10 Data Displays

"I took a survey of pet owners on how many times per day you should treat your dog to a biscuit."

"What do you think?"

"You just couldn't resist participating yourself, could you?"
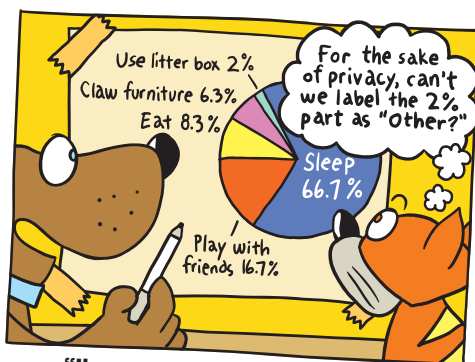
"I've completed a circle graph analyzing what you do each day."

Use litter box 2%
Claw furniture 6.3%
Eat 8.3%
Sleep 66.7%
Play with friends 16.7%

"For the sake of privacy, can't we label the 2% part as "Other?""

Common Core Progression

4th Grade
• Make a line plot with data in fractions of a unit.
• Use line plots to solve problems involving addition and subtraction of fractions.
• Identify line-symmetric figures and draw lines of symmetry.

5th Grade
• Use line plots to solve problems involving operations on fractions.
• Graph ordered pairs in the first quadrant of the coordinate plane.

6th Grade
• Display data on a number line, including line (dot) plots, stem-and-leaf plots, histograms, and box-and-whisker plots.
• Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, and a measure of variation describes how its values vary with a single number.
• Understand that data used to answer statistical questions has a distribution that can be described by center, spread, and shape.
• Choose appropriate measures of center and variation based on shape.

Chapter Summary

Section	Common Core State Standard	
10.1	Applying	6.SP.4
10.2	Learning	6.SP.2, 6.SP.4
10.3	Learning	6.SP.2, 6.SP.4, 6.SP.5d
10.4	Learning	6.SP.2 ★, 6.SP.4 ★, 6.SP.5c ★
★ Teaching is complete. Standard can be assessed.		

Technology for the *Teacher*

BigIdeasMath.com
Chapter at a Glance
Complete Materials List
Parent Letters: English and Spanish

Additional Topics for Review

- Multiplying Fractions and Whole Numbers
- Line (Dot) Plots
- Ordering Whole Numbers and Integers
- Outliers
- Measures of Center
- Measures of Variation
- Quartiles
- Interquartile Range

Try It Yourself

1. orange
2. 11 students
3. 21 students
4. 25 students
5. 8 students
6. 7 students

Record and Practice Journal
Fair Game Review

1. Ranch
2. 8 students
3. 18 students
4. 24 students
5. 9 students
6. 12 students
7. 6 students

Math Background Notes

Vocabulary Review

- Bar Graph
- Circle Graph
- Percent

Analyzing Bar Graphs

- Students should know how to read and draw bar graphs. Even though this is a skill from a previous grade, students may still need practice to achieve mastery.
- **Teaching Tip:** This is an excellent opportunity to review the parts of a bar graph with students. Sample questions could include: What is the title of the graph? What are the categories? What data are being compared? What scale is being used to measure the axes?

Finding Percents

- Students should be able to read and obtain information from a circle graph. Students should also know how to find the percent of a number. They learned this skill in Chapter 5.
- Take time to review the information that is given in a circle graph:
  - The circle represents all of the data.
  - The percents total 100%.
  - Each section represents part of the data.
- If necessary, review the information in Section 5.5 on how to write a percent as a fraction and how to find the percent of a number.
- **Teaching Tip:** When going over the solution of Example 2, in the simplification before the final answer, avoid using terms such as "cancel out." The factors *divide* out. Instilling good habits, such as using correct terminology, will help students later in their mathematical careers. For instance, in algebra, when students are faced with simplifying rational expressions, they are more likely to get correct answers when they look for common factors to divide out instead of common terms to "cancel" out.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Record and Practice Journal • Fair Game Review Skills Review Handbook Lesson Tutorials	Game Closet at *BigIdeasMath.com* Start the next section

What You Learned Before

"Okay, I have the box. But I need your help to complete my box-and-whisker plot."

Analyzing Bar Graphs

Example 1 **The bar graph shows the favorite colors of the students in a class. How many students said their favorite color is blue?**

The height of the bar labeled "Blue" is 8.

⋰ So, 8 students said their favorite color is blue.

Try It Yourself

1. What color was chosen the least?

2. How many students said green or red is their favorite color?

3. How many students did *not* choose yellow as their favorite color?

4. How many students are in the class?
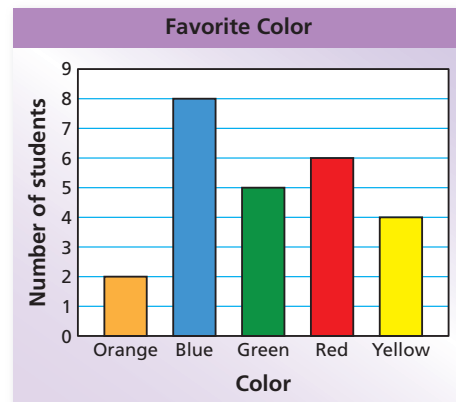
Finding Percents

Example 2 **The circle graph shows the favorite fruits of the students in a class. There are 20 students in the class. How many students said their favorite fruit is an orange?**

Find 25% of 20.

$$25\% \text{ of } 20 = \frac{1}{4} \cdot 20 = \frac{1 \cdot \overset{5}{\cancel{20}}}{\underset{1}{\cancel{4}}} = 5$$

Favorite Fruit

⋰ So, 5 students said their favorite fruit is an orange.

Try It Yourself

5. How many students said their favorite fruit is an apple?

6. How many students said their favorite fruit is a banana?
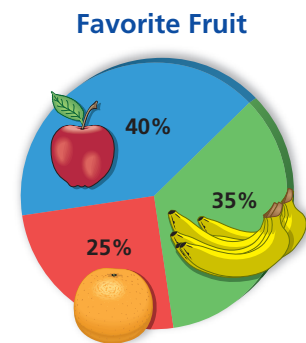
10.1 Stem-and-Leaf Plots

Essential Question How can you use place values to represent data graphically?

1   ACTIVITY: Making a Data Display

Work with a partner. The list below gives the ages of these women when they became first ladies of the United States.

THE WHITE HOUSE
WASHINGTON

Frances Cleveland – 21   Mamie Eisenhower – 56
Caroline Harrison – 56   Jacqueline Kennedy – 31
Ida McKinley – 49   Claudia Johnson – 50
Edith Roosevelt – 40   Patricia Nixon – 56
Helen Taft – 48   Elizabeth Ford – 56
Ellen Wilson – 52   Rosalynn Carter – 49
Florence Harding – 60   Nancy Reagan – 59
Grace Coolidge – 44   Barbara Bush – 63
Lou Hoover – 54   Hillary Clinton – 45
Eleanor Roosevelt – 48   Laura Bush – 54
Elizabeth Truman – 60   Michelle Obama – 45

a. The incomplete data display shows the ages of the first ladies in the left column of the list above.

What do the numbers to the left of the line represent? What do the numbers to the right of the line represent?

Ages of First Ladies

2	1
3	
4	0 4 8 8 9
5	2 4 6
6	0 0

b. This data display is called a *stem-and-leaf plot.* What numbers do you think represent the *stems? leaves?* Explain your reasoning.

c. Complete the stem-and-leaf plot using the remaining ages in the right column. Order the numbers to the right of the line in numerical order.

d. **REASONING** Write a question about the ages of first ladies that would be easier to answer using a stem-and-leaf plot than a dot plot.

Data Displays
In this lesson, you will
• make and interpret stem-and-leaf plots.

Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP4 Model with Mathematics:** Students will construct stem-and-leaf plots to represent and describe features of a data set, including measures of center and measures of variation.

Motivate

- Make a stem-and-leaf plot using sticky notes. As students enter, hand each one a sticky note. On the board draw a vertical line and use stems of 0, 1, 2, and 3. Ask students to write down the day of the month they were born, making their numbers large enough and dark enough to be seen from a distance.
- Ask those students born before the 10th of the month to come forward and put their sticky notes in a row adjacent to the 0 stem.
- Repeat for each of the stems; 10th–19th, 20th–29th, and 30th–31st.
- ❓ "What observations can you make about the data?" most likely there are fewer in the 30s; would expect randomness and balanced distribution over other 3 stems, but it might not be!
- Do not sort—this will be done at the end of the class. Explain that today they will explore a new type of plot similar to what they have made with their sticky notes.

Activity Notes

Activity 1

- The stem-and-leaf plot is fairly simple for students to understand. The motivator should give them a good understanding of how to construct one.
- **FYI:** The ages given are how old these women were when they *first* became first ladies, which was not necessarily on the day of inauguration. For instance, Frances and Grover Cleveland were married on June 2, 1886, during his first term. Also, Woodrow Wilson's second wife, Edith, is not listed. His first wife, Ellen, died while he was in office.
- Students find this data set interesting. Most students know that you must be at least 35 years old to run for president, so they realize that some first ladies were quite young. As an extension to this activity, in Activity 2 students will make a back-to-back stem-and-leaf plot with presidents' and first ladies' ages. The distributions are different!
- ❓ What do the stems represent?" the tens digit of the first ladies' ages
- Note that the leaves are in numerical order. Students may not notice this.
- When students have finished, display a completed plot. Have a few students share their answers to part (d).
- **Big Idea:** A stem-and-leaf plot is very similar to a dot plot, but the stem-and-leaf plot gives additional information. The stem-and-leaf plot uses the ones digit of the data values instead of dots, so you can see the distribution within each group of ten.

Previous Learning

Students should know how to use line plots (or dot plots) to display and analyze data.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

10.1 Record and Practice Journal

Essential Question How can you place values to represent data graphically?

1 ACTIVITY: Making a Data Display

Work with a partner. The list below gives the ages of these women when they became first ladies of the United States.

THE WHITE HOUSE
WASHINGTON

Frances Cleveland – 21	Mamie Eisenhower – 56
Caroline Harrison – 56	Jacqueline Kennedy – 31
Ida McKinley – 44	Claudia Johnson – 50
Edith Roosevelt – 40	Patricia Nixon – 56
Helen Taft – 48	Elizabeth Ford – 56
Ellen Wilson – 52	Rosalynn Carter – 49
Florence Harding – 60	Nancy Reagan – 59
Grace Coolidge – 44	Barbara Bush – 63
Lou Hoover – 54	Hillary Clinton – 45
Eleanor Roosevelt – 48	Laura Bush – 54
Elizabeth Truman – 60	Michelle Obama – 45

a. The incomplete data display shows the ages of the first ladies in the left column of the list above.

Ages of First Ladies

```
2 | 1
3 |
4 | 0 4 8 8 9
5 | 2 4 6
6 | 0 0
```

What do the numbers to the left of the line represent? What do the numbers to the right of the line represent?

left: **tens digit of the ages**
right: **ones digit of the ages**

Kinesthetic

To make the display in Activity 2, copy the data onto individual pieces of paper. Then sort the data into piles based on the stem value and whether the number is a first lady's age or a president's age. For each stem value, cut one of the pieces to separate the stem and leaf. Tape the stems to the board in increasing order from top to bottom. Cut the remaining leaves with the same stem and tape them to the board in ascending order. Place the leaves for the first ladies' ages to the right of the stem and order them from left to right. Place the leaves for the presidents' ages to the left of the stem and order them from right to left.

10.1 Record and Practice Journal

b. This data display is called a *stem-and-leaf plot*. What numbers do you think represent the *stems*? *leaves*? Explain your reasoning.
stems: numbers on left
leaves: numbers on right

c. Complete the stem-and-leaf plot on the previous page using the remaining ages in the right column. Order the numbers to the right of the line in numerical order. **See Additional Answers.**

d. Write a question about the ages of first ladies that would be easier to answer using a stem-and-leaf plot than a dot plot.
Sample answer: **In which age range were there more first ladies, 40s or 50s?**

2 ACTIVITY: Making a Back-to-Back Stem-and-Leaf Plot

Work with a partner. The table below shows the ages of presidents of the United States from 1885 to 2009 on their first inauguration day.

Ages of Presidents									
47	55	54	42	51	56	55	51	54	51	60
62	43	55	56	61	52	69	64	46	54	47

a. On your stem-and-leaf plot from Activity 1(c), draw a vertical line to the left of the display. Represent the ages of the presidents by including numbers to the left of the line. **See Additional Answers.**

b. Find the median ages of both the first ladies and presidents of the United States.
first ladies: 51
presidents: 54

c. Compare the distribution of each data set.
Sample answer: **Distributions are similar, with most ages in the 40s, 50s, and 60s, but presidents appear to be older than first ladies, in general.**

3 ACTIVITY: Conducting an Experiment

Work with a partner. Use two number cubes to conduct the following experiment.
• Toss the cubes and find the product of the resulting numbers.
• Repeat this process 30 times. Record your results.

Toss	1	2	3	4	5	6	7	8	9	10
Product										
Toss	11	12	13	14	15	16	17	18	19	20
Product										
Toss	21	22	23	24	25	26	27	28	29	30
Product										

a. Use a stem-and-leaf plot to organize your results.
Check students' work.

b. Describe the distribution of the data.
Distributions should have more values as the product decreases.

What Is Your Answer?

4. IN YOUR OWN WORDS How can you use place values to represent data graphically?
Sample answer: **Choose digit(s) according to place values and display the data in a stem-and-leaf plot.**

5. How can you display data in a stem-and-leaf plot whose values range from 82 through 129?
Use 8, 9, 10, 11, and 12 as the stems.

Laurie's Notes

Activity 2

• Have students read through the directions to make sure they understand how to make the back-to-back stem-and-leaf plot.
• In part (a), have students decide how to list the leaves (ascending or descending order).
• In addition to finding the median, you could also ask students to find the mean to help compare the data.
• **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Ask students to compare the two distributions. Listen for evidence that supports their description.

Activity 3

• The first two activities are time consuming, so there may not be time to complete Activity 3. You could assign Activity 3 as homework.
? "What is the range of this data set?" $36 - 1 = 35$

What Is Your Answer?

• Have students work in pairs.

Closure

• Return to the sticky note plot made when motivating today's lesson. Ask a volunteer to sort the data. Now ask students to describe the plot, along with finding the median, mode, and range.

2 ACTIVITY: Making a Back-to-Back Stem-and-Leaf Plot

Work with a partner. The table below shows the ages of presidents of the United States from 1885 to 2009 on their first inauguration day.

Ages of Presidents										
47	55	54	42	51	56	55	51	54	51	60
62	43	55	56	61	52	69	64	46	54	47

a. On your stem-and-leaf plot from Activity 1(c), draw a vertical line to the left of the display. Represent the ages of the presidents by including numbers to the left of the line.

b. Find the median ages of both the first ladies and the presidents of the United States.

c. Compare the distribution of each data set.

3 ACTIVITY: Conducting an Experiment

Math Practice

Interpret Results
How can you use the stem-and-leaf plot to interpret your results? Explain.

Work with a partner. Use two number cubes to conduct the following experiment.

- Toss the cubes and find the product of the resulting numbers.
- Repeat this process 30 times. Record your results.

a. Use a stem-and-leaf plot to organize your results.

b. Describe the distribution of the data.

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use place values to represent data graphically?

5. How can you display data in a stem-and-leaf plot whose values range from 82 through 129?

Practice Use what you learned about stem-and-leaf plots to complete Exercises 4 and 5 on page 438.

Key Vocabulary 🔊
stem-and-leaf plot, *p. 436*
stem, *p. 436*
leaf, *p. 436*

🔑 Key Idea

Stem-and-Leaf Plots

A **stem-and-leaf plot** uses the digits of data values to organize a data set. Each data value is broken into a **stem** (digit or digits on the left) and a **leaf** (digit or digits on the right).

A stem-and-leaf plot shows how data are distributed.

Stem	Leaf
2	0 0 1 2 5 7
3	1 4 8
4	2
5	8 9

Key: 2|0 = 20

The *key* explains what the stems and leaves represent.

EXAMPLE ① Making a Stem-and-Leaf Plot

	A	B
1	DATE	MINUTES
2	JULY 9	55
3	JULY 9	3
4	JULY 9	6
5	JULY 10	14
6	JULY 10	18
7	JULY 10	5
8	JULY 10	23
9	JULY 11	30
10	JULY 11	23
11	JULY 11	10
12	JULY 11	2
13	JULY 11	36

Make a stem-and-leaf plot of the length of the 12 cell phone calls.

Step 1: Order the data.

2, 3, 5, 6, 10, 14, 18, 23, 23, 30, 36, 55

Step 2: Choose the stems and the leaves. Because the data values range from 2 to 55, use the *tens* digits for the stems and the *ones* digits for the leaves. Be sure to include the key.

Step 3: Write the stems to the *left* of the vertical line.

Step 4: Write the leaves for each stem to the *right* of the vertical line.

Cell Phone Call Lengths

Order the stems vertically. The stem for data values less than 10 is 0.

Include stems without leaves.

Stem	Leaf
0	2 3 5 6
1	0 4 8
2	3 3
3	0 6
4	
5	5

Write the leaves horizontally.

Key: 1|4 = 14 minutes

🔴 On Your Own

Now You're Ready
Exercises 4–9

1. Make a stem-and-leaf plot of the hair lengths.

Hair Length (centimeters)									
5	1	20	12	27	2	30	5	7	38
40	47	1	2	1	32	4	44	33	23

🔊 Multi-Language Glossary at BigIdeasMath✓com

Laurie's Notes

Introduction

Connect
- **Yesterday:** Students explored the features of a stem-and-leaf plot. (MP3, MP4)
- **Today:** Students will construct and interpret stem-and-leaf plots.

Motivate
- Share a little trivia about cell phones, the context for Example 1.
  - 1973—Dr. Martin Cooper is considered the inventor of the first portable handset. Dr. Cooper, former general manager for the systems division at Motorola, was the first person to make a call on a cell phone.
  - 1977—Cell phones go public. The first trials of cell phone testing began in the city of Chicago with 2000 customers. Japan began testing cellular phone service in 1979.

Lesson Notes

Key Idea
- A common question that students ask is "what if the data are not two-digit numbers?"
  - If there are 3-digit numbers within a small range (i.e., 431–476), the stem can be two digits (43, 44, 45, 46, 47).
  - If there are decimals (i.e., 3.4), the stem can be the whole number portion and the leaf is the decimal portion.

Example 1
- By sorting the data first, the leaves are arranged in order when written to the right of the stem.
- **MP6 Attend to Precision:** Discuss with students the need to have a key that describes how to read the data in the plot.
- **Common Error:** When a data value repeats, remind students that the leaf must be listed again. The number of leaves must equal the number of data in the set.
- **?** "Describe the stem-and-leaf plot. What does the plot tell you about the data?" Data are skewed towards the lower data. It tapers at the upper end. There is a gap from 36 to 55.
- **Big Idea**: Because the stem-and-leaf plot shows how data are distributed, a stem must be included in the plot even if there are no data values in that interval. In this example the stem of "4" is still included to show the gap in the data.

On Your Own
- **Think-Pair-Share:** Students should read the question independently and then work in pairs to answer the question. When they have answered the question, the pair should compare their answers with another group and discuss any discrepancies.
- **?** "Describe what the plot tells you about the data." There are some smaller data values and larger data values with few in the middle.
- **?** "Explain why this might be the case." perhaps girls versus boys

---

Technology for the **Teacher**

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1
Make a stem-and-leaf plot of the length of the eleven fish, in inches.
7, 12, 20, 14, 20, 25, 8, 18, 16, 20, 14

Fish Length

Stem	Leaf
0	7 8
1	2 4 4 6 8
2	0 0 0 5

Key: 1 | 8 = 18 inches

On Your Own

1. **Hair Length**

Stem	Leaf
0	1 1 1 2 2 4 5 5 7
1	2
2	0 3 7
3	0 2 3 8
4	0 4 7

Key: 1 | 2 = 12 cm

Extra Example 2

Use the stem-and-leaf plot of student test scores in Example 2.

a. How many students scored more than 85 points? 8

b. How many students scored at most 75 points? 3

c. What is the median test score? 84

⬤ On Your Own

2. **a.** 9 students

    b. 4 students

Extra Example 3

Use the stem-and-leaf plot in Example 2. Which statement is *not* true? B

A. The range of test scores is 34.

B. The mean test score is 85.

C. The test scores that occur most often are 81, 84, and 88.

D. Most of the test scores are over 80.

⬤ On Your Own

3. yes; From the stem-and-leaf plot, there are plants that are 22 inches, 25 inches, and 36 inches tall.

English Language Learners

Pair Activity

Form pairs of English learners and English speakers. Have them consider different types of data such as golf scores, the number of text messages each person in class sends in a week, the number of sixth, seventh, and eighth graders that ride the school bus, the numbers rolled on a number cube, and the number of hours that students in a class slept last night. Instruct students to discuss whether a stem-and-leaf plot would be an appropriate way to display the data.

Example 2

- **Big Idea**: The leaves in a stem-and-leaf are sorted from least to greatest. The range can be found quickly, along with the median. To find the median, alternate counting in from each end of the data (the least and the greatest).

- ❓ "How many students are represented in the plot?" 18
- ❓ "What is the median? Explain how you decided." 84; There are 18 numbers so the middle will be the average of the 9th and 10th numbers which are both 84.

On Your Own

- **Extension:** Ask students to describe the plot.

Example 3

- This example reviews data analysis concepts from Chapter 9. You may need to remind students how to find the mean, median, mode, and range.

On Your Own

- **Neighbor Check:** Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.

Closure

- Explain how a stem-and-leaf plot is similar to a bar graph and how it differs from a bar graph. *Sample answer:* A stem-and-leaf plot shows the frequency of different intervals for the data similar to a bar graph as well as displaying the distribution of the data. However, all of the data are displayed in a stem-and-leaf plot, but not in a bar graph.

EXAMPLE 2 Interpreting a Stem-and-Leaf Plot

Test Scores

Stem	Leaf
6	6
7	0 5 7 8
8	1 1 3 4 4 6 8 8 9
9	0 2 9
10	0

Key: 9|2 = 92 points

The stem-and-leaf plot shows student test scores. (a) How many students scored less than 80 points? (b) How many students scored at least 90 points? (c) How are the data distributed?

a. There are five scores less than 80 points: 66, 70, 75, 77, and 78.

∴ Five students scored less than 80 points.

b. There are four scores of at least 90 points: 90, 92, 99, and 100.

∴ Four students scored at least 90 points.

c. There are few low test scores and few high test scores. So, most of the scores are in the middle.

On Your Own

Now You're Ready
Exercises 12–15

2. Use the grading scale at the right.

a. How many students received a B on the test?

b. How many students received a C on the test?

> A: 90–100
> B: 80–89
> C: 70–79
> D: 60–69
> F: 59 and below

EXAMPLE 3 Making Conclusions from a Stem-and-Leaf Plot

Which statement is *not* true?

(A) Most of the plants are less than 20 inches tall.

(B) The median plant height is 11 inches.

(C) The range of the plant heights is 35 inches.

(D) The plant height that occurs most often is 11 inches.

Plant Heights

Stem	Leaf
0	1 2 4 5 6 8 9
1	0 1 1 5 7
2	2 5
3	6

Key: 1|5 = 15 inches

There are 15 plant heights. So, the median is the eighth data value, 10 inches.

∴ The correct answer is (B).

On Your Own

3. You are told that three plants are taller than 20 inches. Is the statement true? Explain.

Vocabulary and Concept Check

1. **VOCABULARY** The key for a stem-and-leaf plot is $3|4 = 34$. Which number is the stem? Which number is the leaf?

2. **WRITING** Describe how to make a stem-and-leaf plot of the data values 14, 22, 9, 13, 30, 8, 25, and 29.

3. **WRITING** How does a stem-and-leaf plot show the distribution of data?

Practice and Problem Solving

Make a stem-and-leaf plot of the data.

1 **4.**

Books Read			
26	15	20	9
31	25	29	32
17	26	19	40

5.

Hours Online			
8	12	21	14
18	6	15	24
12	17	2	0

6.

Test Scores (%)				
87	82	95	91	69
88	68	87	65	81
97	85	80	90	62

7.

Points Scored				
58	50	42	71	75
45	51	43	38	71
42	70	56	58	43

8.

Bikes Sold			
78	112	105	99
86	96	115	100
79	81	99	108

9.

Minutes in Line			
4.0	2.6	1.9	3.1
3.6	2.2	2.7	3.8
1.6	2.0	3.1	2.9

10. **ERROR ANALYSIS** Describe and correct the error in making a stem-and-leaf plot of the data.

    51, 25, 47, 42, 55, 26, 50, 44, 55

Stem	Leaf
2	5 6
4	2 4 7
5	0 1 5 5

    Key: $4|2 = 42$

11. **PUPPIES** The weights (in pounds) of eight puppies at a pet store are 12, 24, 17, 8, 18, 31, 24, and 15. Make a stem-and-leaf plot of the data. Describe the distribution of the data.

STEM
VIDEO

Assignment Guide and Homework Check

Level	Day 1 Activity Assignment	Day 2 Lesson Assignment	Homework Check
Basic	4, 5, 19–21	1–3, 8–13, 15	8, 9, 10, 11, 13
Average	4, 5, 19–21	1–3, 9–12, 13, 15, 16–18	9, 12, 16, 17
Advanced	4, 5, 19–21	1–3, 8–18	9, 12, 14, 16, 17

For Your Information
- **Exercises 12–15** A dig is a forearm pass of a hard-driven ball, such as a spike.

Common Errors
- **Exercises 4–9** Students may forget to include the numbers that have zeros in the ones place in the leaf part of the plot. Remind them that they should be able to read the numbers in the data set by reading stem *and* leaf.
- **Exercises 4–9** Students may not include repeats of numbers. Remind them that the plot represents all of the data values, so they should be able to count the values in the leaf part and have all of the data accounted for.
- **Exercises 4–9** Students may forget to include stems that have no data values. Remind them of Example 1. It is necessary to include the stems with no data to help answer questions about the data set.
- **Exercises 12–15** Students may need to be reminded of the definitions of some of the terms so that they can answer the questions. Give an example of each term and how to find it using a stem-and-leaf plot.

10.1 Record and Practice Journal

Make a stem-and-leaf plot of the data.

1.
Class Sizes

12	10	21	28
9	16	19	16
25	32	14	21

```
0 | 9
1 | 0 2 4 6 6 9
2 | 1 1 5 8
3 | 2
```

2.
Minutes Spent on Homework

75	82	91	68
92	86	79	76
75	81	88	60

```
6 | 0 8
7 | 5 5 6 9
8 | 1 2 6 8
9 | 1 2
```

3. The number of text messages from eight phones are 8, 11, 14, 22, 5, 15, 7, and 20. Make a stem-and-leaf plot of the data. Describe the distribution of the data.

```
0 | 5 7 8
1 | 1 4 5
2 | 0 2
```
The data are evenly distributed.

4. The number of minutes seven members spent at band practice are 57, 49, 55, 62, 78, 72, and 75. Make a stem-and-leaf plot of the data. Describe the distribution of the data.

```
4 | 9
5 | 5 7
6 | 2
7 | 2 5 8
```
The data shows that few students do less than 50 minutes of practice.

5. The stem-and-leaf plot shows the numbers of miles students travel to get to school.

```
Stem | Leaf
 0   | 5 7
 1   | 2 4 8
 2   | 0 1 5 7
 3   | 3
```
Key: 1 | 4 = 14 miles

a. How many students travel more than 15 miles?
6 students

b. Find the mean, median, mode, range, and interquartile range of the data.
mean: 18.2   range: 28
median: 19   IQR: 13
mode: none

Vocabulary and Concept Check

1. 3 is the stem.; 4 is the leaf.

2. Write the data set in order from least to greatest. 8, 9, 13, 14, 22, 25, 29, 30 Use the tens digits for the stems and the ones digits for the leaves. Write the stems to the left of the vertical line. Then write the leaves to the right of the vertical line.

```
0 | 8 9
1 | 3 4
2 | 2 5 9
3 | 0
```
Key: 1 | 3 = 13

3. From the leaves, you can see where most of the data lies and whether there are many values that are low or high.

Practice and Problem Solving

4. **Books Read**

Stem	Leaf
0	9
1	5 7 9
2	0 5 6 6 9
3	1 2
4	0

Key: 1 | 5 = 15 books

5. **Hours Online**

Stem	Leaf
0	0 2 6 8
1	2 2 4 5 7 8
2	1 4

Key: 2 | 1 = 21 hours

6–11. See Additional Answers.

T-438

Practice and Problem Solving

12. 6 players

13. mean: 56.6; median: 53;
modes: 41, 43, 63;
range: 56; IQR = 20

14. Most of the data are in the
40s, 50s, and 60s.

15. 97; It increases the mean.

16–17. See Additional Answers.
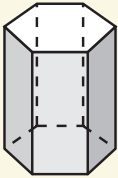
18. See *Taking Math Deeper*.

Fair Game Review

19.
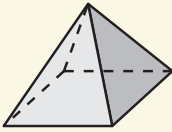
20.

21. B

Mini-Assessment

**The table shows the number of hours
15 students were online this week.**

Hours Online				
21	14	8	13	17
18	9	12	7	21
15	12	21	15	7

1. Make a stem-and-leaf plot of
the data.

Stem	Leaf
0	7 7 8 9
1	2 2 3 4 5 5 7 8
2	1 1 1

Key: 1 | 8 = 18 hours

2. Find the mean, median, mode, and
range of the data. mean = 14,
median = 14, mode = 21, range = 14

Taking Math Deeper

Exercise 18

With many problems involving data analysis, it helps to start by listing the data
in order.

① Order the scores for each golfer in the back-to-back stem-and-leaf plot.

Rich: 35, 37, 41, 42, 43, 44, 45, 48, 50, 55
Will: 42, 43, 44, 44, 46, 47, 47, 48, 49, 50

② Use a table to organize the measures of center and the measures
of variation.

	Mean	Median	Mode	Range	IQR	MAD
Rich	44	43.5	no mode	20	7	4.4
Will	46	46.5	44 and 47	8	4	2.2

③ You can cite reasons for choosing either golfer.

You could choose Rich because he has the
three lowest scores, a lower mean, and a
lower median.

You could choose Will because he is more
consistent. He has a lower range, a lower
interquartile range, and a lower mean
absolute deviation.

*Center and
variation*

Project

Research the meaning of a golfer's *handicap*. Which golfer would you
choose if the tournament was a 4-day, 18-hole tournament based on
handicap? Explain.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Start the next section

VOLLEYBALL The stem-and-leaf plot shows the number of *digs* for the top 15 players at a volleyball tournament.

Stem	Leaf
4	1 1 3 3 5
5	0 2 3 4
6	2 3 3 7
7	5
8	
9	7

Key: 5 | 0 = 50 digs

2 **12.** How many players had more than 60 digs?

13. Find the mean, median, mode, range, and interquartile range of the data.

14. Describe the distribution of the data.

15. Which data value is the outlier? Describe how the outlier affects the mean.

16. **REASONING** Each stem-and-leaf plot below has a mean of 39. Without calculating, determine which stem-and-leaf plot has the lesser mean absolute deviation. Explain your reasoning.

Stem	Leaf
2	3 7
3	0 2 6 9
4	1 2 5 8
5	1 4

Key: 4 | 1 = 41

Stem	Leaf
2	2 4 5 8 9
3	3 8
4	5
5	3 6 7 8

Key: 5 | 3 = 53

17. **TEMPERATURE** The stem-and-leaf plot shows the daily high temperatures (in degrees Fahrenheit) for the first 15 days of a month.

Stem	Leaf
6	7 8
7	0 0 3 4 6 8 9
8	2 3 6 7 8 9

Key: 6 | 7 = 67°F

 a. Find and interpret the mean absolute deviation of the data.

 b. After you include the daily high temperatures for the rest of the month in the stem-and-leaf plot, the mean absolute deviation increases. Where do you think most of the data values for the rest of the month are located in the stem-and-leaf plot? Explain.

18. **Critical Thinking** The back-to-back stem-and-leaf plot shows the 9-hole golf scores for two golfers. Only one of the golfers can compete in a tournament. Use measures of center and measures of variation to give reasons why you would choose each golfer.

Rich		Will
7 5	3	
8 5 4 3 2 1	4	2 3 4 4 6 7 7 8 9
5 0	5	0

Key: 1 | 4 | 2 = 41 and 42 strokes

Fair Game Review *What you learned in previous grades & lessons*

Draw the solid. *(Section 8.1)*

19. square pyramid

20. hexagonal prism

21. **MULTIPLE CHOICE** In a bar graph, what determines the length of each bar? *(Skills Review Handbook)*

Ⓐ frequency   Ⓑ data value   Ⓒ leaf   Ⓓ change in data

10.2 Histograms

Essential Question How can you use intervals, tables, and graphs to organize data?

1 ACTIVITY: Conducting an Experiment

Work with a partner.

a. Roll a number cube 20 times. Record your results in a tally chart.

b. Make a bar graph of the totals.

c. Go to the board and enter your totals in the class tally chart.

d. Make a second bar graph showing the class totals. Compare and contrast the two bar graphs.

Tally Chart	
1	
2	
3	
4	
5	
6	

Key: | = 1   ⫴⫴⫴ = 5

2 ACTIVITY: Using Intervals to Organize Data

Work with a partner. You are judging a paper airplane contest. A contestant flies a paper airplane 20 times. You record the following distances:

20.5 ft, 24.5 ft, 18.5 ft, 19.5 ft, 21.0 ft, 14.0 ft, 12.5 ft, 20.5 ft, 17.5 ft, 24.5 ft,

19.5 ft, 17.0 ft, 18.5 ft, 12.0 ft, 21.5 ft, 23.0 ft, 13.5 ft, 19.0 ft, 22.5 ft, 19.0 ft

a. Complete the tally chart and the bar graph of the distances.

Tally Chart		
Interval	**Tally**	**Total**
10.0−12.9		
13.0−15.9		
16.0−18.9		
19.0−21.9		
22.0−24.9		

Paper Airplane Contest
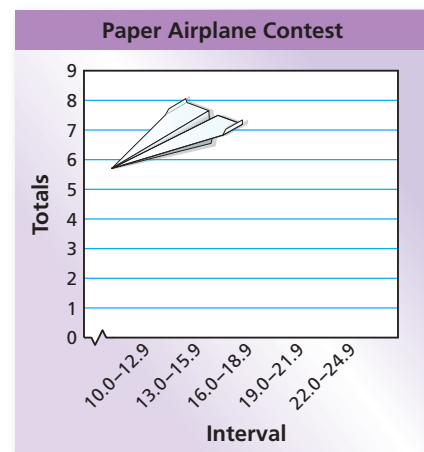
b. Make a different tally chart and bar graph of the distances. Use the following intervals:

10.0–11.9, 12.0–13.9, 14.0–15.9, 16.0–17.9, 18.0–19.9, 20.0–21.9, 22.0–23.9, 24.0–25.9

c. Which graph do you think represents the distances better? Explain.

Data Displays
In this lesson, you will
- make histograms.
- use histograms to analyze data.

Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP4 Model with Mathematics:** Students will construct histograms to represent data sets. Such graphical representation enables students to describe features of the data set.

Motivate

- Ask if students have ever made paper airplanes. If so, how long did their planes usually stay in flight before crashing?

Activity Notes

Activity 1

- ❓ "When you roll a standard number cube, what are the possible outcomes?" 1, 2, 3, 4, 5, 6
- Discuss part (d). Depending upon the number of students, the class results should be close to the theoretical probability for this event, whereas each student's results may not be.
- ❓ **MP4:** "Was it helpful to use a chart and tally marks to keep track of the results from your experiment? Explain." Answers will vary.

Activity 2

- Discuss the context of this problem. Ask for the range of the data. Compare the range to the distances in the classroom to help students learn to judge distances.
- After students finish part (a), ask them to compare the column of tally marks in the chart with the graph at the right known as a histogram. Some might observe that if the tally marks are rotated 90° counterclockwise, the shape resembles the histogram.
- ❓ "How are the intervals in part (b) different from those in part (a)?" Smaller; these intervals have a width of 1.9 versus 2.9.
- ❓ "How will the chart and graph be different?" It's ok for students not to have an answer – it's a big idea!
- **Big Idea:** When the interval is smaller, you have *more* intervals and the tallies become more *spread out*, meaning the number of tallies in each interval will be smaller.
- **MP1 Make Sense of Problems and Persevere in Solving Them:** The size of the interval influences the shape of the histogram. Too many or too few intervals *may* mean that some features of the data go unnoticed.
- ❓ "Looking only at the tally chart or graph, did the person fly the airplane 20.0 feet?" cannot determine
- **Big Idea:** Once the tallies or graph have been made, the actual identity of the data is lost. Only in looking at the raw data can the question above be answered. The interval 19.0–21.9 contains 20.0 but you do not know if it is actually one of the data values.

Common Core State Standards

Common Core State Standards

6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
6.SP.4 Display numerical data in plots on a number line, including . . . histograms . . . .

Previous Learning

Students know how to find the mean, median, and mode and describe the range of a data set.

Technology for the Teacher

Dynamic Classroom

Lesson Plans
Complete Materials List

10.2 Record and Practice Journal

Essential Question How can you use intervals, tables, and graphs to help organize data?

1 ACTIVITY: Conducting an Experiment

Work with a partner. **Check students' work.**

a. Roll a number cube 20 times. Record your results in a tally chart.

	1	2	3	4	5	6
Tally						

Key: | = 1 || = 5

b. Make a bar graph of the totals.

c. Go to the board and enter your totals in the class tally chart.

d. Make a second bar graph showing the class totals. Compare and contrast the two bar graphs.

Auditory

Some students may benefit by hearing how to construct a bar graph. Ask students questions and have them explain how they got their answers. Should you draw a horizontal or vertical bar graph? What increments should you use? What scale should you use and where should it start and end? What is the title of the bar graph?

10.2 Record and Practice Journal

Laurie's Notes

Activity 3

- This activity allows students to collect their own data. It is always more engaging and interesting for students to work with data they generated.
- Use scrap paper and allow only one sheet per student. They need to think before they fold!
- **Management Tip:** Lay tape measures out in a parallel fashion about 6 feet apart on the soccer field or other school grounds.
- Two students at a time fly their planes. They must make a quick read of distance to the nearest foot and call out their distances to their partners who records them.
- Review how to round to the nearest foot before going outside, if necessary.
- Students continue in pairs until all have completed 20 trials. The recorder and pilot switch roles and repeat the process.
- To save class time, you could decrease the number of trials from 20 to, say, 10, but keep in mind that a larger data set is often better for drawing conclusions.
- If the planes hook left or right too far and make a reading difficult, use your judgment to call out a quick measurement.
- **Safety:** Students should fly their planes in the same direction so that no one is accidentally hit in the face.
- **Management Tip:** You may choose to complete this activity inside. If so, it is helpful to mark the floor prior to the start of the activity. For instance, have students measure and mark distances 2 feet apart. When students test their planes, they only have to measure between the marks. This will allow students to review their measuring skills as well as make the activity more time efficient.
- Once the data is collected, complete part (c) inside. To save time, do not have students do any conversions to total inches. The intervals are generally a width of about 2 feet.
- **MP1** and **MP6 Attend to Precision:** Students have fun folding and flying the airplane, but don't forget the mathematics. Raw data does not tell much of a story until it is organized in some fashion.
- ❓ "Where are the mean, median, and mode located in the chart and graph?"
- **Extension:** You are the executive of a paper airplane business. Use the charts and graphs to decide which airplane you would put into production and explain the reasoning of your choice. Answers will vary.

What Is Your Answer?

- Have students share their answers.

Closure

- Explain how the intervals in a tally chart help you organize data. The interval with the most tallies helps you determine the most common answer (the mode).

The tally chart in Activity 2 is also called a *frequency table*. A **frequency table** groups data values into intervals. The **frequency** is the number of values in an interval.

3 ACTIVITY: Developing an Experiment

Work with a partner.

a. Make the airplane shown from a single sheet of $8\frac{1}{2}$-by-11-inch paper. Then design and make your own paper airplane.

1. Fold in half. Then unfold.

2. Fold corners.

3. Fold corners again.

4. Fold in half.

5. Fold wings out on both sides.

6. Fold wing edges up.
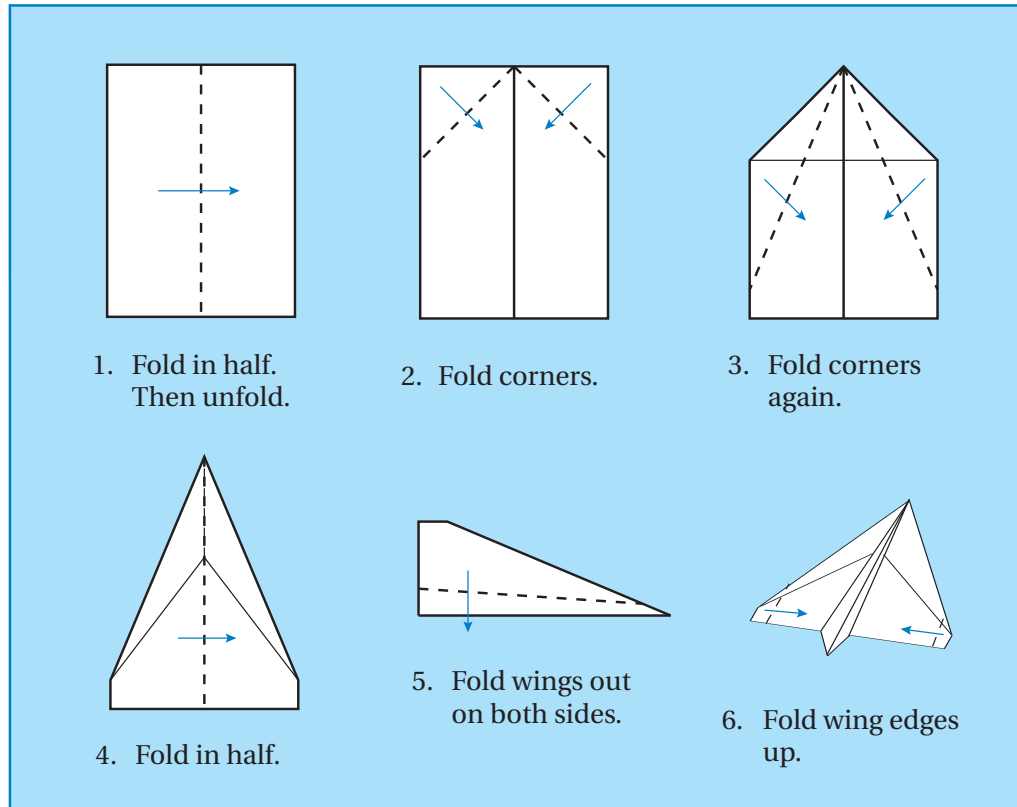
b. **PRECISION** Fly each airplane 20 times. Keep track of the distance flown each time.

c. **MODELING** Organize the results of the flights using frequency tables and graphs. Which airplane flies farther? Explain your reasoning.

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use intervals, tables, and graphs to organize data?

5. What intervals could you use in a graph that displays data whose values range from 40 through 59?

Practice

Use what you learned about organizing data into intervals to complete Exercises 4 and 5 on page 445.

Key Vocabulary
frequency table,
  p. 441
frequency, *p. 441*
histogram, *p. 442*

Key Idea

Histograms

A **histogram** is a bar graph that shows the frequency of data values in intervals of the same size.

The height of a bar represents the frequency of the values in the interval.

CDs Owned

(bar graph: Frequency vs Number of CDs; intervals 0–9, 10–19, 20–29, 30–39)
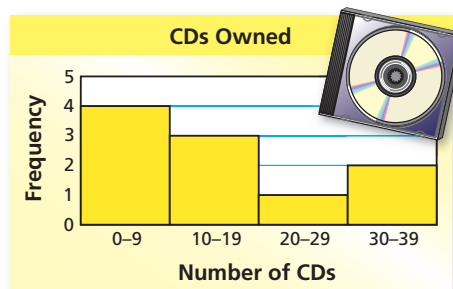
EXAMPLE 1 Making a Histogram

The frequency table shows the numbers of laps that people in a swimming class completed today. Display the data in a histogram.

Step 1: Draw and label the axes.

Step 2: Draw a bar to represent the frequency of each interval.

Number of Laps	Frequency
1–3	11
4–6	4
7–9	0
10–12	3
13–15	6

Laps Completed

(histogram: Frequency vs Number of laps; intervals 1–3, 4–6, 7–9, 10–12, 13–15)
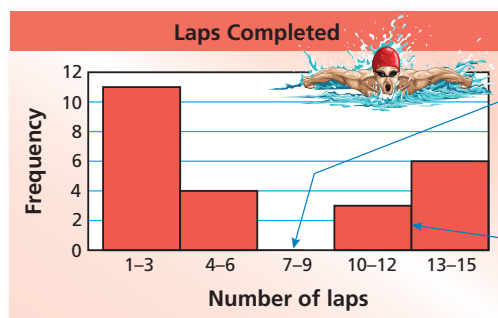
Include any interval with a frequency of 0. The bar height is 0.

There is no space between the bars of a histogram.

On Your Own

Now You're Ready
Exercises 6–8

1. The frequency table shows the ages of people riding a roller coaster. Display the data in a histogram.

Age	10–19	20–29	30–39	40–49	50–59
Frequency	16	11	5	2	4

Laurie's Notes

Introduction

Connect
- **Yesterday:** Students explored how to use tables and graphs to organize data. (MP1, MP4, MP6)
- **Today:** Students will construct and interpret a histogram.

Motivate
- ❓ "How many of you wear swimming caps in a pool?" Answers will vary.
- Competitive swimmers wear swimming caps.
- The largest gathering of people wearing swimming caps was 2,049 at Tybee Island, Georgia's annual Polar Bear Plunge on January 1, 2012.

Lesson Notes

Key Idea
- Students have constructed bar graphs. A histogram is a particular type of bar graph where the data is numeric, and the data is grouped into intervals of equal size. A bar graph includes categorical data (i.e., favorite vegetable) and numeric data.
- ❓ Use the sample shown to ask questions, checking students' understanding of how to read a histogram.
  - "How many people were polled?" 10
  - "How many people polled had 9 or fewer CDs?" 4
  - "How many people polled had 15 CDs?" cannot determine

Example 1
- The intervals for the histogram have been pre-determined. The size of each interval is 3. Students should notice the pattern in the right-end value of each interval: 3, 6, 9, 12, 15.
- Labeling the axes can often present a challenge for students. Students may write a number below the hash mark instead of writing an interval between the hash marks.
- Explain that no space is left between the bars because the intervals are continuous.
- Remind students that the axes are labeled and the histogram is given a title explaining what the data is about.
- Ask questions about the completed histogram.

On Your Own
- Students should recognize that the data is about people ages 10 to 59. It would not be correct to leave a gap for people ages 0 to 9.
- Ask a volunteer to share his or her histogram at the board.

Technology for the **Teacher**

Dynamic Classroom

Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1

The frequency table shows the numbers of T-shirts each person in a class owns. Display the data in a histogram.

T-shirts	Frequency
1–4	3
5–8	6
9–12	13
13–16	0
17–20	4

T-shirts Owned

On Your Own

1.

Riding a Roller Coaster
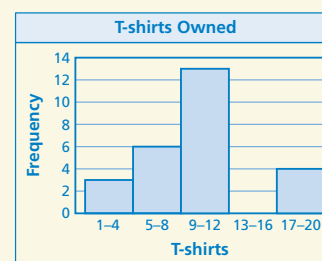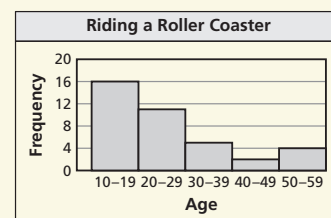
Extra Example 2

The histogram shows the numbers of different instruments each member of a jazz band can play.

Instruments Played
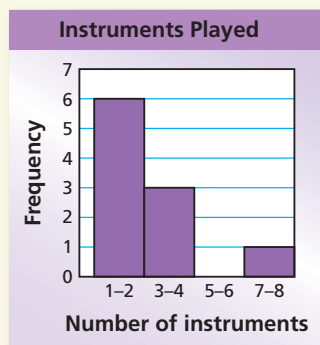
a. Which interval contains the most data values? 1–2

b. How many band members can play more than 2 instruments? 4

c. How many band members can play at most 2 instruments? 6

On Your Own

2. a. 11 students

b. 18 students

Differentiated Instruction

Visual

Give students the following set of data.

1, 2, 2, 4, 5, 7, 7, 8, 9, 11, 13, 14, 14, 16, 17, 19, 20, 20, 21, 22, 24, 24, 25, 26, 26, 29

Instruct half of the class to create a histogram using intervals of length 5 and the other half of the class to create a histogram using intervals of length 10. Compare the shapes of the two graphs and discuss how the choice of intervals affects the interpretation of the data.

Example 2

- The focus of this example is to interpret information given in a histogram. Review the vocabulary *less than*, *at least*, *at most*, and *more than*.
- In addition to the questions posed, ask students to describe the distribution of the data. Also ask about the number of races shown in the graph.
- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Listen for student reasoning as they interpret the histogram.

On Your Own

- This graph is almost symmetric. Ask students what it means for a graph to be symmetric.

EXAMPLE 2 Using a Histogram

The histogram shows the winning speeds at the Daytona 500.
(a) Which interval contains the most data values? **(b)** How many of the
winning speeds are less than 140 miles per hour? **(c)** How many of the
winning speeds are at least 160 miles per hour?

Daytona 500 Winning Speeds
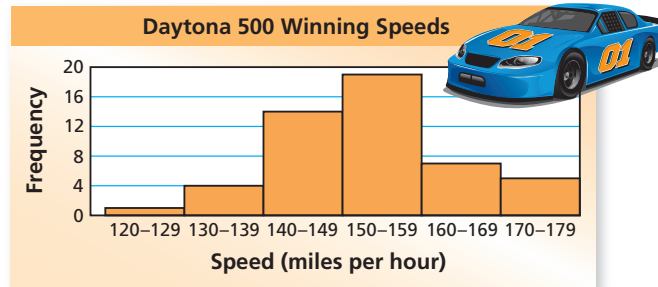
a. The interval with the tallest bar contains the most data values.

⋮⋰ So, the 150–159 miles per hour interval contains the most
data values.

b. One winning speed is in the 120–129 miles per hour interval, and
four winning speeds are in the 130–139 miles per hour interval.

⋮⋰ So, 1 + 4 = 5 winning speeds are less than 140 miles per hour.

c. Seven winning speeds are in the 160–169 miles per hour
interval, and five winning speeds are in the 170–179 miles
per hour interval.

⋮⋰ So, 7 + 5 = 12 winning speeds are at least 160 miles per hour.

On Your Own

2. The histogram shows the numbers of hours that students
in a class slept last night.

 a. How many students slept at least 8 hours?

 b. How many students slept less than 12 hours?

Amount of Sleep

EXAMPLE 3

Comparing Data Displays

The data displays show how many push-ups students in a class completed for a physical fitness test. Which data display can you use to find how many students are in the class? Explain.

Push-Ups

Number of students

| | 0–9 | 10–19 | 20–29 | 30–39 | 40–49 |
Number of push-ups

Push-Ups

40–49 4%
30–39
12%
24%
40%
0–9
10–19
20%
20–29
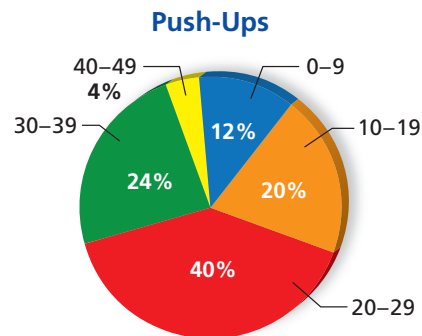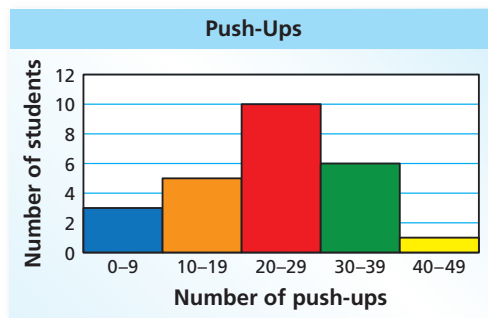
You can use the histogram because it shows the number of students in each interval. The sum of these values represents the number of students in the class. You cannot use the circle graph because it does not show the number of students in each interval.

EXAMPLE 4

Making Conclusions from Data Displays

Which statement _cannot_ be made using the data displays in Example 3?

Ⓐ Twelve percent of the class completed less than 10 push-ups.

Ⓑ Five students completed at least 10 and at most 19 push-ups.

Ⓒ At least one student completed more than 39 push-ups.

Ⓓ Twenty-nine percent of the class completed 30 or more push-ups.

The circle graph shows that 12% completed 0–9 push-ups. So, Statement A can be made.

In the histogram, the bar height for the 10–19 interval is 5, and the bar height for the 40–49 interval is 1. So, Statements B and C can be made.

The circle graph shows that 24% completed 30–39 push-ups, and 4% completed 40–49 push-ups. So, 24% + 4% = 28% completed 30 or more push-ups. Statement D cannot be made.

The correct answer is Ⓓ.

On Your Own

Now You're Ready
Exercises 14 and 15

3. In Example 3, which data display should you use to describe the portion of the entire class that completed 30–39 push-ups?

4. Make two more conclusions from the data displays in Example 3.

Laurie's Notes

Example 3

? "When the sections of a circle graph are labeled with percents, what do you know about the sum of these percents?" They add up to the whole amount, 100% of the data.

- **Connection:** Students should recognize that the colors in each data display represent the same interval of push-ups. Doing 20–29 push-ups was the most frequent amount done by students. The histogram shows that there were 10 students who did 20–29 push-ups, which represents 40% of the class.

? "Could the data set have a mode? Explain." It is possible but you cannot tell for sure. Neither display gives individual values.

Example 4

- **MP3:** Ask different students to read each statement and explain whether or not it can be made using one of the data displays.

? "Can you determine how many students did 15 push-ups from either data display? Explain." no; The histogram only tells how many students did 10–19 push-ups and the circle graph tells the percent of students who did 10–19 push-ups.

- Justify to students that you can add percents using fractions:
$$\frac{24}{100} + \frac{4}{100} = \frac{28}{100} = 28\%.$$

On Your Own

- Have students share the conclusions they wrote for Question 4.

Closure

- Explain how a histogram is similar to a bar graph and how it differs from a bar graph. *Sample answer:* Each bar in a histogram represents the frequency of data values in intervals of the same size. Each bar in a bar graph represents the frequency of a single data value.

1. The *Test Scores* graph is a histogram because the number of students (frequency) achieving the test scores are shown in intervals of the same size (20).

2. The scores falling into the interval 1–20 are outliers because most of the scores are between 41 and 100.

3. No bar is shown on that interval.

Practice and Problem Solving

4–6. See Additional Answers.

7.
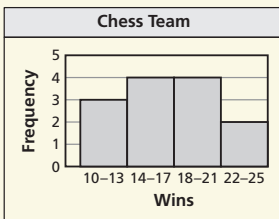
8.
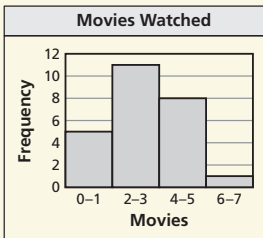
9. There should not be space between the bars of the histogram.
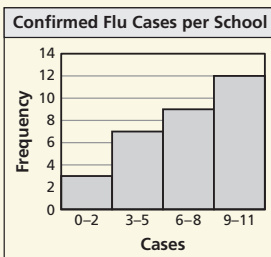
Assignment Guide and Homework Check

Level	Day 1 Activity Assignment	Day 2 Lesson Assignment	Homework Check
Basic	4, 5, 19–23	1–3, 6–11, 13	2, 8–11
Average	4, 5, 19–23	1–3, 7, 9–15	2, 7, 10, 11, 14
Advanced	4, 5, 19–23	1–3, 8, 10–18	2, 11–13, 15

Common Errors

- **Exercises 6–8** Students may struggle with determining how to scale the vertical axis of the histogram. Remind them to use consistent intervals and to base their decision on the frequency of the data. For example, if there is a high frequency, they should count by 5s or 10s, but if there is a low frequency, then they should count by 1s or 2s.

10.2 Record and Practice Journal

Display the data in a histogram.

1.

Birthdays	
Months	Frequency
Jan–Mar	15
Apr–June	9
Jul–Sept	6
Oct–Dec	12

2.

Goals Scored	
Goals	Frequency
0–2	6
3–5	8
6–8	2
9–11	1

1–2. See Additional Answers.

3.

Height Jumped	
Inches	Frequency
0–11	7
12–23	10
24–35	5
36–47	2

4.

Money Spent	
Dollars	Frequency
0–19	3
20–39	8
40–59	8
60–79	15

3–4. See Additional Answers.

5. The histogram shows the times students ran the mile in gym class.

a. Which interval contains the fewest data values?
9–10 minutes

b. How many students are in the class?
34 students

c. What percent of students ran the mile in 12 minutes or less?
about 41%
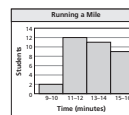
Vocabulary and Concept Check

1. **VOCABULARY** Which graph is a histogram? Explain your reasoning.

2. **REASONING** Describe the outliers in the histogram.

3. **REASONING** How can you tell when an interval of a histogram has a frequency of zero?

Expense Report

Test Scores
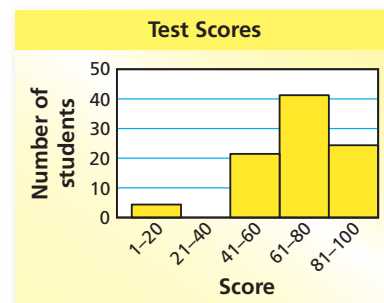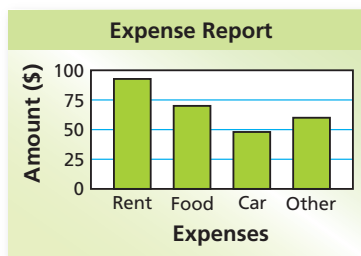
Practice and Problem Solving

Make a tally chart and a bar graph of the data.

4.

Members of Book Clubs			
6	17	13	19
13	9	18	24
11	15	21	14

5.

Points Scored				
42	45	57	39	55
38	48	36	48	46
51	29	45	54	42

Display the data in a histogram.

6.

States Visited	
States	**Frequency**
1–5	12
6–10	14
11–15	6
16–20	3

7.

Chess Team	
Wins	**Frequency**
10–13	3
14–17	4
18–21	4
22–25	2

8.

Movies Watched	
Movies	**Frequency**
0–1	5
2–3	11
4–5	8
6–7	1

9. **ERROR ANALYSIS** Describe and correct the error made in displaying the data in a histogram.

Confirmed Flu Cases per School	
Cases	**Frequency**
0–2	3
3–5	7
6–8	9
9–11	12
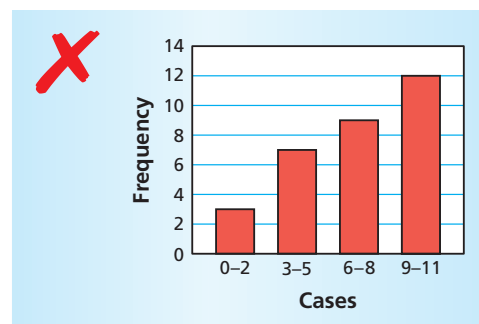
2 **10. MAGAZINES** The histogram shows the number of magazines read last month by the students in a class.

a. Which interval contains the fewest data values?

b. How many students are in the class?

c. What percent of the students read less than six magazines?

d. Can you find the mean or the median of the data? Explain.

Magazines

Amount of Time to Download Songs
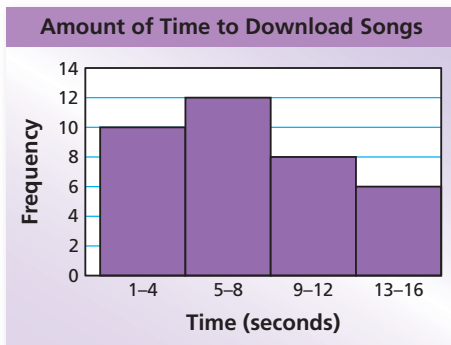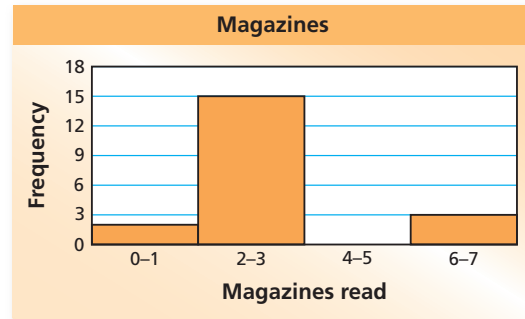
11. ERROR ANALYSIS Describe and correct the error made in reading the histogram.

✗ 12% of the songs took 5–8 seconds to download.

12. VOTING The histogram shows the percent of the voting-age population that voted in a recent presidential election. Explain whether the graph supports each statement.

a. Only 40% of one state voted.

b. In most states, between 50% and 64.9% voted.

c. The mode of the data is between 55 and 59.9.

Presidential Election
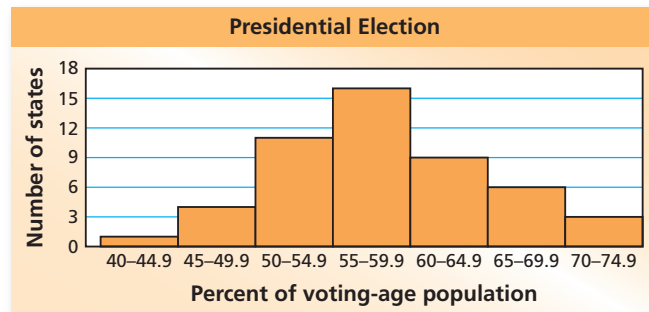
13. PROBLEM SOLVING The histograms show the areas of counties in Pennsylvania and Indiana. Which state do you think has the greater area? Explain.

Pennsylvania
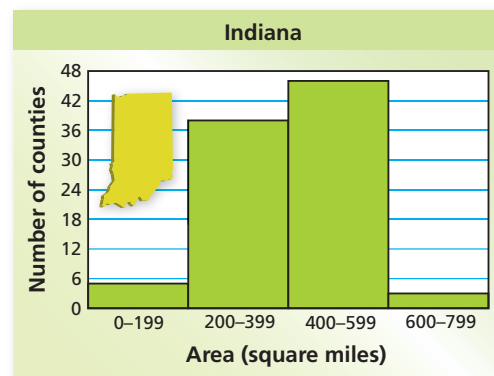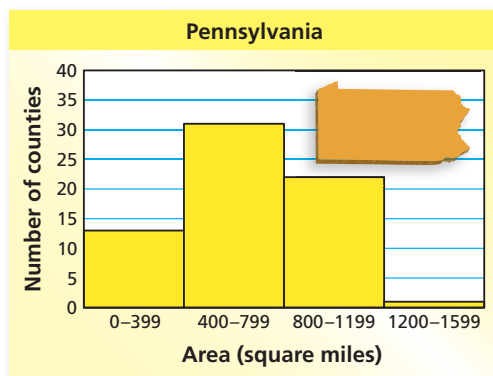
Indiana

For Your Information

- **Exercise 13** Pennsylvania has a total area of 46,055 square miles. Indiana has a total area of 36,418 square miles.

Common Errors

- **Exercise 10** Students may say that the interval 0–1 has the fewest data values, but it is really 4–5. To help them answer this question, ask them to label each interval with the frequency of that interval. Writing the frequency helps students to read the histogram.
- **Exercise 13** Students may only look at the frequency for each graph and say that Indiana has the greater area. Encourage them to look at the intervals as well. The two graphs are drawn using different intervals and cannot be compared strictly by the heights of the bars.

10. a. 4–5

    b. 20 students

    c. 85%

    d. no; You only know what interval each of the data values falls into, not the specific data values.

11. The frequency is the number of songs not the percent of songs. The statement should be "12 of the songs took 5–8 seconds to download."

12. a. no; The histogram shows that only one state fell in the interval of 40–44.9%. This state did not necessarily have 40% of possible voters vote.

    b. yes; 36 states are between 50% and 64.9%.

    c. no; The 55–59.9 interval has the highest frequency, but does not necessarily contain the mode of the data.

13. Pennsylvania; You can see from the intervals and frequencies that Pennsylvania counties are greater in area, which makes up for it having fewer counties.

English Language Learners

Visual

Histograms and bar graphs have a similar appearance. Students may confuse the two. Compare and contrast the two types of data displays. Both use the lengths of bars to represent data. The bars of a histogram represent numerical intervals and are touching. The bars of a bar graph represent categories and are separated.

14. stem-and-leaf plot; You need to know the specific data values, the intervals in the histogram do not give enough information.

15. See Additional Answers.

16. no; You only know what interval each of the data values falls into, not the specific data values.

17. See *Taking Math Deeper*.
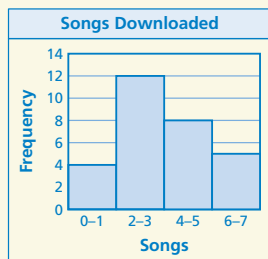
18. 2, 2.5, 3

Fair Game Review

19. 45

20. 27

21. 22.4

22. 51.2

23. D

Mini-Assessment

The table shows the number of songs downloaded last month by your friends.

Songs Downloaded	
Songs	**Frequency**
0–1	4
2–3	12
4–5	8
6–7	5

1. Display the data in a histogram.

Songs Downloaded

2. Which interval contains the most data values? The 2–3 songs interval contains the most data values.

3. Which interval contains the fewest data values? The 0–1 songs interval contains the fewest data values.

4. What is the difference between the 2–3 and 4–5 songs intervals? 4

Taking Math Deeper

Exercise 17

This exercise points out to students that making a histogram is as much an *art* as a *science*. The big question lies in "how many intervals should I use?"

With too many intervals, most distributions will appear flat. With too few intervals, most distributions will have only a few tall spikes.

1 Order the data.

51, 54, 55, 57, 59, 67, 68, 70, 71, 73, 75, 76, 77, 77, 78, 78, 79, 79, 80, 80, 81, 81, 82, 82, 83, 83, 84, 85, 85, 88

2 **a.** Make a frequency distribution and histogram.

51–55	III
56–60	II
61–65	
66–70	III
71–75	III
76–80	IIII IIII
81–85	IIII IIII
86–90	I

Guide Dogs
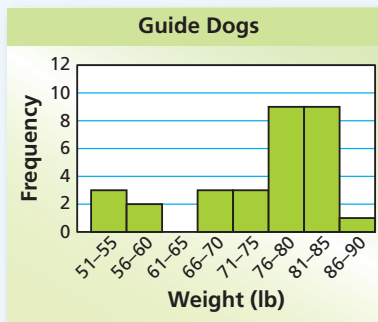
3 **b.** Make a frequency distribution and histogram using different-sized intervals.

51–60	IIII
61–70	III
71–80	IIII IIII II
81–90	IIII IIII

Less bars

Guide Dogs
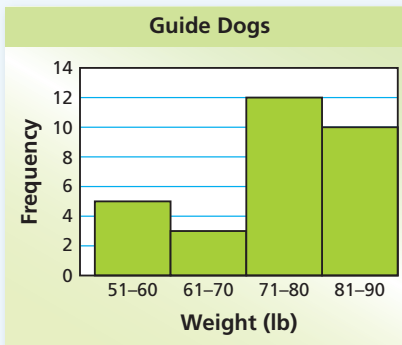
c. The second histogram has four intervals, and it does not have a gap as in the first histogram.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Start the next section

③ **14.** **GARBAGE** The data displays show how many pounds of garbage apartment residents produced in 1 week. Which data display can you use to find how many residents produced more than 25 pounds of garbage? Explain.
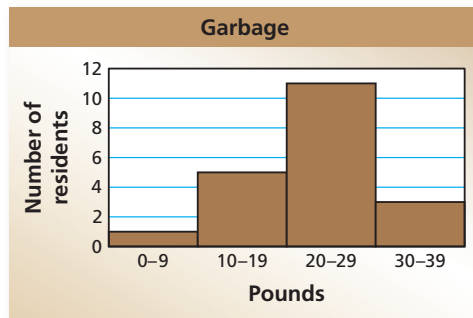
Garbage

Garbage

Stem	Leaf
0	9
1	0 5 8 8 9
2	1 2 5 5 6 7 7 7 9 9 9
3	2 3 3

Key: 1|5 = 15 pounds

15. **REASONING** Determine whether you can make each statement by using the data displays in Exercise 14. Explain your reasoning.

  a. One resident produced 10 pounds of garbage.

  b. Twelve residents produced between 20 and 29 pounds of garbage.

16. **NUMBER SENSE** Can you find the range and the interquartile range of the data in Exercise 7? If so, find them. If you cannot find them, explain why not.

17. **CRITICAL THINKING** The table shows the weights of guide dogs enrolled in a training program.

  a. Make a histogram of the data starting with the interval 51–55.

  b. Make another histogram of the data using different-sized intervals.

  c. Compare and contrast the two histograms.

Weights (pounds)					
81	88	57	82	70	85
71	51	82	77	79	77
83	80	54	80	81	73
59	84	75	76	68	78
83	78	55	67	85	79

18. **Logic** What are the possible values for the median in Exercise 10?

Fair Game Review What you learned in previous grades & lessons

Find the percent of the number. *(Section 5.6)*

19. 25% of 180 **20.** 30% of 90 **21.** 16% of 140 **22.** 64% of 80

23. **MULTIPLE CHOICE** Which is the solution of the inequality represented by "Four times a number n is at least 28"? *(Section 7.7)*

  Ⓐ $n < 7$ Ⓑ $n > 7$ Ⓒ $n \le 7$ Ⓓ $n \ge 7$

You can use a **word magnet** to organize information associated with a vocabulary word. Here is an example of a word magnet for histogram.

Histogram

A histogram is a bar graph that shows the frequency of data values in intervals of the same size.

The height of a bar represents the frequency of the values in the interval.

You can make a histogram from a frequency table. A frequency table groups data values into intervals. The frequency is the number of data values in an interval.

The histogram shows the number of times students were absent from homeroom this year.

Homeroom Absences

Frequency
15
10
5
0
0–1 2–3 4–5 6–7
Number of absences

On Your Own

Make a word magnet to help you study this topic.

1. stem-and-leaf plot

After you complete this chapter, make word magnets for the following topics.

2. shapes of distributions

3. box-and-whisker plot

4. Choose three other topics that you studied earlier in this course. Make a word magnet for each topic.

Wags tail | **BEAGLE** | ♥'s Treats
Cool | | Genius
4 Legs | | 0 Fleas

I see "humble" didn't make the list.
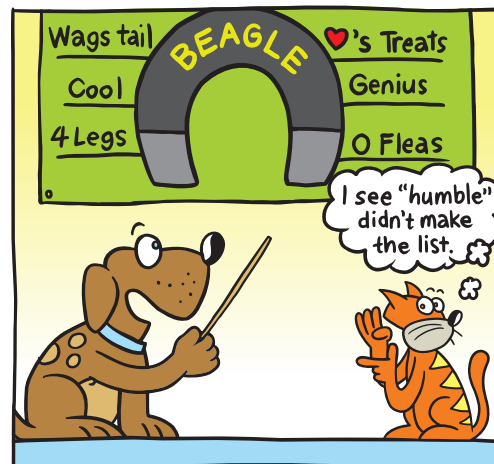
"How do you like the word magnet I made for 'Beagle'?"

Sample Answer

1.
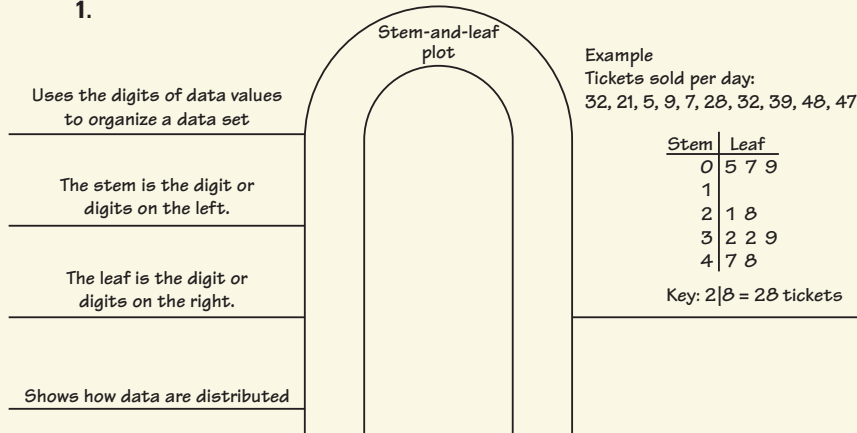
Uses the digits of data values to organize a data set

The stem is the digit or digits on the left.

The leaf is the digit or digits on the right.

Shows how data are distributed

Stem-and-leaf plot

Example
Tickets sold per day:
32, 21, 5, 9, 7, 28, 32, 39, 48, 47

Stem	Leaf
0	5 7 9
1	
2	1 8
3	2 2 9
4	7 8

Key: 2|8 = 28 tickets

About this Organizer

A **Word Magnet** can be used to organize information associated with a vocabulary word or term. As shown, students write the word or term inside the magnet. Students write associated information on the blank lines that "radiate" from the magnet. Associated information can include, but is not limited to: other vocabulary words or terms, definitions, formulas, procedures, examples, and visuals. This type of organizer serves as a good summary tool because any information related to a topic can be included.

Technology for the *Teacher*

Editable Graphic Organizer

1. Cans Collected Each Month

Stem	Leaf
5	5 8 9
6	1 8
7	6 9
8	0 3 4
9	0 2

Key: 6 | 1 = 61 cans

2. Miles Driven Each Day

Stem	Leaf
0	9
1	0 1 2 5 6 6 8
2	0 0 1 8
3	5 7
4	
5	0

Key: 3 | 5 = 35 miles

3. Age of Tortoises

Stem	Leaf
8	3 5 6
9	2 9
10	0 4
11	0 5
12	4 9
13	0

Key: 9 | 2 = 92 years

4. Kilometers Run Each Day

Stem	Leaf
2	0 5
3	0 1 1 5 8 9
4	5
5	0 6
6	0 1 2 2

Key: 2 | 0 = 2.0 kilometers

5–10. See Additional Answers.

Technology for the ***Teacher***

Online Assessment
Assessment Book
ExamView® Assessment Suite

Alternative Quiz Ideas

100% Quiz	Math Log
Error Notebook	Notebook Quiz
Group Quiz	Partner Quiz
Homework Quiz	Pass the Paper

Group Quiz

Students work in groups. Give each group a large index card. Each group writes five questions that they feel evaluate the material they have been studying. On a separate piece of paper, students solve the problems. When they are finished, they exchange cards with another group. The new groups work through the questions on the card.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Lesson Tutorials *BigIdeasMath.com*	Resources by Chapter • Enrichment and Extension • Technology Connection Game Closet at *BigIdeasMath.com* Start the next section

Check It Out
Progress Check
BigIdeasMath.com

Make a stem-and-leaf plot of the data. *(Section 10.1)*

1.

Cans Collected Each Month			
80	90	84	92
76	83	79	59
68	55	58	61

2.

Miles Driven Each Day				
21	18	12	16	10
16	9	15	20	28
35	50	37	20	11

3.

Ages of Tortoises			
86	99	100	124
92	85	110	130
115	129	83	104

4.

Kilometers Run Each Day				
6.0	5.6	6.2	3.0	2.5
3.5	2.0	5.0	3.9	3.1
6.2	3.1	4.5	3.8	6.1

Display the data in a histogram. *(Section 10.2)*

5.

Soccer Team Goals	
Goals per Game	Frequency
0–1	5
2–3	4
4–5	0
6–7	1

6.

Minutes Practiced	
Minutes	Frequency
0–19	8
20–39	10
40–59	11
60–79	2

7.

Poems Written for Class	
Poems	Frequency
0–4	6
5–9	16
10–14	4
15–19	2
20–24	2

8. WEIGHTS The weights (in ounces) of nine packages are 7, 22, 16, 12, 6, 18, 15, 13, and 25. Make a stem-and-leaf plot of the data. Describe the distribution of the data. *(Section 10.1)*

9. REBOUNDS The histogram shows the number of rebounds per game for a middle school basketball player this season. *(Section 10.2)*

a. Which interval contains the most data values?

b. How many games did the player play this season?

c. What percent of the games did the player have 4 or more rebounds?

Rebounds per Game
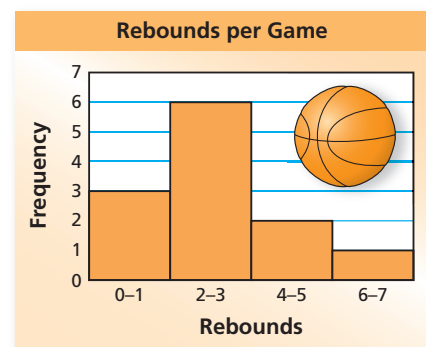
Stem	Leaf
0	6 8 8 9
1	0 1 2 3 7 8
2	0

Key: 0|9 = 9 hours

10. STAGE CREW The stem-and-leaf plot shows the number of hours 11 stage crew members spent building sets. Find the mean, median, mode, range, and interquartile range of the data. *(Section 10.1)*

10.3 Shapes of Distributions

Essential Question How can you describe the shape of the distribution of a data set?

1 ACTIVITY: Describing the Shape of a Distribution

Work with a partner. The lists at the left show the last four digits of a set of phone numbers in a phone book.

```
-7253      -8678
-7290      -2063
-7200      -2911
-1192      -2103
-1142      -4328
           -7826
-3500      -7957
-2531      -7246
-2079      -2119
-5897      -7845
-5341      -1109
-1392      -9154
-5406
-7875
-7335
-0494
-9018
-2184
-2367
```

a. Create a list that represents the last digit of each phone number shown. Make a dot plot of the data.

b. In your own words, how would you describe the shape of the distribution? What single word do you think you can use to identify this type of distribution? Explain your reasoning.

2 ACTIVITY: Describing the Shape of a Distribution

Work with a partner. The lists at the right show the first three digits of a set of phone numbers in a phone book.

```
538-      664-
438-      664-
664-      538-
761-      855-
868-      664-
          538-
735-      654-
694-      654-
599-      725-
725-      538-
556-      799-
555-      764-
456-
736-
664-
576-
664-
664-
725-
```

a. Create a list that represents the first digit of each phone number shown. Make a dot plot of the data.

b. In your own words, how would you describe the shape of the distribution? What single word do you think you can use to identify this type of distribution? Explain your reasoning.

c. In your dot plot, draw a vertical line through the middle of the data set. What do you notice?

d. Repeat part (c) for the dot plot you constructed in Activity 1. What do you notice? Compare the distributions from Activities 1 and 2.

Data Displays
In this lesson, you will
• describe shapes of distributions.

Laurie's Notes

Introduction

Standards for Mathematical Practice
- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Students will describe the shape of a distribution and make a connection to the measures of center and variation studied in Chapter 9.

Motivate
- ❓ "Who said, 'Mr. Watson, come here; I want to see you'?" Alexander Graham Bell
- ❓ When did Alexander Graham Bell utter those words?" March 10, 1876
- Have a general discussion about the impact of telephones, and now, cellular phones, on society.

Activity Notes

Activity 1
- Ask a volunteer to read the problem.
- **Management Tip:** Have one student read the numbers from the lists and one student fill in the dot plot. At this point, students should not be overwhelmed by the amount of data. Each value is a single digit and students have been constructing line/dot plots now for several grades.
- Discuss telephone numbers. The area code and exchanges for a particular area are not randomly distributed. The last digits of the given telephone numbers, which can be considered random, are evenly distributed. So, the dot plot appears flat or rectangular.

Activity 2
- This activity is related to Activity 1 and involves exchanges. A telephone book will have a limited number of exchanges so the dot plot will look different than that of Activity 1. Some numbers (0, 1, 2, 3, and 9 in this activity) do not appear at all.
- **MP3:** Listen to student reasoning as they compare the distributions from Activities 1 and 2.
- Review lines of symmetry to extend students' knowledge of symmetric shapes. Also, review the discussion in Section 9.1 regarding the basic characteristics of a distribution such as peaks, gaps, and clusters.
- For homework, you could have students repeat Activities 1 and 2 using a sample of 100 phone numbers.

Previous Learning

Students should know how to use line plots (or dot plots), stem-and-leaf plots, and histograms to display and analyze data.

Technology for the **Teacher**

Dynamic Classroom

Lesson Plans
Complete Materials List

10.3 Record and Practice Journal

Differentiated Instruction

Inclusion

In this section, students are expected to make histograms. Provide a resource page of blank axes with horizontal lines and vertical lines. Students add the vertical and horizontal scales and labels and color the bars to represent the data.

Activity 3

- Discuss the meaning of *skewed*. Look around the classroom for an example of something that is skewed.
- ❓ "What does an age of 0 mean?" a fairly new cell phone
- After students finish the activity, ask a volunteer to share his or her dot plot with the class.
- **MP3:** Listen to student reasoning regarding why this type of distribution is called a *skewed distribution*.
- ❓ "Can you think of other data sets that have skewed distributions?" Answers will vary.
- Mention that distributions can have many different shapes. Point out that the first three activities identify some of the more common shapes.
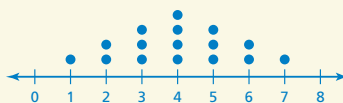
Activity 4

- This activity could be optional if you are not teaching Extension 10.3.
- Review mean and median from Chapter 9.
- Encourage students to recognize and use shortcuts to find the means. For instance, in Activity 1, because there are 3 of each number, students can multiply each value by 3 and then add each product to find the sum of the data. For the data in Activity 2, ask whether anyone can tell "by inspection" that the mean is 6.
- ❓ "How can you find the median in a dot plot?" Because the data values in a dot plot are in order, students should describe moving from the least and greatest numbers toward the center of the distribution.
- In answering part (c), students may notice that the mean and median are equal for the data sets in Activities 1 and 2. If students are stuck trying to come up with an answer, then suggest that the mean uses all of the data values in its calculation, so it could be considered more reliable. In Activity 3, as well as the median being equal to 1, the mode is 1 and most of the data are clustered around 1. The mean is more affected by the outliers than the median, so the median should be used.
- Student answers to part (c) will influence their answers to part (d).
- To help students with part (d), review interquartile range and mean absolute deviation from Chapter 9.

What Is Your Answer?

- Question 6 helps students understand that a distribution has a visual component in addition to numeric values.

Closure

- Sketch a dot plot that has a symmetric distribution and a mean of 4.
  Sample answer:

10.3 Record and Practice Journal

b. In your own words, how would you describe the shape of the distribution? What single word do you think you can use to identify this type of distribution? Explain your reasoning.
symmetric or reflective

c. In your dot plot, draw a vertical line through the middle of the data set. What do you notice?
The line divides the data set into two halves that are mirror images of each other.

d. Repeat part (c) for the dot plot you constructed in Activity 1. What do you notice? Compare the distributions from Activities 1 and 2.
You can draw a vertical line through the middle of the data to get two halves that are mirror images of each other.

3 ACTIVITY: Describing the Shape of a Distribution

Work with a partner. The table shows the ages of cellular phones owned by a group of students.

a. Make a dot plot of the data.

Ages of Cellular Phones (years)					
0	1	0	6	4	
2	3	5	1	1	
0	1	2	3	1	
0	0	1	1	1	
7	1	4	2	2	
0	2	0	1	2	

b. In your own words, how would you describe the shape of the distribution? Compare it to the distributions in Activities 1 and 2.
Most of the data are on the left.

c. Why do you think this type of distribution is called a *skewed distribution*?
The distribution is "slanted" in a certain direction.

4 ACTIVITY: Finding Measures of Center

Work with a partner.

a. Find the mean and median of the data sets in Activities 1–3.

Activity 1	Activity 2	Activity 3
mean: 4.5	mean: 6	mean: 1.8
median: 4.5	median: 6	median: 1

b. What do you notice about the means and medians of the data sets and the shapes of the distributions? Explain.
symmetric – Mean and median are the same.
skewed – Mean and median will vary.

c. Which measure of center do you think best describes the data set in Activity 2? in Activity 3? Explain your reasoning.
Activity 2: mean
Activity 3: median

d. Using your answers to part (c), decide which measure of variation you think best describes the data set in Activity 2. Which measure of variation do you think best describes the data set in Activity 3? Explain your reasoning.
Activity 2: MAD
Activity 3: IQR

What Is Your Answer?

5. IN YOUR OWN WORDS How can you describe the shape of the distribution of a data set?
identify peaks, gaps, and outliers; look at the distribution

6. Name two other ways you can describe the distribution of a data set.
You can describe the center and the variation of a data distribution.

The Meaning of a Word ● Skewed

When something is **skewed**, it has a slanted direction or position.

3 ACTIVITY: Describing the Shape of a Distribution

Work with a partner. The table shows the ages of cellular phones owned by a group of students.

Ages of Cellular Phones (years)				
0	1	0	6	4
2	3	5	1	1
0	1	2	3	1
0	0	1	1	1
7	1	4	2	2
0	2	0	1	2

a. Make a dot plot of the data.

b. In your own words, how would you describe the shape of the distribution? Compare it to the distributions in Activities 1 and 2.

c. Why do you think this type of distribution is called a *skewed distribution*?

4 ACTIVITY: Finding Measures of Center

Math Practice

Use Prior Results

How is the distribution of the data related to the mean and the median?

Work with a partner.

a. Find the means and the medians of the data sets in Activities 1–3.

b. What do you notice about the means and the medians of the data sets and the shapes of the distributions? Explain.

c. Which measure of center do you think best describes the data set in Activity 2? in Activity 3? Explain your reasoning.

d. Using your answers to part (c), decide which measure of variation you think best describes the data set in Activity 2. Which measure of variation do you think best describes the data set in Activity 3? Explain your reasoning.

What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you describe the shape of the distribution of a data set?

6. Name two other ways you can describe the distribution of a data set.

Practice Use what you learned about shapes of distributions to complete Exercises 3 and 4 on page 454.

10.3 Lesson

You can use dot plots and histograms to identify shapes of distributions.

Key Ideas

Symmetric and Skewed Distributions

Study Tip

If all the dots of a dot plot or bars of a histogram are about the same height, then the distribution is a *flat*, or *uniform*, distribution. A uniform distribution is also symmetric.
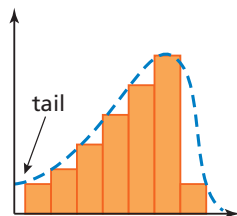
Skewed left

- The "tail" of the graph extends to the left.
- Most data are on the right.
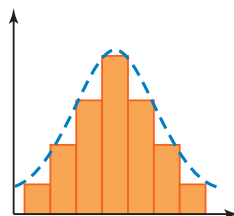
tail

Symmetric

- The left side of the graph is a mirror image of the right side of the graph.
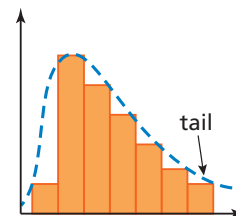
Skewed right

- The "tail" of the graph extends to the right.
- Most data are on the left.

tail

EXAMPLE 1 Describing the Shapes of Distributions

Describe the shape of each distribution.

a. **Daily Snowfall Amounts**
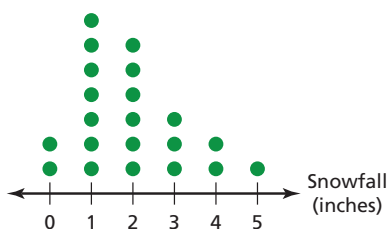
Snowfall (inches)
0 1 2 3 4 5

Most of the data are on the left, and the tail extends to the right.

So, the distribution is skewed right.

b. **Passes Thrown**

Frequency
20
15
10
5
0
0–9 10–19 20–29 30–39 40–49 50–59 60–69
Number of passes
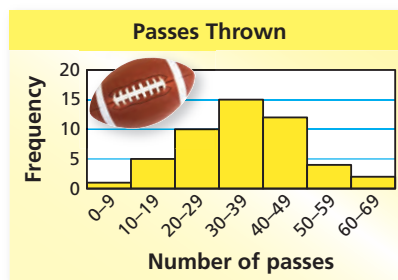
The left side of the graph is approximately a mirror image of the right side of the graph.

So, the distribution is symmetric.

On Your Own

1. Describe the shape of the distribution.

Daily Spam Emails Received

Number of emails
1 2 3 4 5 6
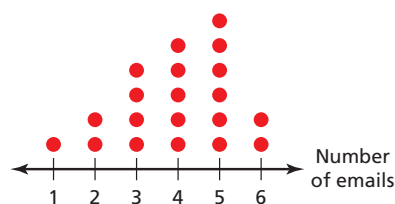
Laurie's Notes

Introduction

Connect
- **Yesterday:** Students made dot plots and explored the shapes of distributions. (MP3)
- **Today:** Students will describe and compare shapes of distributions.

Motivate
- **Story time:** Tell the students that yesterday in the teacher's lounge, three teachers were describing the results of a recent test they gave.
  - **Teacher A:** Lots of low scores; Ds and Fs, some Cs, a few Bs, and one A
  - **Teacher B:** Lots of high scores; As and Bs, some Cs, a few Ds, and one F
  - **Teacher C:** Lots of average scores; Cs, some Bs and Ds, and a few Fs and As
- Ask students to quickly sketch a histogram for each of the three sets of test scores. They should sketch something similar to the three distributions in the Key Ideas.
- Discuss the attributes of a histogram: similar to a bar graph, intervals are all the same size, and frequencies of data values are displayed in the vertical direction by the bar heights.

Lesson Notes

Key Ideas
- Draw a sketch of each type of distribution and label it: skewed left, symmetric, and skewed right.
- Connect the distributions to the test score descriptions above.
- Explain that a dashed line can be drawn to help identify the greatest frequency and to help highlight the basic shape of a distribution.
- **?** "Do all distributions fall into one of these three categories?" no; There are many different types of distributions.
- Skewed left is also called *negatively* skewed and skewed right is also called *positively* skewed. This may help students remember by relating skewness to the number line: the negative direction is to the left and the positive direction is to the right.

Example 1
- **?** "How many days are represented by the distribution in part (a)?" 21
- **MP4 Model with Mathematics:** Ask students to make a statement about the data represented by the histogram in part (b). Answers will vary.

On Your Own
- **Neighbor Check:** Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.
- **Extension:** If time allows, ask questions about the mean, median, mode, and number of pieces of data.

Goal Today's lesson is describing and comparing shapes of distributions.

Extra Example 1

Describe the shape of each distribution.

a. **Monthly Rainfall Amounts**

skewed left

b.

Home Runs Hit
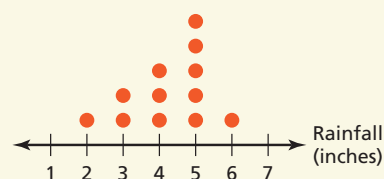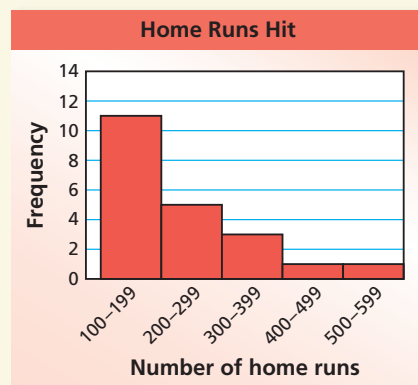
skewed right

On Your Own

1. skewed left

### English Language Learners

Notebook Development

Provide English language learners with a handout of a simplified version of the notes from this lesson. This will allow the student to focus on the concepts being taught. Students should highlight information and add additional notes to the handout during the classroom discussion. They can refer to the page as they work through the exercises.

Extra Example 2

The frequency table shows the quiz scores (in percent) in a science class. Display the data in a histogram. Describe the shape of the distribution.
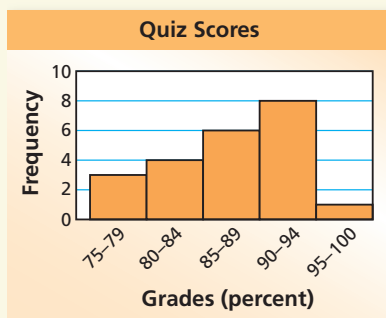
Scores (percent)	Frequency
75–79	4
80–84	6
85–89	11
90–94	7
95–100	4
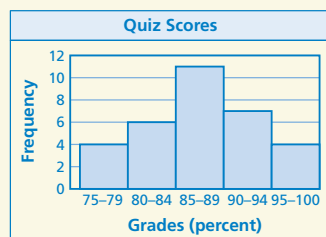
Quiz Scores

symmetric distribution

Extra Example 3

The histogram shows the quiz scores (in percent) in a social studies class in the same school as in Extra Example 2.

Quiz Scores

a. Describe the shape of the distribution. skewed left

b. Which class has the better grade? the social studies class

On Your Own

2. See Additional Answers.

Laurie's Notes

Example 2

- This is a good review of drawing a histogram. The intervals are given, which are often difficult for students to determine.
- Ask for a volunteer to read the values in the frequency table as you write them. Have students describe the process for creating the histogram.
- Ask a few students to do their work on transparencies if a document camera is not available.

Example 3

- Discuss the data represented in the histogram.
- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** The explanations that students offer provide additional practice with constructing viable arguments. Students should mention that the intervals in both histograms are the same.

On Your Own

- After students have finished, ask a volunteer to share his or her histogram at the board or under a document camera.
- Students should observe that the intervals are different in Question 2 than in Examples 2 and 3.

Closure

- **Exit Ticket:** Return to the example in the Motivate section. A fourth teacher describes the results of a test: lots of As and Fs, some Bs and Ds, and a few Cs. Sketch a histogram for the test scores. How would you describe the distribution? *Sample answer:* The distribution is symmetric, but most of the data does not lie near the mean.

EXAMPLE **2**

Describing the Shape of a Distribution

Ages	Frequency
10–13	1
14–17	3
18–21	7
22–25	12
26–29	20
30–33	18
34–37	3

The frequency table shows the ages of people watching a comedy in a theater. Display the data in a histogram. Describe the shape of the distribution.

Draw and label the axes. Then draw a bar to represent the frequency of each interval.

Most of the data are on the right, and the tail extends to the left.

∴ So, the distribution is skewed left.

Comedy Attendance
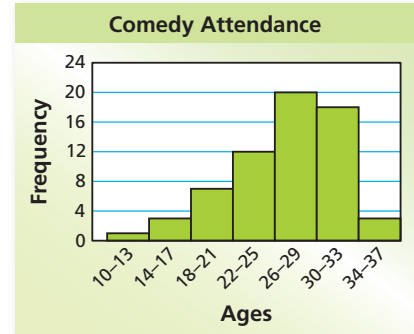
EXAMPLE **3**

Comparing Shapes of Distributions

The histogram shows the ages of people watching an animated movie in the same theater as in Example 2.

a. Describe the shape of the distribution.

Most of the data are on the left, and the tail extends to the right.

∴ So, the distribution is skewed right.

Animated Movie Attendance

b. Which movie has an older audience?

The intervals in the histograms are the same. Most of the data for the animated movie are on the left, while most of the data for the comedy are on the right. This means that the people watching the comedy are generally older than the people watching the animated movie.

∴ So, the comedy has an older audience.
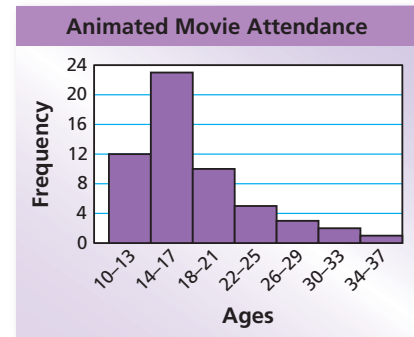
On Your Own

Now You're Ready
Exercise 9

2. The frequency table shows the ages of people watching a historical movie in a theater.

Ages	10−19	20−29	30−39	40−49	50−59	60−69
Frequency	3	18	36	40	14	5

a. Display the data in a histogram. Describe the shape of the distribution.

b. Compare the distribution of the data to the distributions in Examples 2 and 3. What can you conclude?
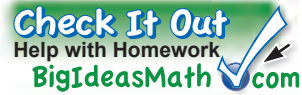
✓ Vocabulary and Concept Check

1. **VOCABULARY** How does the shape of a symmetric distribution differ from the shape of a skewed distribution?

2. **VOCABULARY** For a distribution that is skewed right, which direction does the tail extend? Where do most of the data lie?

✎ Practice and Problem Solving

Make a dot plot of the data. In your own words, how would you describe the shape of the distribution?
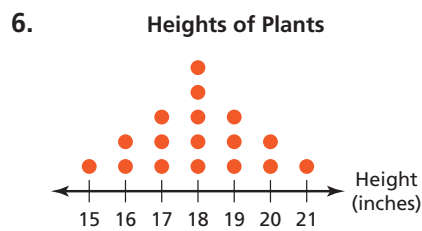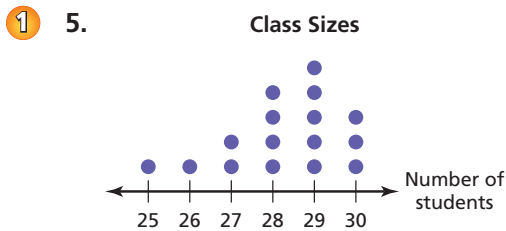
3.

Miles Run per Day									
1	4	2	0	3	2	1	2	4	2	3
2	1	6	3	2	4	0	5	3	1	5

4.

Raffle Tickets Sold							
15	12	16	15	13	14	16	13
13	16	14	12	15	12	14	

Describe the shape of each distribution.

① 5.

Class Sizes

6.

Heights of Plants

7.

Travel Time to School

8.

Ages of People at a Concert
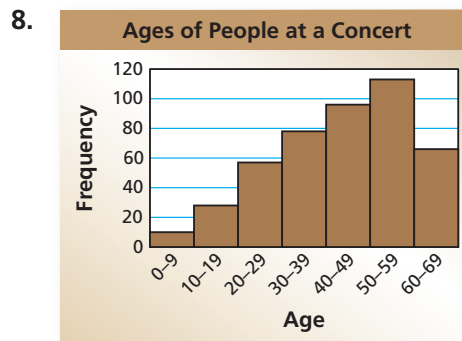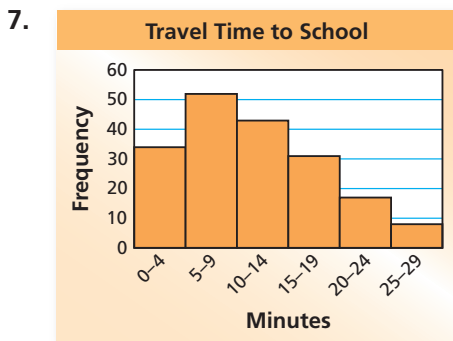
② ③ 9. **POLICE** The frequency table shows the years of service for the police officers of Jones County and Pine County. Display the data for each county in a histogram. Describe the shape of each distribution. Which county's police force has less experience? Explain.

Years of Service	0–3	4–7	8–11	12–15	16–19	20–23	24–27
Frequency for Jones County	7	15	17	12	8	5	3
Frequency for Pine County	3	5	9	14	10	6	2

Assignment Guide and Homework Check

Level	Day 1 Activity Assignment	Day 2 Lesson Assignment	Homework Check
Basic	3, 4, 15–17	1, 2, 5–13 odd	5, 7, 9, 11
Average	3, 4, 15–17	1, 2, 5–9 odd, 10–14	5, 7, 12, 13
Advanced	3, 4, 15–17	1, 2, 5–14	6, 7, 8, 11, 13

Common Errors

- **Exercises 3–10** Students may confuse "skewed left" and "skewed right." Point out that the "tail" of the graph of a distribution determines the direction of the skew. That is, in a skewed left distribution, the tail extends to the left, and in a skewed right distribution, the tail extends to the right.

10.3 Record and Practice Journal

Describe the shape of each distribution.

1. **Gift Bags**

Number of bags
13 14 15 16 17

symmetric

2. **Sidewalks**

Length (feet)
30 31 32 33 34 35

skewed right

3. **Concert Length**

Frequency
0–59 60–119 120–179 180–239
Minutes

skewed right

4. **Heights of Trees**

Frequency
0–2 3–5 6–8 9–11 12–14 15–17
Height (feet)

symmetric

5. The frequency table shows the number of months each person has been a member of a golf league. Display the data in a histogram. Describe the shape of the distribution.

Months as a Member	0–4	5–9	10–14	15–19	20–24	25–29	30–34
Frequency	4	6	8	10	12	12	8

skewed left;
See Additional Answers for the histogram.

10. symmetric; The data on the left is a mirror image of the data on the right.

11–13. See Additional Answers.

14. See *Taking Math Deeper*.

Fair Game Review

15. median = 70; $Q_1 = 65.5$; $Q_3 = 75$; IQR = 9.5

16. median = 37.5; $Q_1 = 30$; $Q_3 = 44$; IQR = 14

17. A

Mini-Assessment

The frequency table shows the ages of people in attendance at a baseball game.

Ages	Frequency
0–9	25
10–19	30
20–29	58
30–39	60
40–49	32
50–59	21
60–69	16
70–79	8
80–89	4

1. Display the data in a histogram.

Baseball Attendance
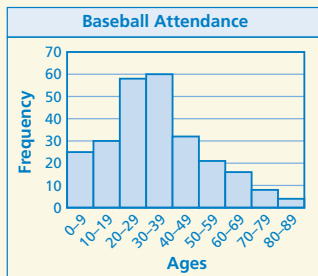
2. Describe the shape of the distribution. skewed right

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Taking Math Deeper

Exercise 14

The key to this problem is distinguishing between the mean and the median in the two skewed distributions.

1 Identify the mode of each data set.

The mode is the value that occurs most often. So, the tallest bar in a bar graph represents the mode. The tallest bar is represented by the letter A in part (a) and by the letter C in part (b).

2 Identify the mean and the median of each data set.

In part (a), there are 71 values in the data set. The median is the 36th value, which occurs in the third bar and is represented by the letter B. The letter C represents the mean because the greater values on the tail extending to the right lead to a mean that is greater than the median.

In part (b), there are 107 values in the data set. The median is the 54th value, which occurs in the seventh bar and is represented by the letter B. The letter A represents the mean because the lesser values on the tail extending to the left lead to a mean that is less than the median.

3 Answer the question.

a. Most of the data are on the left, and the tail extends to the right. So, the distribution is skewed right.

  A: Mode   B: Median   C: Mean

b. Most of the data are on the right, and the tail extends to the left. So, the distribution is skewed left.

  A: Mean   B: Median   C: Mode

Both are skewed.

Project

Does the tail of the data in a skewed distribution "pull" the median and mean towards the tail or away from it? Which measure does it pull more? Provide data to justify your reasoning.

Reteaching and Enrichment Strategies

| If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Start the next section

10. REASONING What is the shape of the distribution of the restaurant waiting times? Explain your reasoning.

Restaurant Waiting Times

11. LOGIC Are all distributions either approximately symmetric or skewed? Explain. If not, give an example.

12. REASONING Can you use a stem-and-leaf plot to describe the shape of a distribution? Explain your reasoning.
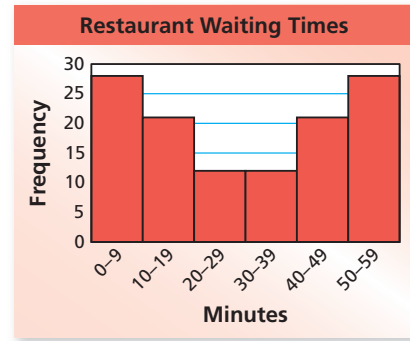
13. CHARITY The table shows the donation amounts received by a charity in one day.

Donations (dollars)												
20	15	40	70	20	5	25	50	47	20	62	55	40
10	50	18	20	100	40	80	60	20	80	3	30	50
25	30	10	33	20	50	7	35	40	25	70		

a. Make a histogram of the data starting with the interval 0–14. Describe the shape of the distribution.

b. A company adds $5 to each donation. Make another histogram starting with the same first interval as in part (a). Compare the shape of this distribution with the distribution in part (a). Explain any differences in the distributions.

14. **Critical Thinking** Describe the shape of the distribution of each bar graph. Match the letters A, B, and C with the mean, the median, and the mode of the data set. Explain your reasoning.

a.

b.
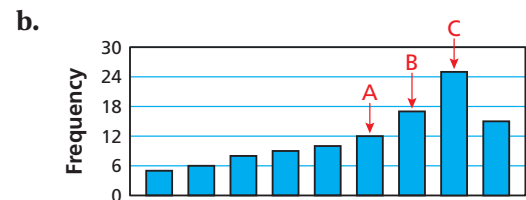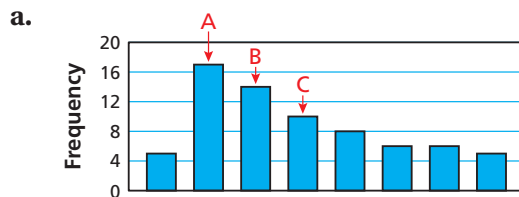
![Fair Game Review](pencil icon) **Fair Game Review** *What you learned in previous grades & lessons*

Find the median, first quartile, third quartile, and interquartile range of the data. *(Section 9.4)*

15. 68, 74, 67, 72, 63, 70, 78, 64, 76

16. 39, 48, 33, 24, 30, 44, 36, 41, 28, 53

17. MULTIPLE CHOICE Sixty people participate in a trivia contest. How many four-person teams can be formed? *(Section 7.3)*

(A) 15   (B) 56   (C) 64   (D) 240

You can use a measure of center and a measure of variation to describe the distribution of a data set. The shape of the distribution can help you choose which measures are the most appropriate to use.

🔑 Key Idea

Choosing Appropriate Measures

The mean absolute deviation (MAD) uses the mean in its calculation. So, when a data distribution is *symmetric*,

- use the mean to describe the center and
- use the MAD to describe the variation.

The interquartile range (IQR) uses quartiles in its calculation. So, when a data distribution is *skewed*,

- use the median to describe the center and
- use the IQR to describe the variation.

EXAMPLE 1 **Choosing Appropriate Measures**

The dot plot shows the average number of hours students in a class sleep each night.

Amount of Sleep
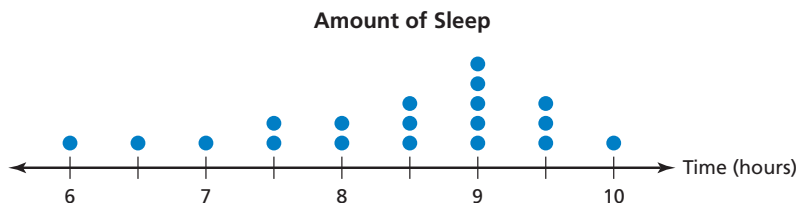
a. What are the most appropriate measures to describe the center and the variation?

Most of the data values are on the right clustered around 9, and the tail extends to the left. The distribution is skewed left.

So, the median and the interquartile range are the most appropriate measures to describe the center and the variation.

b. Describe the center and the variation of the data set.

The median is 8.5 hours. The first quartile is 7.5, and the third quartile is 9. So, the interquartile range is $9 - 7.5 = 1.5$ hours.

The data are centered around 8.5 hours. The middle half of the data varies by no more than 1.5 hours.

Laurie's Notes

Introduction

Connect
- **Yesterday:** Students described and compared shapes of distributions. (MP3, MP4)
- **Today:** Students will use the shape of a distribution to choose the most appropriate measures to describe the center and the variation.

Motivate
- ❓ "How many states does [insert your state] border?" Answers will vary.
- ❓ "How many states border 0 or 1 other states? Explain." 3; Hawaii and Alaska border 0 other states and Maine borders 1 other state.
- Explain that one of the examples in today's lesson will explore a data set involving state borders.

Discuss
- Introduce this lesson by asking how students can describe the distribution of a data set. Listen for an understanding of the measure of center (mean, median, mode), the measure of variation (mean absolute deviation, interquartile range), and the shape of the distribution (symmetric, skewed).
- Acknowledge the need to keep track of lots of vocabulary and definitions.
- In this lesson, the goal is to use the shape of a distribution to choose the most appropriate measures to describe the center and the variation.

Lesson Notes

Key Idea
- Write the Key Idea. In choosing the appropriate measures, first determine the shape of the distribution, symmetric or skewed. Students should know that the shape does not need to be either of these, but only these types of shapes are considered here.
- **MP2 Reason Abstractly and Quantitatively:** Discuss how outliers can affect mean and median. Both symmetric and skewed distributions can have outliers.

Example 1
- ❓ "What is the average number of hours of sleep you get each night?" Answers will vary.
- Sketch the dot plot shown.
- ❓ "How many students does this plot represent?" 19 "What is the range?" 4 hours "What is the mode?" 9 hours
- ❓ "What are the most appropriate measures to describe the center and variation? Explain." median and interquartile range; The distribution is skewed (left).
- Work through the steps in finding the median and the interquartile range. Summarize the results as shown.

Goal Today's lesson is choosing the most appropriate measures to describe the center and the variation.

Technology for the **Teacher**

Dynamic Classroom
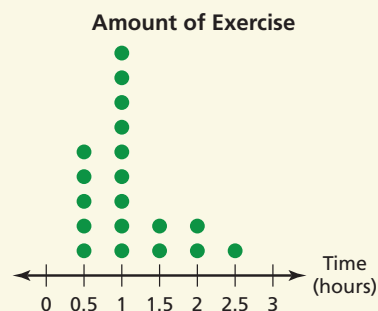
Lesson Tutorials
Lesson Plans
Answer Presentation Tool

Extra Example 1
The dot plot shows the average number of hours students in a class exercise each day.

Amount of Exercise

a. What are the most appropriate measures to describe the center and the variation? median and interquartile range
b. Describe the center and the variation of the data set. The data are centered around 1 hour. The middle half of the data vary by no more than 1 hour.

Record and Practice Journal Extension 10.3 Practice
1. The distribution is skewed left.; median and interquartile range; median: $19.5; interquartile range: $2

2–5. See Additional Answers.

Extra Example 2

The frequency table shows the gas mileages of several vehicles made by a company.

Mileage (miles per gallon)	Frequency
15–17	2
18–20	4
21–23	3
24–26	2
27–29	1

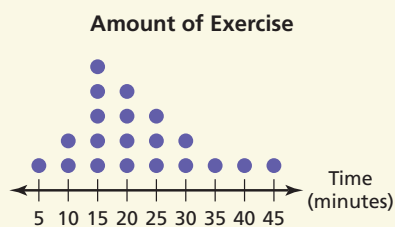**a.** Display the data in a histogram.
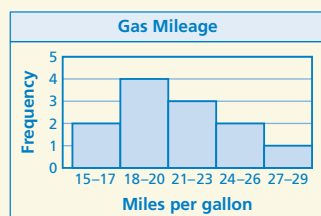
Gas Mileage

b. What are the most appropriate measures to describe the center and the variation? median and interquartile range

Practice

1–5. See Additional Answers.

Mini-Assessment

The dot plot shows the average number of minutes students in a class exercise every day.

Amount of Exercise

1. What are the most appropriate measures to describe the center and the variation? median and interquartile range

2. Describe the center and the variation of the data set. The data are centered around 20 minutes. The middle half of the data vary by no more than 12.5 minutes.

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Laurie's Notes

Example 2

- Draw the frequency table.
- ❓ "Where does [insert your state] fall in the frequency table?" Answers will vary.
- ❓ "How many values does the data set have?" 50
- **FYI:** Tennessee and Missouri both border 8 other states.
- Ask students to construct the corresponding histogram.
- ❓ "What are the most appropriate measures to describe the center and variation? Explain." mean and mean absolute deviation; The distribution is symmetric.
- **MP2:** Students should be able to explain why it is not possible to find exact values of the mean and the mean absolute deviation for the data. Question 3 in the Practice Exercises asks students to explain whether they can find these values.

Practice

- **Think-Pair-Share:** Students should read each question independently and then work in pairs to answer the questions. When they have answered the questions, the pair should compare their answers with another group and discuss any discrepancies.

Closure

- Summarize how to use the shape of a distribution to choose the most appropriate measures to describe the center and the variation. When the data is symmetric, use the mean and the mean absolute deviation. When the data is skewed, use the median and interquartile range.
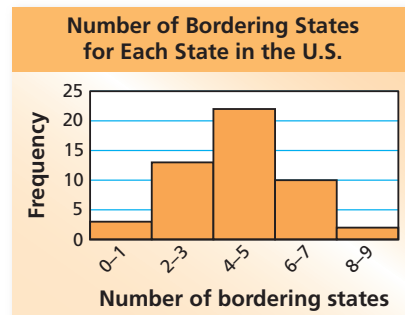
EXAMPLE **2** **Choosing Appropriate Measures**

Bordering States	Frequency
0–1	3
2–3	13
4–5	22
6–7	10
8–9	2

The frequency table shows the number of states that border each state in the United States.

a. **Display the data in a histogram.**

Draw and label the axes. Then draw a bar to represent the frequency of each interval.

Number of Bordering States for Each State in the U.S.

b. **What are the most appropriate measures to describe the center and the variation?**

The left side of the graph is approximately a mirror image of the right side of the graph. The distribution is symmetric.

∴ So, the mean and the mean absolute deviation are the most appropriate measures to describe the center and the variation.

Practice

Choose the most appropriate measures to describe the center and the variation. Find the measures you chose.

1. **Prices of Jeans**
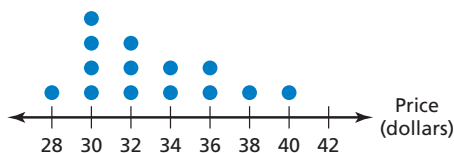
2. **Weekly Biking Times**
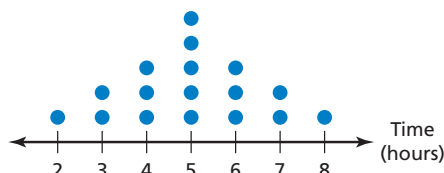
3. **REASONING** Can you find the exact values of the mean and the mean absolute deviation for the data in Example 2? Explain.

4. **GAS MILEAGE** The frequency table shows the gas mileages of several vehicles made by a company.

   a. What are the most appropriate measures to describe the center and the variation?

   b. What conclusions can you make?

Mileage (miles per gallon)	Frequency
10–14	2
15–19	1
20–24	6
25–29	8
30–34	10
35–39	3

5. **OPEN-ENDED** Construct a dot plot for which the mean is the most appropriate measure to describe the center of the distribution.

10.4 Box-and-Whisker Plots

Essential Question How can you use quartiles to represent data graphically?

### 1 ACTIVITY: Drawing a Box-and-Whisker Plot

Work with a partner.

The numbers of pairs of footwear owned by each student in a sixth grade class are shown.

Numbers of Pairs of Footwear			
2	5	12	3
7	2	4	6
14	10	6	28
5	3	2	4
9	25	4	10
8	15	5	8

a. Order the data set from least to greatest. Then write the data on a strip of grid paper with 24 boxes.

b. Use the strip of grid paper to find the median, the first quartile, and the third quartile. Identify the least value and the greatest value in the data set.

c. Graph the five numbers that you found in part (b) on the number line below.
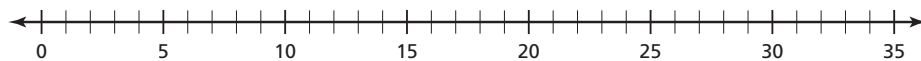
d. The data display shown below is called a *box-and-whisker plot*. Fill in the missing labels and numbers. Explain how a box-and-whisker plot uses quartiles to represent the data.
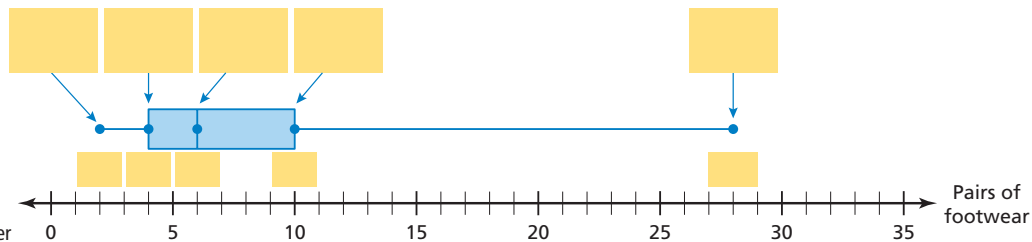
e. Using only the box-and-whisker plot, which measure(s) of center can you find for the data set? Which measure(s) of variation can you find for the data set? Explain your reasoning.

f. Why do you think this type of data display is called a box-and-whisker plot? Explain.

Laurie's Notes

Introduction

Standards for Mathematical Practice

- **MP3a Construct Viable Arguments:** Students create box-and-whisker plots to display data sets. Multiple box-and-whisker plots can be displayed on the same number line, therefore students are asked to construct viable arguments in comparing data sets.

Motivate

- Collect class data about the numbers of minutes it took your students to commute to school this morning, from the time they left their front doors until they walked into the school. If this is awkward data to collect, change the question. Data can be collected on slips of paper.
- Record the data on the board and leave it for later. You may want to take time to have students make comments about the data set.

Activity Notes

Discuss

- The box-and-whisker plot is used to display information about very large data sets. However, in order to make learning the technique manageable for students, the actual data sets used today are small.
- Students quickly recognize that box-and-whisker plots involve finding five key values. Constructing the plot is not difficult. Analyzing the plot is the challenging part for students.
- **Connection:** When discussing the meaning of percentiles and quartiles with my students, I had one boy who said, "It's like at the doctor's office when they tell me what percentile I'm in for height and weight." The connection was immediate for the class and *they got it*!

Activity 1

- Explain that today they are going to construct a box-and-whisker plot, a data display that is generally used for very large data sets. Data are *not* graphed, but characteristics of the data set are still conveyed. For instance, the results of a state test for all 6th graders could be displayed using a box-and-whisker plot.
- The box-and-whisker plot uses a number line to visually represent the data.
- **Big Idea:** The 5 numbers graphed summarize the entire data set. The least and greatest values are the boundaries. The median separates the data into two parts. The first (or lower) quartile is the median of the lower half. The third (or upper) quartile is the median of the upper half. The box encloses the middle 50% of the data.
- **?** "What percent of the data is represented by each whisker?" 25%
  "How many data values are in each whisker?" 6
- **MP2 Reason Abstractly and Quantitatively:** The above question provides students with the opportunity to reason quantitatively.

Technology for the **Teacher**

Dynamic Classroom

Lesson Plans
Complete Materials List

10.4 Record and Practice Journal

Essential Question How can you use quartiles to represent data graphically?

1 ACTIVITY: Drawing a Box-and-Whisker Plot

Work with a partner.

The numbers of pairs of footwear owned by each student in a sixth grade class are shown.

A box-and-whisker plot uses a number line to represent the data visually.

| Numbers of Pairs of Footwear			
2	5	12	3
7	2	4	6
14	10	6	28
5	3	2	4
9	25	4	10
8	15	5	8

a. Order the data set from least to greatest. Then write the data on a strip of grid paper with 24 boxes.

| 2 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 5 | 6 | 6 | 7 | 7 | 8 | 8 | 9 | 10 | 10 | 12 | 14 | 15 | 25 | 28 |

b. Use the strip of grid paper to find the median, the first quartile, and the third quartile. Identify the least value and the greatest value in the data set.

median: 6   **least value: 2**
first quartile: 4   **greatest value: 28**
third quartile: 10

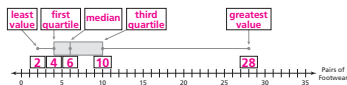c. Graph the five numbers that you found in part (b) on the number line below.
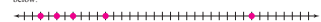
10.4 Record and Practice Journal

d. The data display shown below is called a *box-and-whisker plot.* Fill in the missing labels and numbers. Explain how the box-and-whisker plot uses quartiles to represent the data.

least value | first quartile | median | third quartile | greatest value

e. Using only the box-and-whisker plot, which measure(s) of center can you find for the data set? Which measure(s) of variation can you find for the data set? Explain your reasoning.
median; range and interquartile range

f. Why do you think this type of data display is called a box-and-whisker plot? Explain.
Rectangle represents the "box."
Line segments represent the "whiskers."

2 ACTIVITY: Conducting a Survey

Have your class conduct a survey. Each student will write on the chalkboard the number of pairs of footwear that he or she owns.

Now, work with a partner to draw a box-and-whisker plot of the data.
Check students' work.

3 ACTIVITY: Reading a Box-and-Whisker Plot

Work with a partner. The box-and-whisker plots show the test score distributions of two sixth grade achievement tests. The same group of students took both tests. The students took one test in the fall and the other in the spring.
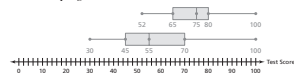
a. Compare and contrast the test results.
Upper plot shows higher scores.

b. Decide which box-and-whisker plot represents the results of which test. How did you make your decision?
spring; fall

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use quartiles to represent data graphically?
You can display how the data are distributed, using the quartiles, the median, and the range.

5. Describe who might be interested in test score distributions like those shown in Activity 3. Explain why it is important for such people to know test score distributions.
Sample answer: **teachers; if they know when scores are lower, they can determine the reason and address it.**

Laurie's Notes

Activity 2

- Explain that you want to practice making a box-and-whisker plot with data collected from students. You will gather and record information about the number of pairs of footwear each of your students owns.
- Students should follow the steps from Activity 1 to construct this plot. If there is an odd number of students in the class, the median is the middle value of the sorted data. To find the first quartile, exclude the median and find the median of the lower half of the data. To find the third quartile, exclude the median and find the median of the upper half of the data.
- As students are making the plot, make the same plot at the board for discussion purposes.
- Ask questions about the plot: median, range, number of data values considered versus number of data values graphed.

Activity 3

- One advantage of box-and-whisker plots is that multiple plots can be displayed and analyzed using the same number line. For instance, state test scores for 5 different schools could be displayed on the same number line.
- Read the information given and analyze the two plots.
- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Students should be listening to the analysis offered by their partners or other classmates and critiquing their reasoning.
- **?** "Which test is represented by which plot? Explain." Listen for students discussing the location of the median and the third quartile for each plot. The spring test is the top plot.
- **?** "True or false: 50% of the top plot is greater than 75% of the bottom plot." true

What Is Your Answer?

- **Think-Pair-Share:** Students should read each question independently and then work in pairs to answer the questions. When they have answered the questions, the pair should compare their answers with another group and discuss any discrepancies.

Closure

- Make a box-and-whisker plot of the data collected at the beginning of class. Write one or two observations about the plot.

ACTIVITY: Conducting a Survey

Have your class conduct a survey. Each student will write on the chalkboard the number of pairs of footwear that he or she owns.

Now, work with a partner to draw a box-and-whisker plot of the data.

3 **ACTIVITY: Reading a Box-and-Whisker Plot**

Math Practice

View as Components
What do the different components of a box-and-whisker plot represent?

Work with a partner. The box-and-whisker plots show the test score distributions of two sixth grade achievement tests. The same group of students took both tests. The students took one test in the fall and the other in the spring.

a. Compare and contrast the test results.

b. Decide which box-and-whisker plot represents the results of which test. How did you make your decision?

52   65   75 80   100

30   45   55   70   100

Test score
0   10   20   30   40   50   60   70   80   90   100
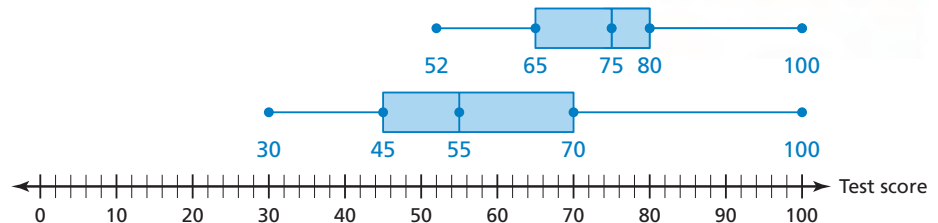
What Is Your Answer?

4. IN YOUR OWN WORDS How can you use quartiles to represent data graphically?

5. Describe who might be interested in test score distributions like those shown in Activity 3. Explain why it is important for such people to know test score distributions.

Practice

Use what you learned about box-and-whisker plots to complete Exercise 4 on page 463.

Key Vocabulary 🔊
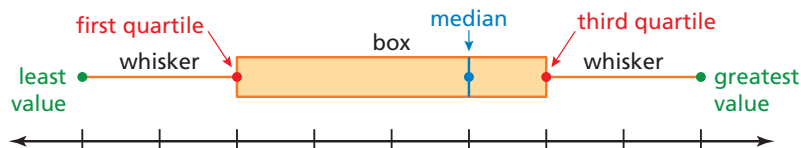box-and-whisker plot,
  p. 460
five-number
  summary, *p. 460*

🔑 Key Idea

Box-and-Whisker Plot

A **box-and-whisker plot** represents a data set along a number line by using the least value, the greatest value, and the quartiles of the data. A box-and-whisker plot shows the *variability* of a data set.
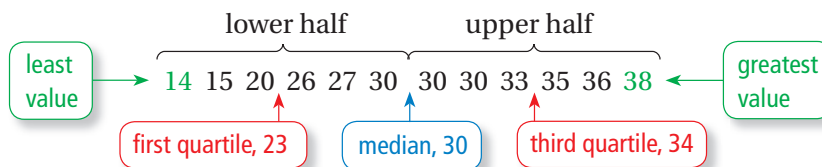

The five numbers that make up the box-and-whisker plot are called the **five-number summary** of the data set.

EXAMPLE 1 Making a Box-and-Whisker Plot

Make a box-and-whisker plot for the ages (in years) of the spider monkeys at a zoo:

15, 20, 14, 38, 30, 36, 30, 30, 27, 26, 33, 35

Step 1: Order the data. Find the median and the quartiles.

least value
→ 14 15 20 26 27 30 30 30 33 35 36 38 ←
greatest value

lower half   upper half

first quartile, 23   median, 30   third quartile, 34

Step 2: Draw a number line that includes the least and greatest values. Graph points above the number line that represent the five-number summary.

Step 3: Draw a box using the quartiles. Draw a line through the median. Draw whiskers from the box to the least and the greatest values.

Age (years)
14 16 18 20 22 24 26 28 30 32 34 36 38

🔴 On Your Own
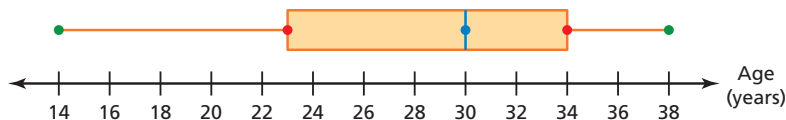
Now You're Ready
Exercises 5–8

1. A group of friends spent 1, 0, 2, 3, 4, 3, 6, 1, 0, 1, 2, and 2 hours online last night. Make a box-and-whisker plot for the data.

🔊 Multi-Language Glossary at BigIdeasMath√com

Laurie's Notes

Introduction

Connect
- **Yesterday:** Students gained a general understanding of how a box-and-whisker plot is constructed. (MP2, MP3)
- **Today:** Students will construct and analyze box-and-whisker plots.

Motivate
- The physical involvement of making a human box-and-whisker plot makes a lasting impression on students.
- **Preparation:** Give each student an index card with a number written on it. Include an outlier or two on one end of the data.
- **?** "What is the first step in making a box-and-whisker plot?" sort the data
- Students should stand up and sort themselves. Have the median, the first and third quartiles, and the least and greatest data values take one step forward. If there is an even number of data values, the middle two students must figure out how to represent the mean and take a step forward.
- Make a number line on the floor or on the board. Position the 5 key values. If the plot is done on the floor, use string to form the whiskers. Students will have to visualize the box.
- **MP2 Reason Abstractly and Quantitatively:** Discuss features of the plot. If the plot includes an outlier, the length of the string becomes an instant topic of conversation. Students recognize that the same number of data values (about 25% of the class) is being represented by each whisker, yet the lengths of string are very different.

Lesson Notes

Key Idea
- Define a box-and-whisker plot. Draw the sample plot and discuss the process and vocabulary.
- **Discuss:** The box-and-whisker plot shows the *variability* of the data. Refer to this idea in each example done today.

Example 1
- Point out that ordering the data from least to greatest makes it possible to find the least value, greatest value, and quartiles.
- **?** "How many data values are there?" 12 "When there are 12 data values, how do you find the median?" Find the mean of the 6th and 7th data values.
- There are six data values in the lower half and six data values in the upper half. The first quartile and third quartile are the mean of the middle two data values in each half.

On Your Own
- **Common Error:** Students may forget to order the data before creating the box-and-whisker plot.

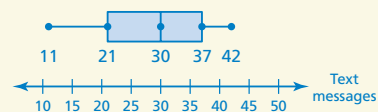**Technology** for the **Teacher**

Dynamic Classroom

Lesson Tutorials
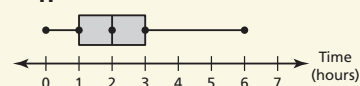Lesson Plans
Answer Presentation Tool

Extra Example 1

Make a box-and-whisker plot for the numbers of text messages received over several days.

32, 18, 11, 28, 42, 33, 40, 21, 37, 24

On Your Own

1.

Discuss
- The five key values plotted in a box-and-whisker plot divide the data into four parts.
- ❓ "What fraction of the data in the box are to the left of the median? to the right?" about $\frac{1}{4}$; about $\frac{1}{4}$
- Relate the definition of interquartile range to the box-and-whisker plot, indicating that it represents the "length" of the box.

Example 2
- **FYI:** Body Mass Index (BMI) is a numerical value determined using a person's weight and height. For most people, it provides a reliable indication of body "fatness" and can be used to screen for health problems.
- ❓ "Do you know from the box-and-whisker plot how many students are in the class?" no
- It is important for students to remember that you cannot determine the number of data points from a given box-and-whisker plot. You can only determine the five-number summary points and of those, only two *must* be values in the data set (the least value and the greatest value).
- ❓ "What is the range of the data and what does it mean in the context of the problem?" $28 - 17 = 11$; This means that the difference between the highest and lowest BMIs is 11.
- ❓ "What does the length of each part of the box-and-whisker plot tell you about the distribution of the data?" *Sample answers:* About 50% of the BMIs are between 17 and 21, and about 50% are between 21 and 28.
- In part (c), the students should be focusing on the middle half of the data. The IQR is 3.

On Your Own
- **MP3a Construct Viable Arguments:** Ask different students to explain their answers. Check for correct language and valid arguments.
- **Common Error:** Students may read the box-and-whisker plot incorrectly and find the range instead of the interquartile range.

Extra Example 2

The box-and-whisker plot shows the times that the students in a gym class spent on an obstacle course.

Time (seconds)
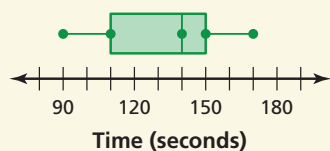
a. What fraction of the class spent 110 seconds or less on the obstacle course? about $\frac{1}{4}$

b. Are the data more spread out below the first quartile or above the third quartile? Explain. neither; Both whiskers are the same length.

c. Find and interpret the interquartile range of the data. 40; The middle half of the times varies by no more than 40 seconds.

On Your Own

2. **a.** about $\frac{3}{4}$

   b. neither; The data are evenly spread out below and above the median.

   c. 80; The middle half of the heights varies by no more than 80 feet.
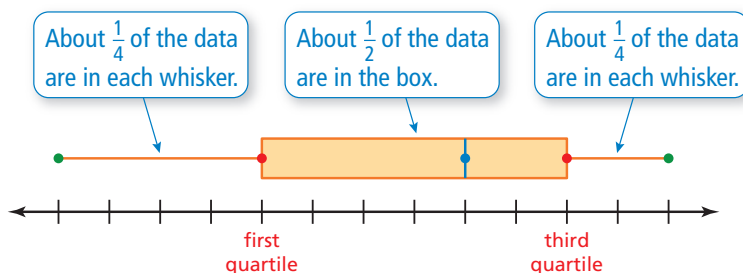
Differentiated Instruction

Auditory
Remind students of other words that have the same root as *quartile:* quarter and quart, for example. Define the words as *four* or *fourths*. Mention that one-fourth is the same as 25%.

The figure shows how data are distributed in a box-and-whisker plot.

About $\frac{1}{4}$ of the data are in each whisker.

About $\frac{1}{2}$ of the data are in the box.

About $\frac{1}{4}$ of the data are in each whisker.

first quartile

third quartile

EXAMPLE 2 Analyzing a Box-and-Whisker Plot

The box-and-whisker plot shows the body mass index (BMI) of a sixth grade class.

BMI

17 18 19 20 21 22 23 24 25 26 27 28

a. What fraction of the students have a BMI of at least 22?

The right whisker represents students who have a BMI of at least 22.

So, about $\frac{1}{4}$ of the students have a BMI of at least 22.

b. Are the data more spread out below the first quartile or above the third quartile? Explain.

The right whisker is longer than the left whisker.

So, the data are more spread out above the third quartile than below the first quartile.

c. Find and interpret the interquartile range of the data.

$$\text{interquartile range} = \text{third quartile} - \text{first quartile}$$
$$= 22 - 19 = 3$$

So, the middle half of the students' BMIs varies by no more than 3.

⬤ **On Your Own**

2. The box-and whisker plot shows the heights of the roller coasters at an amusement park. (a) What fraction of the roller coasters are between 120 feet tall and 220 feet tall? (b) Are the data more spread out below or above the median? Explain. (c) Find and interpret the interquartile range of the data.

Height (feet)

80 100 120 140 160 180 200 220 240 260
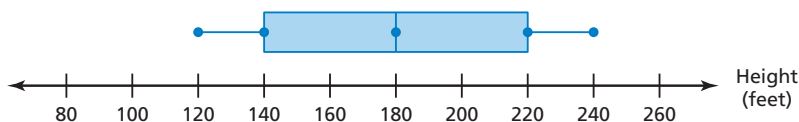
A box-and-whisker plot also shows the shape of a distribution.

🔑 Key Ideas

Shapes of Box-and-Whisker Plots

Skewed left

- Left whisker longer than right whisker
- Most data on the right

Symmetric

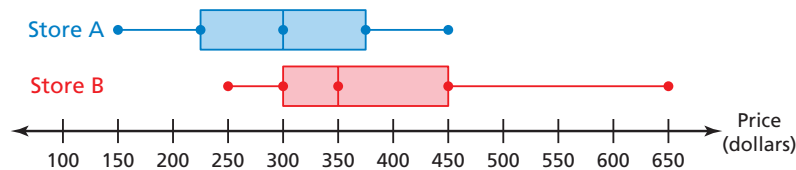- Whiskers about same length
- Median in the middle of the box

Skewed right

- Right whisker longer than left whisker
- Most data on the left

EXAMPLE 3 — **Comparing Box-and-Whisker Plots**

The double box-and-whisker plot represents the prices of snowboards at two stores.

a. Identify the shape of each distribution.

For Store A, the whisker lengths are equal. The median is in the middle of the box. The data on the left are the mirror image of the data on the right. So, the distribution is symmetric.

For Store B, the right whisker is longer than the left whisker, and most of the data are on the left side of the display. So, the distribution is skewed right.

b. Which store's prices are more spread out? Explain.

Both boxes appear to be the same length. So, the interquartile range of each data set is equal. However, the range of the prices in Store B is greater than the range of the prices in Store A. So, the prices in Store B are more spread out.

⬤ On Your Own

3. The double box-and-whisker plot represents the life spans of crocodiles and alligators at a zoo. Identify the shape of each distribution. Which reptile's life spans are more spread out? Explain.
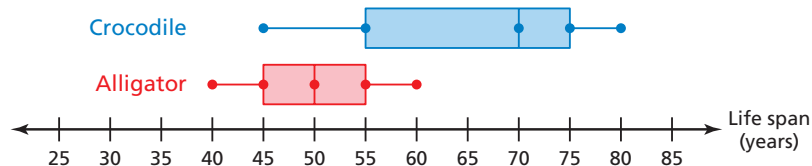
Laurie's Notes

Key Ideas

- Sketch each of the box-and-whisker plots. Discuss what the shape of the plot implies about the distribution of the data.
- ❓ "If the box-and-whisker plots represent the prices of jeans at three different stores and the range is the same for all three plots, what does the shape tell you about the distribution of the prices at all three stores?" Listen for student understanding of the spread of the data, referencing percentages and relative price.

Example 3

- One advantage of a box-and-whisker plot is that multiple data sets can be shown in the same display, as with double bar graphs and double line graphs. The data sets can also have different numbers of data values.
- Part (a) asks students to identify the shape. A student might comment that a high price in Store B could be the reason for it being skewed. Without the outlier it could be a symmetric graph.
- **MP2:** Supplement the discussion in part (b) with quantitative comparisons, such as 50% of the prices of snowboards in Store A are as low as, or lower than, the lowest 25% of the prices of snowboards in Store B.

On Your Own

- **Neighbor Check:** Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.
- Ask others quantitative questions about the two box-and-whisker plots.

Closure

- **Exit Ticket:**
  - What are the 5 key values that are graphed in a box-and-whisker plot? least value, first quartile, median, third quartile, greatest value
  - How does an outlier affect a box-and-whisker plot? *Sample answer:* increases the length of one of the whiskers
  - Explain why two data sets of different sizes can be graphed on the same number line. Box-and-whisker plots show the distribution of the data, not individual data points.

Extra Example 3

The double box-and-whisker plot represents the length of time (in minutes) it takes students in your gym class and your friend's gym class to run 1 mile.

Your Gym Class

Your Friend's Gym Class

Time (minutes)
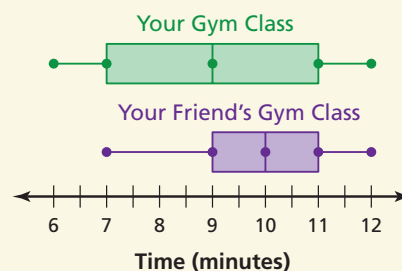
a. Identify the shape of each distribution. For your class, the whisker lengths are equal. The median is in the middle of the box. The data appear to be evenly distributed on both sides of the median. So, the distribution is symmetric. For your friend's class, the left whisker is longer than the right whisker, and most of the data are on the right side of the display. So, the distribution is skewed left.

b. Which lengths of time are more spread out? Explain. The range of the times in your class is greater than the range in your friend's class. Also, because the box for your class is longer than the box for your friend's class, the interquartile range is also greater. So, the times in your class are more spread out.

On Your Own

3. Crocodiles: skewed left, alligators: symmetric; crocodile; The box for crocodiles is longer than the box for alligators. So, the IQR for crocodiles is greater than the IQR for alligators. Also, the range of life spans for crocodiles is greater than the range of life spans for alligators.

Vocabulary and Concept Check

1. Order the data. The first number is the *least value* and the last number is the *greatest value*. The middle value is the *median*. The middle value of the lower half of the data is the *first quartile*. The middle value of the upper half of the data is the *third quartile*.

2. $\frac{3}{4}$

3. Is the distribution skewed right?; yes; no

Practice and Problem Solving

4. *Sample answer:* The attendance for Beach 2 is more spread out than the attendance for Beach 1. Beach 1 seems to be more popular because its daily attendance is higher, on average, than Beach 2.

5–10. See Additional Answers.

11. **a.** about $\frac{1}{2}$

   b. The right whisker is longer than the left whisker. So the data are more spread out above the third quartile than below the first quartile.

   c. 150; The middle half of the data varies by no more than 150 gallons.

Assignment Guide and Homework Check

Level	Day 1 Activity Assignment	Day 2 Lesson Assignment	Homework Check
Basic	4, 27–30	1–3, 5–17 odd	5, 7, 9, 11
Average	4, 27–30	1–3, 7–14, 17–23 odd	7, 8, 10, 12
Advanced	4, 27–30	1–3, 6, 8, 9, 10–26 even	6, 8, 10, 12, 14

Common Errors

- **Exercise 2** Students may have trouble corresponding fractions of the data with quartiles. Remind them that they can obtain this information by studying the Key Idea on page 460 as well as the figure at the top of page 461. Also, a quick review of the definition of *quartiles* can be helpful.
- **Exercises 5–8** Students may have difficulty creating the box-and-whisker plots. Remind them of the five-number summary of a data set.

10.4 Record and Practice Journal
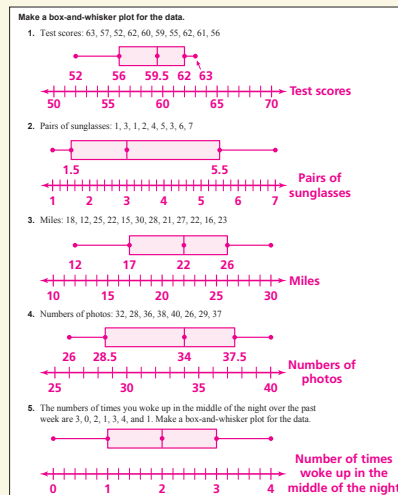
## Vocabulary and Concept Check

1. **VOCABULARY** Explain how to find the five-number summary of a data set.

2. **NUMBER SENSE** In a box-and-whisker plot, what fraction of the data is greater than the first quartile?

3. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Is the distribution skewed right?

Is the left whisker longer than the right whisker?

Are the data more spread out below the first quartile than above the third quartile?

Does the lower fourth of the data vary more than the upper fourth of the data?
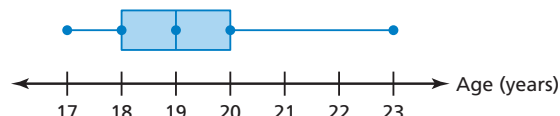
Practice and Problem Solving

4. The box-and-whisker plots represent the daily attendance at two beaches during July. Compare and contrast the attendances for the two beaches.
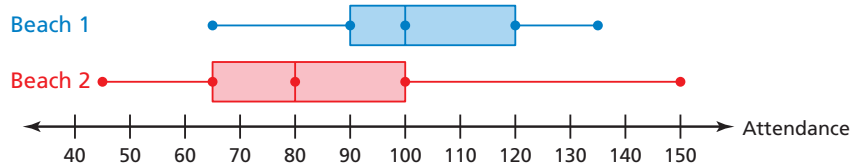
Make a box-and-whisker plot for the data.

5. Ages of teachers (in years): 30, 62, 26, 35, 45, 22, 49, 32, 28, 50, 42, 35

6. Quiz scores: 8, 12, 9, 10, 12, 8, 5, 9, 7, 10, 8, 9, 11

7. Donations (in dollars): 10, 30, 5, 15, 50, 25, 5, 20, 15, 35, 10, 30, 20

8. Ski lengths (in centimeters): 180, 175, 205, 160, 210, 175, 190, 205, 190, 160, 165, 195

9. **ERROR ANALYSIS** Describe and correct the error in making a box-and-whisker plot for the data.

2, 6, 3, 3, 7, 4, 6, 9, 6, 8, 5, 8
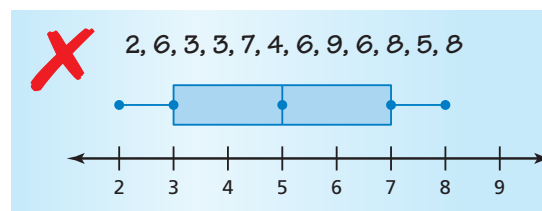
10. **CAMPING** The numbers of days 12 friends went camping during the summer are 6, 2, 0, 10, 3, 6, 6, 4, 12, 0, 6, and 2. Make a box-and-whisker plot for the data. What is the range of the data?

2 **11. DUNK TANK** The box-and-whisker plot represents the numbers of gallons of water needed to fill different types of dunk tanks offered by a company.


350   450 500   600   750

Water (gallons)

350 400 450 500 550 600 650 700 750
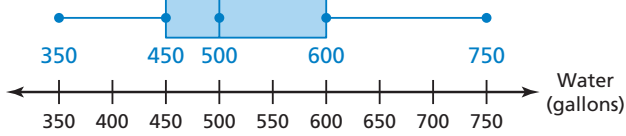
   a. What fraction of the dunk tanks require at least 500 gallons of water?

   b. Are the data more spread out below the first quartile or above the third quartile? Explain.

   c. Find and interpret the interquartile range of the data.

12. BUILDINGS The box-and-whisker plot represents the heights (in meters) of the tallest buildings in Chicago.


250 261   293   345   442

Height (meters)

250   275   300   325   350   375   400   425   450
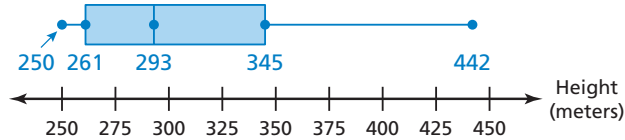
   a. What percent of the buildings are no taller than 345 meters?

   b. Is there more variability in the heights above 345 meters or below 261 meters? Explain.

   c. Find and interpret the interquartile range of the data.

Identify the shape of the distribution. Explain.

3 **13.**

14.

15.

16.
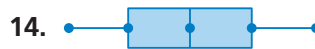
17. RECESS The double box-and-whisker plot represents the start times of recess for two schools.

School 1

School 2

Recess start time

10 A.M.   11 A.M.   12 P.M.   1 P.M.   2 P.M.
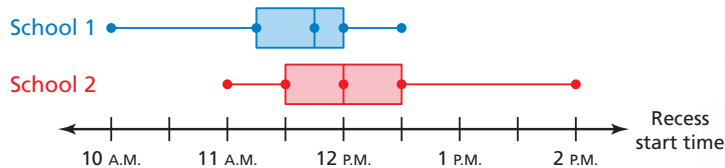
   a. Identify the shape of each distribution.

   b. Which school's start times for recess are more spread out? Explain.

   c. Which school is more likely to have recess before lunch? Explain.

Common Errors

- **Exercises 10–12** Students may confuse the range and interquartile range of a data set. Remind them of these definitions.
- **Exercises 13–16** When identifying the shape of the distribution, students may confuse the meanings of skewed left and skewed right. Tell them that the shape is determined by the longer whisker.

12. a. about 75%

b. The right whisker is longer than the left whisker. So there is more variability in the heights above 345 meters than the heights below 261 meters.

c. 84; The middle half of the data varies by no more than 84 meters.

13. skewed left; The left whisker is longer than the right whisker, and most of the data are on the right.

14. symmetric; The whiskers are about the same length, and the median is in the middle of the box.

15. symmetric; The whiskers are about the same length, and the median is in the middle of the box.

16. skewed right; The right whisker is longer than the left whisker, and most of the data are on the left.

17. a. School 1 is skewed left and School 2 is skewed right.

b. School 2; The range for School 2 is a half hour greater than the range for School 1. Also, the IQR of School 2 is greater than the IQR of School 1.

c. School 1; School 1 has more data on the left than School 2. So, School 1 is more likely to have recess before lunch.

Differentiated Instruction

Kinesthetic

To make ordering data easier, have students write each data value on a sticky note. Using their desks or notebook paper, students can move the notes to order the data and divide them into quartiles.

18–20. See Additional Answers.

21. Use the median to describe the center and the interquartile range to describe the variation.

22. See *Taking Math Deeper.*

23. When the least value and the first quartile are equal, there is no whisker on the left. When the greatest value and the third quartile are equal, there is no whisker on the right.

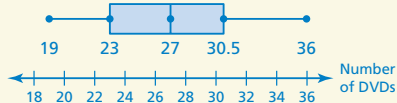**24–26.** See Additional Answers.

Fair Game Review

27. >   **28.** <

29. >   **30.** B

Mini-Assessment

The list represents the numbers of DVDs owned by the students in your class.

25, 31, 27, 36, 19, 22, 20, 24, 30, 32, 29, 27

1. Make a box-and-whisker plot for the data.

19   23   27   30.5   36

Number of DVDs

18 20 22 24 26 28 30 32 34 36

2. What is the range of the data? **17**

3. What fraction of students owns at least 23 DVDs? **about $\frac{3}{4}$**

4. Find and interpret the interquartile range of the data. **7.5; The middle half of the numbers of DVDs varies by no more than 7.5.**

Taking Math Deeper

Exercise 22

It is important for students to make a plan for solving this exercise rather than simply jumping in and trying numbers.

① Summarize what you know about the structure of the data set.
  • The data set has 12 values.
 • The quartiles break the data set up into 4 parts. Each part has 3 values.

Mirror image

② Draw a symmetric box-and-whisker plot using a simple scale on a number line. Indicate where the 12 values in the data set are represented in the box-and-whisker plot.

X   X   X   X   X   X   X   X   X   X   X   X

5   10   15   20   25   30   35   40   45   50   55
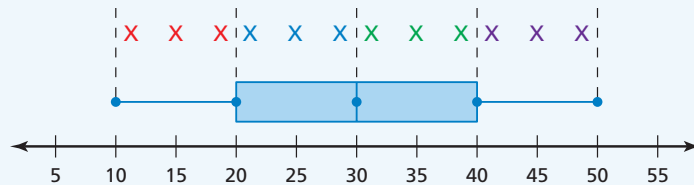
③ Begin finding values for the Xs.

  • The least value is 10 and the greatest value is 50.

  • Be careful choosing adjacent Xs that have different colors. The mean of each of these three pairs of values must equal the associated quartile. One way to do this is to make each value in the pair equal to the quartile. For example, the first quartile is 20. So, you can let the third and fourth values be 20.

10   15   20   20   25   30   30   35   40   40   45   50

5   10   15   20   25   30   35   40   45   50   55
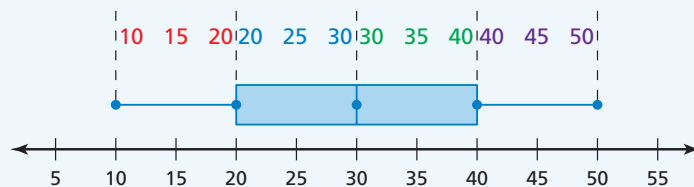
So, one possible data set is 10, 15, 20, 20, 25, 30, 30, 35, 40, 40, 45, 50.

Reteaching and Enrichment Strategies

| If students need help. . .	If students got it. . .

Resources by Chapter
  • Practice A and Practice B
  • Puzzle Time
Record and Practice Journal Practice
Differentiating the Lesson
Lesson Tutorials
Skills Review Handbook
	Resources by Chapter
	  • Enrichment and Extension
	  • Technology Connection
	Start the next section

Make a box-and-whisker plot for the data.

18. Temperatures (in °C): 5, 1, 4, 0, 9, 0, −8, 5, 2, 4, −1, 10, 7, −5

19. Checking account balances (in dollars): 30, 0, 50, 20, 90, −15, 40, 100, 45, −20, 70, 0

20. **REASONING** The data set in Exercise 18 has an outlier. Describe how removing the outlier affects the box-and-whisker plot.

21. **CHOOSE TOOLS** What are the most appropriate measures to describe the center and the variation of the distribution in Exercise 12?

22. **OPEN-ENDED** Write a data set with 12 values that has a symmetric box-and-whisker plot.

23. **CRITICAL THINKING** When would a box-and-whisker plot *not* have one or both whiskers?

24. **STRUCTURE** Draw a histogram that could represent the distribution shown in Exercise 15.

25. **REASONING** The double box-and-whisker plot represents the runs scored per game by two softball teams during a 32-game season.
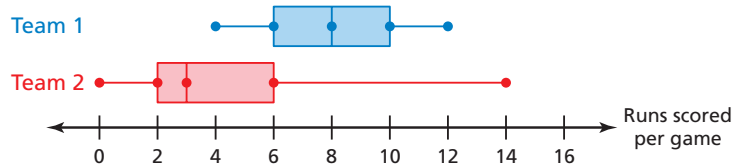
a. Which team is more consistent at scoring runs? Explain.

b. In how many games did Team 2 score 6 runs or less?

c. Team 1 played Team 2 once during the season. Which team do you think won? Explain.

d. Which team do you think has the greater mean? Explain.

26. **Choose Tools** A market research company wants to summarize the variability of the SAT scores of graduating seniors in the United States. Do you think the company should use a stem-and-leaf plot, a histogram, or a box-and-whisker plot? Explain.

Fair Game Review What you learned in previous grades & lessons

Copy and complete the statement using < or >. *(Section 6.3)*

27. $-\dfrac{2}{3}$ ▢ $-\dfrac{3}{4}$

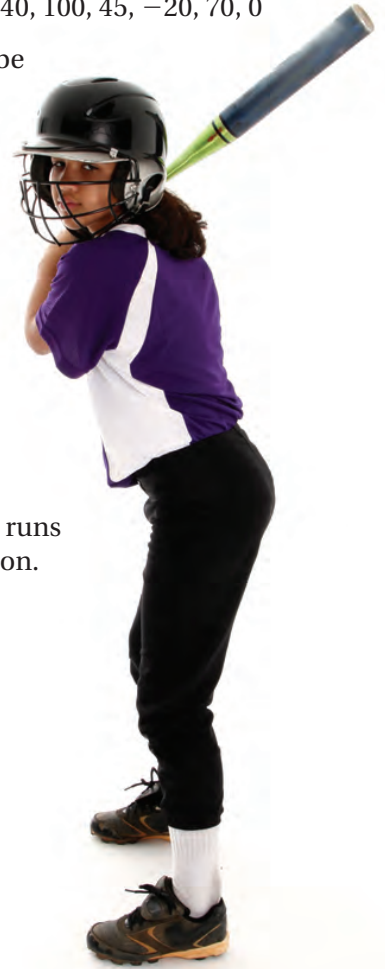**28.** $-2\dfrac{1}{5}$ ▢ $-2\dfrac{1}{6}$

29. -5.3 ▢ -5.5

30. **MULTIPLE CHOICE** Which of the following items is most likely represented by a rectangular prism with a volume of 1785 cubic inches? *(Section 8.4)*
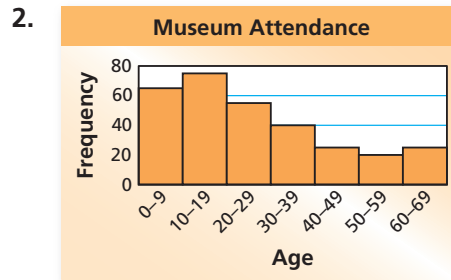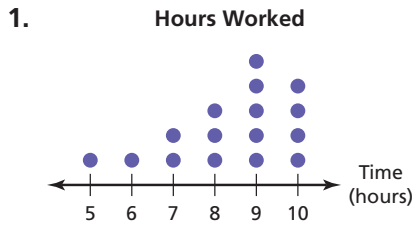
- Ⓐ closet
- Ⓑ computer tower
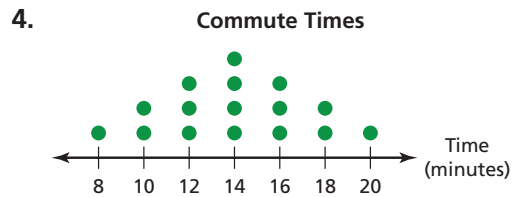- Ⓒ filing cabinet
- Ⓓ your math book

Describe the shape of each distribution. *(Section 10.3)*

1. **Hours Worked**

Time (hours)

5 6 7 8 9 10

2. **Museum Attendance**

**Choose the most appropriate measures to describe the center and the variation. Find the measures you chose.** *(Section 10.3)*

3. **Prices of Shoes**

Price (dollars)

20 24 28 32 36 40 44 48

4. **Commute Times**

Time (minutes)

8 10 12 14 16 18 20

Make a box-and-whisker plot for the data. *(Section 10.4)*

5. Science test scores: 85, 76, 99, 84, 92, 95, 68, 100, 93, 88, 87, 85

6. Shoe sizes: 12, 8.5, 9, 10, 9, 11, 11.5, 9, 9, 10, 10, 10.5, 8

7. **MOVIES** The box-and-whisker plot represents the lengths (in minutes) of movies being shown at a theater. *(Section 10.4)*
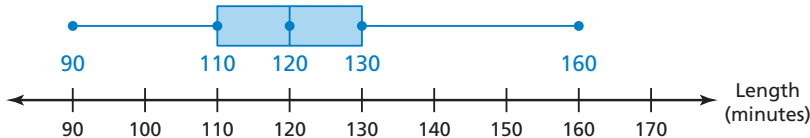
a. What percent of the movies are no longer than 120 minutes?

b. Is there more variability in the movie lengths longer than 130 minutes or shorter than 110 minutes? Explain.

c. Find and interpret the interquartile range of the data.

8. **EXPERIENCE** The frequency table shows the years of experience of employees at two branches of a company. Display the data for each branch in a histogram. Describe the shape of each distribution. Which branch has less experience? Explain. *(Section 10.3)*

Years of Experience	0–2	3–6	7–10	11–14	15–18	19–22	23–26
Frequency at Branch A	10	25	14	20	8	5	2
Frequency at Branch B	3	6	8	10	15	25	8

Alternative Assessment Options

Math Chat

Student Reflective Focus Question

Structured Interview

Writing Prompt

Student Reflective Focus Question

Ask students to use their own words to summarize the similarities and differences among line plots, stem-and-leaf plots, histograms, and box-and-whisker plots. Be sure that they include examples. Select students at random to present their summaries to the class.
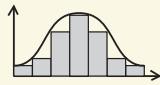
Study Help Sample Answers

Remind students to complete Graphic Organizers for the rest of the chapter.

2.

**3–4.** Available at *BigIdeasMath.com.*

Reteaching and Enrichment Strategies

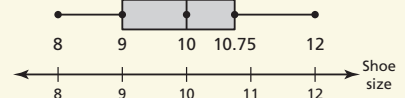| If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Lesson Tutorials *BigIdeasMath.com*	Resources by Chapter • Enrichment and Extension • Technology Connection Game Closet at *BigIdeasMath.com* Start the Chapter Review

1. skewed left   **2.** skewed right

3. median and IQR;
   median = 40, IQR = 12

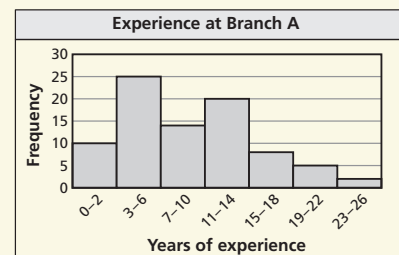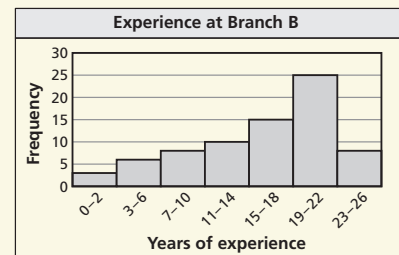4. mean and MAD; mean = 14, MAD = 2.5

5.

6.
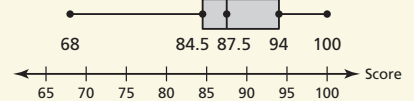
7. **a.** about 50%

   b. longer than 130 minutes, because the right whisker is longer

   c. 20 minutes; The middle half of the data vary by no more than 20 minutes.

8.

skewed right

skewed left

Branch A; The distribution of Branch A is skewed right, so most of the data values are on the left.

Answers

1. **Hats Sold Each Day**

Stem	Leaf
0	5 8 9
1	2 2 3 4 5 8
2	1 5
3	0

Key: 2 | 1 = 21 hats

2. **Ages of Park Volunteers**

Stem	Leaf
1	3 3 5 7 9
2	0 1
3	
4	0 8
5	2 5
6	0

Key: 1 | 3 = 13 years old

3. 6 tuna

4. 87 pounds

Review of Common Errors

Exercises 1 and 2
- Students may forget to include the zeros in the ones place as leaves or forget to include repeats of numbers. Remind them that *all* of the numbers in a data set should be represented in a stem-and-leaf plot.
- Students may forget to include stems that do not have leaves. Remind them that a stem-and-leaf plot shows how data are distributed, so it is important to include all of the stems in a range of data.

Exercise 3
- Students may misinterpret the question and count the data values represented with a stem of 9. Point out that neither of the values represented with this stem is less than 90.

Exercise 4
- Students may forget how to find a median, or confuse the median with the mean or mode. Remind them how to find the median of a data set.

Exercises 5 and 6
- Students may have difficulty determining how to scale the vertical axis of the histogram. Suggest that they find the interval with the greatest frequency and scale the axis so that this interval will fit.

Exercise 7
- Students may confuse "skewed left" and "skewed right." Point out that the "tail" of the graph of a distribution determines the direction of the skew. That is, in a skewed left distribution, the tail extends to the left, and in a skewed right distribution, the tail extends to the right.

Exercise 8
- Students may not recognize the distribution as symmetric and subsequently use the wrong measures to describe the center and the variation. Review with students the different types of distributions.

Exercises 9 and 10
- Students may have difficulty creating the box-and-whisker plots. Remind them of the five-number summary of a data set.

Review Key Vocabulary

stem-and-leaf plot, *p. 436*   frequency table, *p. 441*   box-and-whisker plot, *p. 460*
stem, *p. 436*   frequency, *p. 441*   five-number summary, *p. 460*
leaf, *p. 436*   histogram, *p. 442*

Review Examples and Exercises

10.1 Stem-and-Leaf Plots *(pp. 434–439)*

Make a stem-and-leaf plot of the number of DVDs rented each day at a store.

Day	DVDs Rented
Sun	50
Mon	19
Tue	25
Wed	28
Thu	39
Fri	53
Sat	50

Step 1: Order the data. 19, 25, 28, 39, 50, 50, 53

Step 2: Choose the stems and the leaves. Because the data range from 19 to 53, use the *tens* digits for the stems and the *ones* digits for the leaves. Be sure to include the key.

Step 3: Write the stems to the *left* of the vertical line.

Step 4: Write the leaves for each stem to the *right* of the vertical line.

Order the stems vertically. The stem for data values less than 10 is 0.

Include stems without leaves.

Write the leaves horizontally.

DVDs Rented

Stem	Leaf
1	9
2	5 8
3	9
4	
5	0 0 3

Key: 2|5 = 25 DVDs

Exercises

Make a stem-and-leaf plot of the data.

1.

Hats Sold Each Day			
5	18	12	15
21	30	8	12
13	9	14	25

2.

Ages of Park Volunteers			
13	17	40	15
48	21	19	52
13	55	60	20

The stem-and-leaf plot shows the weights (in pounds) of yellowfin tuna caught during a fishing contest.

3. How many tuna weigh less than 90 pounds?

4. What is the median weight of the tuna?

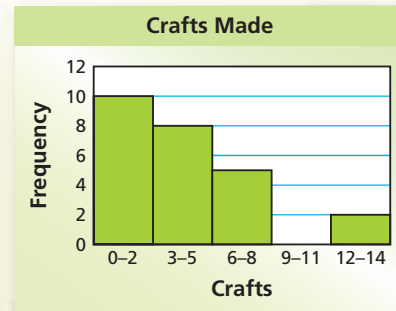| Stem	Leaf
7	6
8	0 2 5 7 9
9	5 6
10	2

Key: 8|5 = 85 pounds

Histograms *(pp. 440–447)*

The frequency table shows the number of
crafts each member of a craft club made for
a fundraiser. Display the data in a histogram.

Crafts	Frequency
0–2	10
3–5	8
6–8	5
9–11	0
12–14	2

Step 1: Draw and label
the axes.

Step 2: Draw a bar to
represent the
frequency of
each interval.
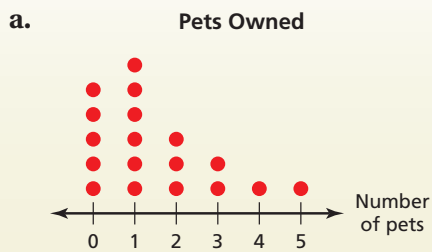
Crafts Made

Exercises

Display the data in a histogram.

5.

Heights of Gymnasts	
Heights (in.)	**Frequency**
50–54	1
55–59	8
60–64	5
65–69	2

6.

Minutes Studied	
Minutes	**Frequency**
0–19	5
20–39	9
40–59	12
60–79	3

10.3 **Shapes of Distributions** *(pp. 450–457)*

Describe the shape of each distribution.

a. Pets Owned

Most of the data are on the
left, and the tail extends to
the right.

∴ So, the distribution is
skewed right.

b. Biology Test Scores
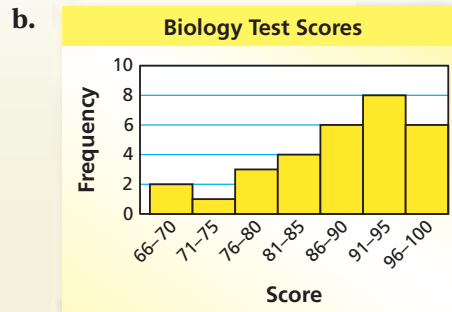
Most of the data are on the
right, and the tail extends to
the left.

∴ So, the distribution is
skewed left.

Review Game

Sampling and Predictions

Materials per group:
- pencils
- paper

Directions:

Divide the class into groups of 2 to 4 students. Each group comes up with a survey question. For instance, a group's survey question could be "how many pets does a student have?" Make sure each group has a different question.

Each group surveys 25% (or some other predetermined percentage) of the class and makes a display of the results, choosing the most appropriate data display. Groups then use their displays to predict the results for the entire class.

Each group then surveys the entire class and makes a display of these results, using the same type of data display chosen to analyze the sample. Groups then write a short paragraph in which they compare their actual results for their entire class to the predicted results for their entire class.

The larger the sample size, the more accurate the predictions may be. If possible, arrange to involve more students, in addition to the students in your class, in the surveys.
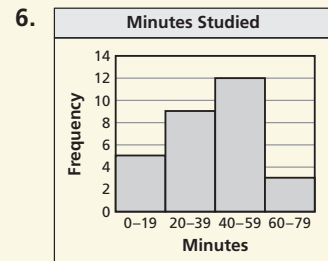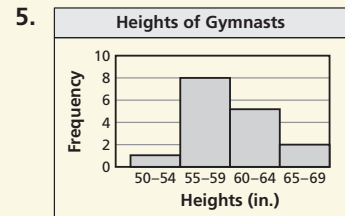
Who Wins?

The group whose actual results are closest to the predicted results wins.

Answers

5.

Heights of Gymnasts

6.

Minutes Studied

7. skewed left

8. mean, MAD; mean = 60, MAD ≈ 0.9

9.
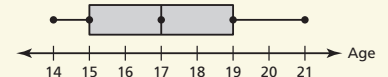
10.
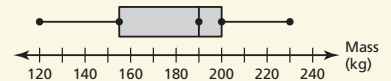
My Thoughts on the Chapter

What worked. . .

What did not work. . .

What I would do differently. . .

7. Describe the shape of the distribution.

Daily High Temperatures
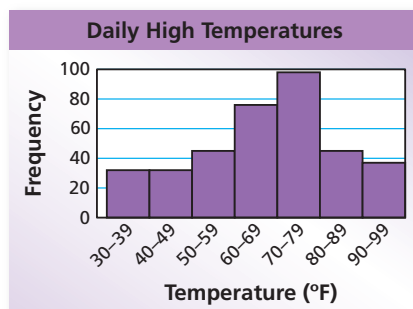
8. Choose the most appropriate measures to describe the center and the variation. Find the measures you chose.

Students' Heights
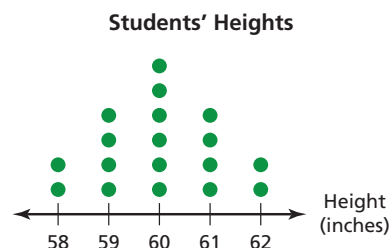
10.4 **Box-and-Whisker Plots** *(pp. 458–465)*

Make a box-and-whisker plot for the weights (in pounds) of pumpkins sold at a market.

16, 20, 14, 15, 12, 8, 8, 19, 14, 10, 8, 16

Step 1: Order the data. Find the median and the quartiles.

lower half   upper half

least value → 8 8 8 10 12 14 14 15 16 16 19 20 ← greatest value

first quartile, 9   median, 14   third quartile, 16

Step 2: Draw a number line that includes the least and the greatest values. Graph points above the number line that represent the five-number summary.

Step 3: Draw a box using the quartiles. Draw a line through the median. Draw whiskers from the box to the least and the greatest values.

Weight (pounds)

8 9 10 11 12 13 14 15 16 17 18 19 20
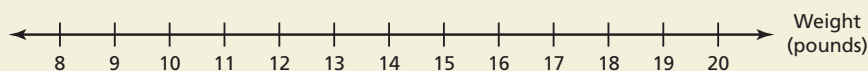
Exercises

Make a box-and-whisker plot for the data.

9. Ages of volunteers at a hospital: 14, 17, 20, 16, 17, 14, 21, 18

10. Masses (in kilograms) of lions: 120, 200, 180, 150, 200, 200, 230, 160

Make a stem-and-leaf plot of the data.

1.

Quiz Scores (%)			
96	88	80	72
80	94	92	100
76	80	68	90

2.

CDs Sold Each Day				
45	31	29	38	38
67	40	62	45	60
40	39	60	43	48

3. Find the mean, median, mode, range, and interquartile range of the data.

Cooking Time (minutes)

Stem	Leaf
3	5 8
4	0 1 8
5	0 4 4 4 5 9
6	0

Key: 4|1 = 41 minutes

4. Display the data in a histogram.

Television Watched Per Week	
Hours	**Frequency**
0–9	14
10–19	16
20–29	10
30–39	8

5. WATER The dot plot shows the number of glasses of water that the students in a class drink in one day.

   a. Describe the shape of the distribution.

   b. Choose the most appropriate measures to describe the center and the variation. Find the measures you chose.
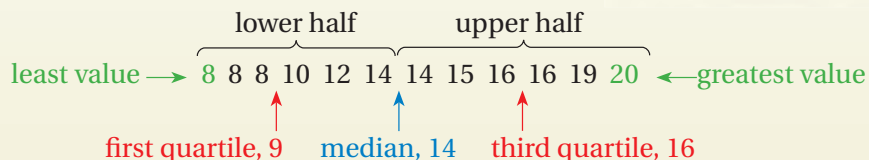
Water Consumed
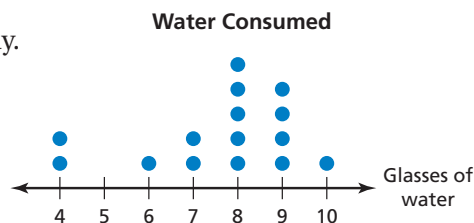
Make a box-and-whisker plot for the data.

6. Ages (in years) of dogs at a vet's office: 1, 3, 5, 11, 5, 7, 5, 9

7. Lengths (in inches) of fish in a pond: 12, 13, 7, 8, 14, 6, 13, 10

8. Hours practiced each week: 7, 6, 5, 4.5, 3.5, 7, 7.5, 2, 8, 7, 7.5, 6.5

9. CELL PHONES The double box-and-whisker plot compares the battery life (in hours) of two brands of cell phones.

   a. What is the range of the upper 75% of each brand?

   b. Which battery has a longer battery life? Explain.
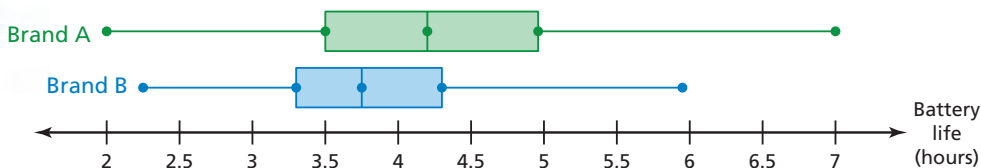
Test Item References

Chapter Test Questions	Section to Review	Common Core State Standards
1–3	10.1	6.SP.4
4	10.2	6.SP.2, 6.SP.4
5	10.3	6.SP.2, 6.SP.4, 6.SP.5d
6–9	10.4	6.SP.2, 6.SP.4, 6.SP.5c

Test-Taking Strategies

Remind students to quickly look over the entire test before they start so that they can budget their time. On this test, students are asked to display as well as analyze data. It can be difficult for students to determine how to begin. So, remind students to use the **Stop** and **Think** strategy before they answer a question.

Common Errors

- **Exercises 1 and 2** Students may forget to include the zeros in the ones place as leaves, or forget to include repeats of numbers. Remind them that *all* of the numbers in a data set should be represented in a stem-and-leaf plot.
- **Exercise 2** Students may forget to include 5 as a stem because it does not have leaves. Remind them that a stem-and-leaf plot shows how data are distributed, so it is important to include all of the stems in a range of data.
- **Exercise 3** Students may need to be reminded of the definitions of some of the terms so that they can answer the questions. Give an example of each term and how to find it using a stem-and-leaf plot.
- **Exercise 4** Students may have difficulty determining how to scale the vertical axis of the histogram. Suggest that they find the interval with the greatest frequency and scale the axis so that this interval will fit.
- **Exercise 5** Students may not recognize the distribution as skewed left and subsequently use the wrong measures to describe the center and the variation. Review with students the different types of distributions.
- **Exercises 6–8** Students may have difficulty creating the box-and-whisker plots. Remind them of the five-number summary of a data set.

Reteaching and Enrichment Strategies

If students need help. . .	If students got it. . .
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials *BigIdeasMath.com* Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Game Closet at *BigIdeasMath.com* Start Cumulative Assessment

Answers

1. Quiz Scores

Stem	Leaf
6	8
7	2 6
8	0 0 0 8
9	0 2 4 6
10	0

Key: 7│2 = 72%

2. CDs Sold Each Day

Stem	Leaf
2	9
3	1 8 8 9
4	0 0 3 5 5 8
5	
6	0 0 2 7

Key: 4│0 = 40 CDs

3. mean: 49; median: 52; mode: 54; range: 25; IQR: 14

4.

Television Watched Per Week (histogram)
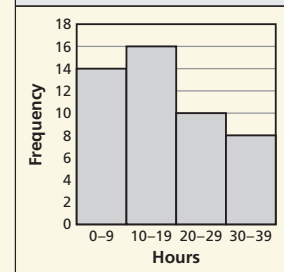
5. a. skewed left
   b. median, IQR; median = 8, IQR = 2

6.
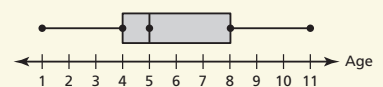
7–9. See Additional Answers.

Test-Taking Strategies

Available at *BigIdeasMath.com*

After Answering Easy Questions, Relax
Answer Easy Questions First
Estimate the Answer
Read All Choices before Answering
Read Question before Answering
Solve Directly or Eliminate Choices
Solve Problem before Looking at
    Choices
Use Intelligent Guessing
Work Backwards

About this Strategy

When taking a multiple choice test, be sure to read each question carefully and thoroughly. Look closely for words that change the meaning of the question, such as *not*, *never*, *all*, *every*, and *always*.

Answers

1. D
2. F
3. C

Item Analysis

1. **A.** The student misreads the plot, thinking that the median is the number at its right end.

   B. The student misreads the plot, thinking that 8 is the median.

   C. The student misreads the plot, thinking that the lower whisker represents 50% of the data.

   D. Correct answer

2. **F.** Correct answer

   G. The student does not correctly compare the unit cost in answer choice G, $0.325, with the unit cost in answer choice F, $0.30.

   H. The student does not correctly compare the unit cost in answer choice H, $0.375, with the unit cost in answer choice F, $0.30.

   I. The student does not correctly compare the unit cost in answer choice I, $0.33, with the unit cost in answer choice F, $0.30.

3. **A.** The student rewrites the divisor as $\frac{17}{3}$ but does not find the reciprocal.

   B. The student only takes the reciprocal of the fraction part of the divisor and rewrites it as $5\frac{3}{2}$, which the student then rewrites as $\frac{13}{2}$.

   C. Correct answer

   D. The student only takes the reciprocal of the fraction part of the divisor and ignores the whole number part.

1. Research scientists are measuring the number of days lettuce seeds take to germinate. In a study, 500 seeds were planted. Of these, 473 seeds germinated. The box-and-whisker plot summarizes the number of days it took the seeds to germinate. What can you conclude from the box-and-whisker plot?
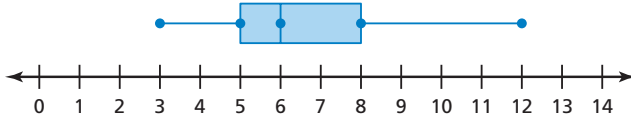
A. The median number of days for the seeds to germinate is 12.

B. 50% of the seeds took more than 8 days to germinate.

C. 50% of the seeds took less than 5 days to germinate.

D. The median number of days for the seeds to germinate was 6.

2. You are comparing the costs of buying bottles of water at the supermarket. Which of the following has the least cost per liter?

F. six 1-liter bottles for $1.80

G. one 2-liter bottle for $0.65

H. eight $\frac{1}{2}$-liter bottles for $1.50

I. twelve $\frac{1}{2}$-liter bottles for $1.98

3. What number belongs in the box to make the equation true?

$$3\frac{1}{2} \div 5\frac{2}{3} = \frac{7}{2} \times \boxed{}$$

A. $\frac{17}{3}$

B. $\frac{13}{2}$

C. $\frac{3}{17}$

D. $\frac{3}{2}$

4. What is the mean number of seats?

Seats in an SUV

F. 2.4 seats

G. 5 seats

H. 6.5 seats

I. 7 seats

5. On Wednesday, the town of Mims received 17 millimeters of rain. This was x millimeters more rain than the town received on Tuesday. Which expression represents the amount of rain, in millimeters, the town received on Tuesday?

A. $17x$

B. $17 - x$

C. $x + 17$

D. $x - 17$

6. One of the leaves is missing in the stem-and-leaf plot.

The median of the data set represented by the stem-and-leaf plot is 38. What is the value of the missing leaf?

Stem	Leaf
1	3 4
2	
3	4 5 7 7 7 ? 9
4	0 1 1 4
5	0 2 3

Key: 1|4 = 14

7. Which property is demonstrated by the equation below?

$$723 + (884 + 277) = 723 + (277 + 884)$$

F. Associative Property of Addition

G. Commutative Property of Addition

H. Distributive Property

I. Identity Property of Addition

Item Analysis (continued)

4. **F.** The student adds the frequencies and divides by 5 to find the mean.

   G. The student thinks mean is mode.

   H. Correct answer.

   I. The student chooses the middle row.

5. **A.** The student misinterprets "more than" as meaning multiplication.

   B. Correct answer

   C. The student misinterprets the inverse relationship in the problem, thinking that the town received more rain on Tuesday than on Wednesday.

   D. The student subtracts in the wrong order and writes an expression that represents 17 millimeters less rain than the difference of the rain amounts on Tuesday and Wednesday.

6. **Gridded Response**: Correct answer: 7

   Common Error: The student thinks that the missing leaf represents the ones digit of the actual median, getting an answer of 8.

7. **F.** The student does not recognize the equation as an example of the Commutative Property of Addition.

   G. Correct answer

   H. The student does not recognize the equation as an example of the Commutative Property of Addition.

   I. The student does not recognize the equation as an example of the Commutative Property of Addition.
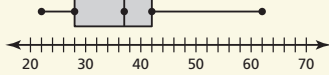
Answers

4. H

5. B

6. 7

7. G

8. C

9. 14 students

10. *Part A* 22, 24, 25, 28, 28, 30, 31, 37, 37, 39, 40, 40, 44, 51, 58, 62

Part B 37

Part C

Item Analysis (continued)

8. A. The student thinks mean is mode.

   B. The student finds the mean of the four scores (91) and adds 1 to 92 because 91 is 1 less than 92.

   C. Correct answer

   D. The student makes an arithmetic error finding the sum.

9. Gridded Response: Correct answer: 14 students

Common Error: The student only considers the bar immediately to the left of 10, getting an answer of 6.

10. 4 points The student demonstrates a thorough understanding of how to make a box-and-whisker plot. The data is ordered correctly. The median of 37, first quartile of 28, third quartile of 42, least value of 22, and greatest value of 62 are identified correctly and graphed accurately.

3 points The student demonstrates an essential but less than thorough understanding of how to make a box-and-whisker plot. The data is ordered correctly and the median is found correctly. But, there may be a small mistake made subsequently, e.g., when finding the third quartile or when trying to accurately match the plot to the number line given.

2 points The student demonstrates a partial understanding of how to make a box-and-whisker plot. The student's work and explanations demonstrate a lack of essential understanding. For example, the data is ordered correctly, but the quartiles are misidentified.

1 point The student demonstrates very limited understanding of making a box-and-whisker plot. E.g., the only key value that the student can accurately find is the median, and the data may not be correctly ordered.

0 points The student provides no response, a completely incorrect or incomprehensible response, or a response that demonstrates insufficient understanding of making a box-and-whisker plot.

8. A student took 5 tests this marking period and had a mean score of 92. Her scores on the first 4 tests were 90, 96, 86, and 92. What was her score on the fifth test?

  A. 92

  B. 93

  C. 96

  D. 98

9. At the end of the school year, your teacher counted up the number of absences for each student. The results are shown in the histogram below.

Student Absences

Frequency

Number of absences
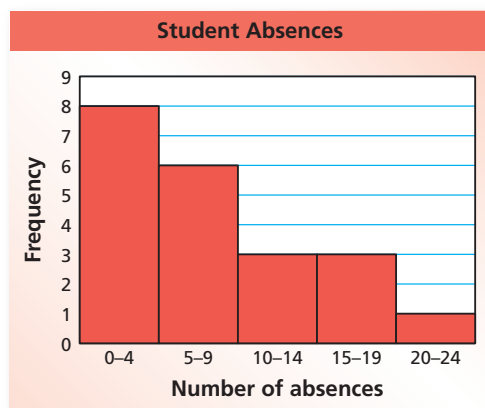
Based on the histogram, how many students had fewer than 10 absences?

10. The 16 members of a camera club have the ages listed below.

  40, 22, 24, 58, 30, 31, 37, 25, 62, 40, 39, 37, 28, 28, 51, 44

Part A Order the ages from least to greatest.

Part B Find the median of the ages.

Part C Make a box-and-whisker plot for the ages of the camera club members.