner of the round earns one point.

Who Wins? The player with the most points after the five rounds.

Round 1: Whole Number Operations
Direction Simplify the expressions.
Who Wins? The player with the greatest total wins.

A. 195 + 217 **B.** 569 − 497 **C.** 12 × 36 **D.** 583 ÷ 11

Round 2: Powers and Exponents
Direction Simplify the expressions.
Who Wins? The player with the least total wins.

E. 3^5 **F.** 5^3 **G.** 4^4 **H.** 6^3

Round 3: Order of Operations
Direction Simplify the expressions.
Who Wins? The player with the greatest total wins.

I. 3 + (5 • 4) **J.** 17 − 5 • 6 ÷ 10
K. 36 ÷ (7 − 1) • (15 ÷ 3) + 4 **L.** 7 − 4 + 5 • 7

Round 4: Greatest Common Factor
Direction Determine the greatest common factor of the numbers listed.
Who Wins? The player with the greatest total wins.

M. 16, 64 **N.** 42, 63 **P.** 18, 45 **Q.** 9, 39, 42

Round 5: Least Common Multiple
Direction Determine the least common multiple of the numbers listed.
Who Wins? The player with the least total wins.

R. 4, 6 **S.** 7, 4 **T.** 3, 8 **U.** 5, 6, 12

For the Student Additional Practice

- Lesson Tutorials
- Multi-Language Glossary
- Self-Grading Progress Check
- *BigIdeasMath.com*
  Dynamic Student Edition
  Student Resources

Answers

8. 16
9. 2
10. 8
11. 1, 28; 2, 14; 4, 7
12. 1, 44; 2, 22; 4, 11
13. 1, 63; 3, 21; 7, 9
14. $2 \times 3 \times 7$
15. 2×5^2
16. $2 \times 3 \times 11$
17. 9
18. 6
19. 4
20. 6
21. 4
22. 8
23. 28
24. 60
25. 84
26. 90
27. 60
28. 54
29. $\frac{15}{28}$
30. $\frac{67}{72}$
31. $1\frac{11}{30}$
32. $1\frac{8}{21}$ gal

My Thoughts on the Chapter

Teacher Tip

Not allowed to write in your teaching edition? Use sticky notes to record your thoughts.

What worked. . .

What did not work. . .

What I would do differently. . .

Exercises

Find the GCF of the numbers using lists of factors.

17. 27, 45
18. 30, 48
19. 28, 48, 64

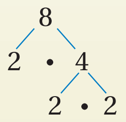
Find the GCF of the numbers using prime factorizations.

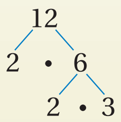
20. 24, 90
21. 52, 68
22. 32, 56, 96

1.6 Least Common Multiple *(pp. 36–43)*

a. Find the LCM of 8 and 12.

Make a factor tree for each number.





Write the prime factorization of each number. Circle each different factor where it appears the greater number of times.

$8 = \textcircled{2} \cdot \textcircled{2} \cdot \textcircled{2}$ 2 appears more often here, so circle all 2s.

$12 = 2 \cdot 2 \cdot \textcircled{3}$ 3 appears once. Do not circle the 2s again.

$2 \cdot 2 \cdot 2 \cdot 3 = 24$ Find the product of the circled factors.

So, the LCM of 8 and 12 is 24.

b. Find $\frac{1}{2} + \frac{1}{3}$.

The LCM of 2 and 3 is 6. So, the LCD is 6.

$$\frac{1}{2} + \frac{1}{3} = \frac{1 \cdot 3}{2 \cdot 3} + \frac{1 \cdot 2}{3 \cdot 2} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

Exercises

Find the LCM of the numbers using lists of multiples.

23. 4, 14
24. 6, 20
25. 12, 28

Find the LCM of the numbers using prime factorizations.

26. 6, 45
27. 10, 12
28. 18, 27

Add or subtract. Write the answer in simplest form.

29. $\frac{2}{7} + \frac{1}{4}$
30. $\frac{5}{9} + \frac{3}{8}$
31. $3\frac{5}{6} - 2\frac{7}{15}$

32. **WATER PITCHER** A water pitcher contains $\frac{2}{3}$ gallon of water. You add $\frac{5}{7}$ gallon of water to the pitcher. How much water does the pitcher contain?

1 Chapter Test

Check It Out
Test Practice
BigIdeasMath.com

Find the value of the expression. Use estimation to check your answer.

1. 3963 + 2379

2. 6184 – 2348

3. 184 × 26

4. 207 ÷ 23

Find the value of the power.

5. 2^3

6. 15^2

7. 5^4

Evaluate the expression.

8. $11 \times 8 - 6 \div 2$

9. $5 + 2^3 \div 4 - 2$

10. $6 + 4(11 - 2) \div 3^2$

List the factor pairs of the number.

11. 52

12. 66

Write the prime factorization of the number.

13. 46

14. 28

Find the GCF of the numbers using lists of factors.

15. 24, 54

16. 16, 32, 72

Find the GCF of the numbers using prime factorizations.

17. 52, 65

18. 18, 45, 63

Find the LCM of the numbers using lists of multiples.

19. 14, 21

20. 9, 24

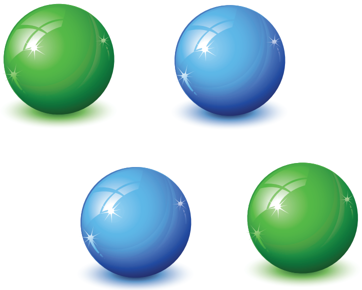
Find the LCM of the numbers using prime factorizations.

21. 26, 39

22. 6, 12, 14

23. **BRACELETS** You have 16 yellow beads, 20 red beads, and 24 orange beads to make identical bracelets. What is the greatest number of bracelets that you can make using all the beads?

24. **MARBLES** A bag contains equal numbers of green and blue marbles. You can divide all the green marbles into groups of 12 and all the blue marbles into groups of 16. What is the least number of each color of marble that can be in the bag?



25. **SCALE** You place a $3\frac{3}{8}$-pound weight on the left side of a balance scale and a $1\frac{1}{5}$-pound weight on the right side. How much weight do you need to add to the right side to balance the scale?

Test Item References

Chapter Test Questions	Section to Review	Common Core State Standards
1–4	1.1	6.NS.2
5–7	1.2	6.EE.1
8–10	1.3	6.EE.1
11–14	1.4	6.NS.4
15–18, 23	1.5	6.NS.4, 6.EE.2b
19–22, 24, 25	1.6	6.NS.4

Test-Taking Strategies

Remind students to quickly look over the entire test before they start so that they can budget their time. When they receive their tests, students should jot down simple examples of finding the greatest common factor and least common multiple on the back of the test. By doing this they will not become confused when they are under pressure. Have students use the **Stop** and **Think** strategy before they answer each question.

Common Errors

- **Exercises 5–7** Students may make the mistake of writing the exponent as a factor. Again, remind them that the exponent is the number of times the base is used as a factor. You may want to demonstrate this point with a quick example. For instance, point out that $5^2 = 5 \times 5 = 25$ but $5 \times 2 = 10$.
- **Exercises 13, 14, 17, 18, 21, 22** Students may think that smaller numbers have fewer factors. Remind students to keep finding factors until all factors are prime numbers.
- **Exercises 21, 22** After writing the prime factorizations, students may struggle over which factors to use to find the LCM. If needed, students should draw a Venn diagram to clarify the problem.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials *BigIdeasMath.com* Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Game Closet at *BigIdeasMath.com* Start Cumulative Assessment

Answers

1. 6342

2. 3836

3. 4784

4. 9

5. 8

6. 225

7. 625

8. 85

9. 5

10. 10

11. 1, 52; 2, 26; 4, 13

12. 1, 66; 2, 33; 3, 22; 6, 11

13. 2×23

14. $2^2 \times 7$

15. 6

16. 8

17. 13

18. 9

19. 42

20. 72

21. 78

22. 84

23. 4 bracelets

24. 48 of each color of marble

25. $2\frac{7}{40}$ lb

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Test-Taking Strategies

Available at *BigIdeasMath.com*

After Answering Easy Questions, Relax
Answer Easy Questions First
Estimate the Answer
Read All Choices before Answering
Read Question before Answering
Solve Directly or Eliminate Choices
Solve Problem before Looking at Choices
Use Intelligent Guessing
Work Backwards

About this Strategy

When taking a multiple choice test, be sure to read each question carefully and thoroughly. Before answering a question, determine exactly what is being asked, then eliminate the wrong answers and select the best choice.

Answers

1. A
2. I
3. C
4. 98

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Item Analysis

1. **A.** Correct answer

   B. The student finds the GCF of 24 and 30 instead of the GCF of 24, 30, and 40.

   C. The student finds the GCF of 24 and 40 instead of the GCF of 24, 30, and 40.

   D. The student finds the GCF of 30 and 40 instead of the GCF of 24, 30, and 40.

2. **F.** The student thinks the area is equal to the side length.

   G. The student doubles the side length instead of squaring the side length.

   H. The student calculates the perimeter instead of the area.

   I. Correct answer

3. **A.** The student does not follow the correct order of operations and subtracts 8 from 2^3 first.

   B. The student does not follow the correct order of operations and subtracts 8 from $3 \cdot 2^3$ instead of dividing 8 by 4.

   C. Correct answer

   D. The student does not follow the correct order of operations and calculates $(3 \cdot 2)^3$ instead of $3 \cdot 2^3$.

4. **Gridded response:** Correct answer: 98

   Common error: The student calculates the GCF instead of the LCM.

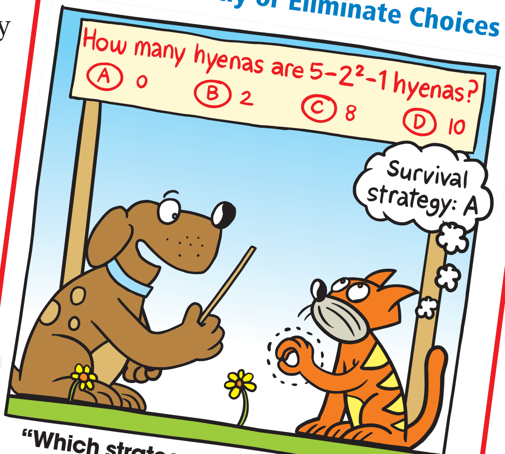
Cumulative Assessment Icons

Gridded Response
Think Solve Explain — Short Response (2-point rubric)
Think Solve Explain — Extended Response (4-point rubric)

1 Cumulative Assessment

Test-Taking Strategy

Solve Directly or Eliminate Choices



"Which strategy would you use on this one: solve directly or eliminate choices?"

1. You are making identical bagel platters using 40 plain bagels, 30 raisin bagels, and 24 blueberry bagels. What is the greatest number of platters that you can make if there are no leftover bagels?

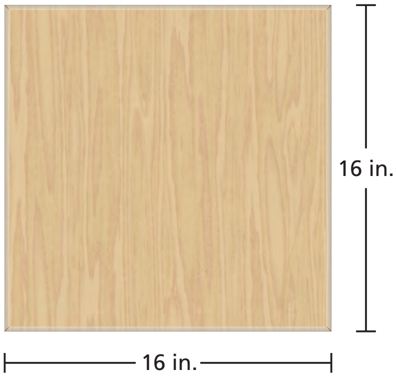
A. 2

B. 6

C. 8

D. 10

2. The top of an end table is a square with a side length of 16 inches. What is the area of the tabletop?



F. 16 in.^{2}

G. 32 in.^{2}

H. 64 in.^{2}

I. 256 in.^{2}

3. Which number is equivalent to the expression below?

$$3 \cdot 2^3 - 8 \div 4$$

A. 0

B. 4

C. 22

D. 214

4. What is the least common multiple of 14 and 49?

5. Which number is equivalent to the expression 7059 ÷ 301?

F. 23

H. $23\frac{136}{301}$

G. $23\frac{136}{7059}$

I. 136

6. You are building identical displays for the school fair using 65 blue boxes and 91 yellow boxes. What is the greatest number of displays you can build using all the boxes?

A. 13

C. 91

B. 35

D. 156

7. You hang the two strands of decorative lights shown below.

Strand 1: changes between red and blue every 15 seconds

Strand 2: changes between green and gold every 18 seconds

Both strands just changed color. After how many seconds will the strands change color at the same time again?

F. 3 seconds

H. 90 seconds

G. 30 seconds

I. 270 seconds

8. Which expression is equivalent to $\frac{29}{63}$?

A. $\frac{28}{60} + \frac{1}{3}$

C. $\frac{5}{21} + \frac{2}{9}$

B. $\frac{4}{27} + \frac{25}{36}$

D. $\frac{22}{47} + \frac{7}{16}$

9. Which expression is *not* equivalent to 32?

F. $6^2 - 8 \div 2$

H. $30 + 4^2 \div (2 + 6)$

G. $30 \div 2 + 5^2 - 8$

I. $8^2 \div 4 - 2$

10. Which number is equivalent to the expression 148 × 27?

A. 3696

C. 3946

B. 3896

D. 3996

Item Analysis (continued)

5. **F.** The student does not include the fractional part of the answer.
   G. The student incorrectly uses the dividend when writing the fractional part of the answer.
   H. Correct answer
   I. The student confuses the remainder with the quotient.

6. **A.** Correct answer
   B. The student multiplies 5 (a factor of 65) by 7 (a factor of 91) instead of finding the GCF of 65 and 91.
   C. The student chooses the greater of 65 and 91.
   D. The student adds 65 and 91.

7. **F.** The student finds the GCF instead of the LCM.
   G. The student incorrectly multiplies 2, 3, and 5 to find the LCM.
   H. Correct answer
   I. The student incorrectly multiplies 15 and 18 to find the LCM.

8. **A.** The student incorrectly adds the numerators and adds the denominators.
   B. The student incorrectly adds the numerators and adds the denominators.
   C. Correct answer
   D. The student incorrectly adds the numerators and adds the denominators.

9. **F.** The student does not apply the correct order of operations.
   G. The student does not apply the correct order of operations.
   H. The student does not apply the correct order of operations.
   I. Correct answer

10. **A.** The student does not carry the 3 to the hundreds place when multiplying 148 by 7.
    B. The student does not carry the 1 to the hundreds place when multiplying 148 by 20.
    C. The student does not carry the 5 to the tens place when multiplying 148 by 7.
    D. Correct answer

Answers

5. H
6. A
7. H
8. C
9. I
10. D

Answers

11. 6

12. H

13. 5, 1, 3; The GCF of the three numbers must be 1 because two of the numbers do not have any factors in common except 1.

14. C

15. I

16. B

Item Analysis (continued)

11. Gridded response: Correct answer: 6

Common error: The student does not find the GCF correctly.

12.
- **F.** The student does not apply the correct order of operations.
- **G.** The student does not apply the correct order of operations.
- **H.** Correct answer
- **I.** The student does not apply the correct order of operations.

13. **2 points** The student demonstrates a thorough understanding of the GCF of the pairs of numbers and of the set of three numbers. GCF(10, 15) = 5, GCF(10, 21) = 1, GCF(15, 21) = 3, GCF(10, 15, 21) = 1; The GCF of the three numbers must be 1, because two of the numbers, 10 and 21, have only 1 as a common factor.

1 point The student demonstrates a partial understanding of GCF. For instance, the student does not find the GCF of the three numbers.

0 points The student demonstrates insufficient understanding of the GCF of a set of numbers.

14.
- **A.** The student performs a calculation incorrectly or does not understand the concept of a perfect square.
- **B.** The student performs a calculation incorrectly or does not understand the concept of a perfect square.
- **C.** Correct answer
- **D.** The student performs a calculation incorrectly or does not understand the concept of a perfect square.

15.
- **F.** The student multiplies 4 by 12 instead of finding the LCM.
- **G.** The student multiplies 6 by 8 instead of finding the LCM.
- **H.** The student finds a common multiple of 8 and 24, but not the LCM.
- **I.** Correct answer

16.
- **A.** The student does not follow the correct order of operations and uses the Distributive Property incorrectly, calculating 3 • 6 instead of 3 times the quantity $6 + 2^2$.
- **B.** Correct answer
- **C.** The student does not follow the correct order of operations and calculates, $3 \cdot 6 + (3 \cdot 2)^2$.
- **D.** The student does not follow the correct order of operations and incorrectly adds 6 + 2 before squaring.

11. You have 60 nickels, 48 dimes, and 42 quarters. You want to divide the coins into identical groups with no coins left over. What is the greatest number of groups that you can make?

12. Erica was evaluating the expression in the box below.

$$\begin{aligned} 56 \div (2^3 - 1) \times 4 &= 56 \div (8 - 1) \times 4 \\ &= 56 \div 7 \times 4 \\ &= 56 \div 28 \\ &= 2 \end{aligned}$$

What should Erica do to correct the error that she made?

F. Divide 56 by 8 because operations are performed left to right.

G. Multiply 1 by 4 because multiplication is done before subtraction.

H. Divide 56 by 7 because operations are performed left to right.

I. Divide 56 by 8 and multiply 1 by 4 because division and multiplication are performed before subtraction.

13. Find the greatest common factor for each pair of numbers.

Think Solve Explain

10 and 15 10 and 21 15 and 21

What can you conclude about the greatest common factor of 10, 15, and 21? Explain your reasoning.

14. Which number is *not* a perfect square?

A. 64

B. 81

C. 96

D. 100

15. Which number pair has a least common multiple of 48?

F. 4, 12

G. 6, 8

H. 8, 24

I. 16, 24

16. Which number is equivalent to the expression below?

$$\frac{3(6 + 2^2) + 2}{8}$$

A. 3

B. 4

C. 7

D. $24\frac{1}{4}$